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ATOMIC SPECTRA

In two volumes

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ATOMIC SPECTRA AND THE VECTOR MODEL

BY

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*Sometime Scholar of Trinity College,
Cambridge*

'There is one thing I would be glad to ask you. When a mathematician engaged in investigating physical actions and results has arrived at his own conclusions, may they not be expressed in common language as fully, clearly and definitely as in mathematical formulae? If so, would it not be a great boon to such as we to express them so—translating them out of their hieroglyphics that we also might work upon them by experiment.'

Letter from MICHAEL FARADAY to CLERK MAXWELL

VOLUME II
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CONTENTS

VOLUME II. COMPLEX SPECTRA

<i>Chap.</i> XII	Displaced terms	<i>page</i> 1
XIII	Combination of several electrons	9
XIV	Short periods	21
XV	Long periods	45
XVI	Rare earths	75
XVII	Intensity relations	90
XVIII	Sum rules and (J) coupling	122
XIX	Series limit	158
XX	Hyperfine structure	166
XXI	Quadrupole radiation	215
XXII	Fluorescent crystals	228
<i>App</i> V	Key to references	247
VI	Bibliography	248
	A Books of reference	248
	B The spectra of the elements	248
	C The hyperfine structure of the elements	261
	<i>Subject index</i>	267
	<i>Author index</i>	275

LIST OF PLATES

5	Fluorescent spectrum of a chromium phosphore and a multiplet from the iron arc	<i>facing page 62</i>
6	Absorption spectra of two samarium compounds	86
7.	Hyperfine structure of various lines	168
8	Fluorescent spectra of samarium and a multiplet from the iron arc	240

CHAPTER XII

DISPLACED TERMS

1. The alkaline earths

Only about half of the bright lines of the calcium arc spectrum are accounted for by the simple terms described in a previous chapter. Most of the remaining lines, however, can be explained by the 'dashed' or 'displaced' terms introduced by Götze* in 1921, terms which Russell and Saunders† were later to attribute to the activity of two electrons (Fig. 12.2).

As an example of these terms consider the group of six bright lines occurring round 4300 Å.

Terms	$2^3P_2'$		$2^3P_1'$		$2^3P_0'$
2^3P_2			4280.363 (10) 23300.100 52.20		
2^3P_1	4307.718 (15) 23207.53	47.23	4298.080 (30) 23264.76 105.85	86.77	4283.008 (30) 23344.53 105.87
2^3P_0			4318.648 (15) 23148.01	86.75	4302.526 (60) 23275.06

Fig. 12.1. Wave lengths and wave numbers of the $2^3P' - 2^3P$ multiplet of calcium. The numbers in brackets are intensities.

The intervals 52.2 and 105.9 are recognised as the intervals of the well known 2^3P terms and these terms are therefore written in the left hand column. The intervals 47.2 and 86.8 do not occur in the earlier analysis, but Landé's interval rule suggests that an empirical ratio of 1.84 should arise from a 3P term, for the ideal interval ratios of 3P and 3D terms are 2 and 1.5 respectively.

If the combinations not observed are those forbidden by the J selection rule, then the empirical terms must have J values of 0, 1 and 2. Of the nine possible combinations, two are then forbidden because they have $\Delta J = 2$, and a third because the electron

* Götze, *AP*, 1921, **68** 285. A key to letters used in referring to periodicals is given in Appendix iv.

† Russell and Saunders, *AJ*, 1925, **51** 38. The authors write the terms 1^3P and 1^1P instead of 2^3P and 2^1P used here.

would have to jump from $J=0$ to $J=0$. The intersystem line $^3P_0 \rightarrow ^1S_0$ is noticeably absent in the mercury spectrum

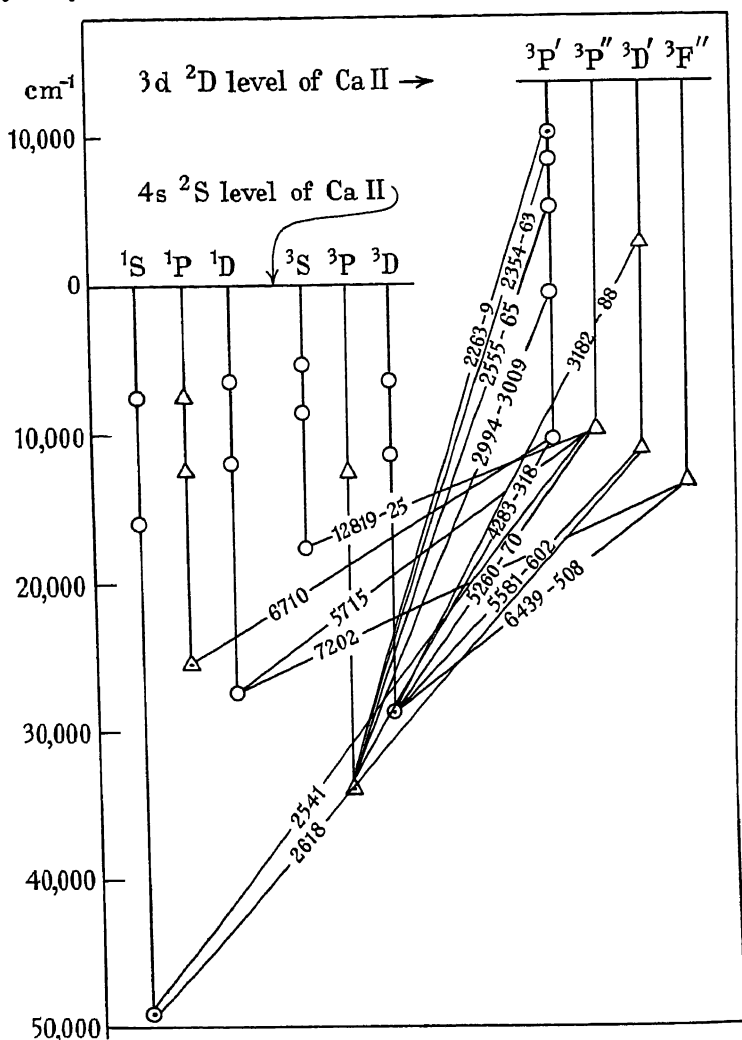


Fig 12.2 Level diagram of calcium, extended to show the displaced terms. Even terms are shown by a circle, odd terms by a triangle

A critic might suggest that the above argument really only determines the values of J , but the Zeeman effect has also been examined and this, connecting as it does L , J and S , serves to fix L , when J and S are known. Every indication therefore

suggests that the empirical terms should be written 4P , or, since they are not of the usual 3P series, $^3P'$.

So far no use has been made of the series laws, but five terms of the $^3P'$ series have been observed, and if these are fitted to a Rydberg formula, they indicate that the limit of the series is some 13961 cm^{-1} above the common limit of the earlier known series of the calcium atom. Indeed, the $4^3P'$, $5^3P'$ and $6^3P'$ terms are all above the limit, and contain energies of 0.68, 1.03 and 1.24 volts in excess of that required to ionise the atom. This surprisingly large energy content seems to be open to only one explanation. In the levels of this displaced series two electrons are excited, and when the atom returns to one of the normal states both electrons jump and both contribute energy to a single quantum of radiation.

This deduction may be still further refined. The second deepest term 3D of the calcium ion is 13711 cm^{-1} or 1.72 volts above the basic 3S term, and this is not very different from the amount by which the limit of the P' series is raised above the normal limit. If then the two optical electrons are assumed to have orbital moments l_1, l_2 , l_1 must be assigned the value 2.

The difficulty of explaining why the P' terms combine with normal P terms is then surmounted by the hypothesis that l_1 and l_2 combine vectorially to give a resultant L . L combines in the usual way with S itself the sum of the s_1, s_2 of the individual electrons, to give J , but the quantum transitions allowed depend on l_1, l_2 individually and only indirectly on L . The reasons already given for writing the unknown term $^4P'$ first determine J , and then require L to be 1, but l_2 is still unassigned. If the old rule $\Delta L = \pm 1$ is to be written now $\Delta L = 0, 1$, then the fact that the $^3P'$ term combines with a P term shows that l_2 is 0 or 2, and the former is ruled out, because if $l_1 = l_2 = L$ it is inconsistent with the previous allotment of L and l_1 .

Apply the same arguments to a so called $^3D'$ term found by Russell and Saunders, which has the J properties of a D term but combines with $^3D, ^3S$ and 1D . Clearly the J properties indicate $L = 2$, while the combining terms show that $l_2 = 1$, and as the $^3D'$ series has the same displaced limit as the $^3P'$ series, l_1 is 2.

When two electrons are excited, the atom as a whole can no longer be said to have a chief quantum number, though each electron has. Thus in the normal state of calcium, two electrons lie outside the closed shell of argon, and these occupy $4s$ orbits, so that the ground state is $4s^2\ ^1S_0$. When one of these moves to a $4p$ orbit the configuration becomes $4s4p\ ^3P$, this is the 2^3P term of the normal spectrum. In the lowest $3^3P'$ term both electrons have moved to $3d$ orbits, so that the configuration is $3d^2\ ^3P'$ *

Often the chief quantum number does not need to be stressed, and then $3d^2\ ^3P'$ is abbreviated to $d^2\ ^3P'$, or if the two d electrons have different chief quantum numbers, as in $3d\ 4d\ ^3P'$, then we write $d\ d\ ^3P'$. Again as there is only one $3d\ nd\ ^3P$ series, the omission of the dash introduces no ambiguity, but in fact it is often retained as it gives a key to the transitions permitted.

What are these? Or in other words, what is the selection rule governing the combination of displaced terms? Heisenberg stated, on the basis of the quantum mechanics, that when two electrons jump simultaneously one is bound by the condition $\Delta l_1 = \pm 1$ and the second by $\Delta l_2 = 0$ or ± 2 . Of this rule the transitions from the displaced to the normal terms of the alkaline earths offer a first example.

A very important consequence of this selection rule is that when an atom emits or absorbs a quantum of radiation the sum of the orbital quantum numbers of the individual electrons Σl must change by an odd number, if the sum was even it must become odd, or if it was odd it must become even. Consequently all possible terms may be divided into two groups, an even group, written in accord with the suggestion of a number of physicists† S, P, D, \dots , and an odd group, written $S^\circ, P^\circ, D^\circ, \dots$. A term of the one group can then combine only with a member of the other group.

Another notation adopted for several years by many spectroscopists differed somewhat from this. The even group of terms

* The $3d^2\ ^3P'$ term is the $3^3P'$ term of this description, though the $3^3P'$, $4^3P'$ and $5^3P'$ terms arise from $3d\ nd$ configurations, the $2^3P'$ term arises from $4p^2$, it was wrongly assigned by Russell and Saunders.

† Russell, Shenstone and Turner, *PR*, 1929, 33 900

was written S, P', D, F', . and the odd series S', P, D', F, This notation was convenient because it was in agreement with the notation of the earlier chapters of this book and could be extended. But to-day it seems more important to be able to distinguish odd and even terms at sight, for the group to which an empirical term belongs can easily be determined, and thus the dashed notation does not facilitate

In simple spectra only one electron is excited, and for it the S, D terms have L even, while the P, F terms have L odd, and so to be quite accurate should be written P° , F° . Again, the displaced P', D' terms of Ca I have the sum $(l_1 + l_2)$ equal to 4 and 3 respectively, so that they are d^2^3P and $p\ d^3D^\circ$

Having dealt with two displaced series at considerable length, the other series found by Russell and Saunders, and written $^3P''$, $^3D''$ and $^3F''$, can be considered more briefly. All the series have the same elevated limit and consequently in every state one electron must occupy a d orbit. The double dash signified that these terms, unlike $^3P'$ and $^3D'$, do not combine with the normal terms of the same letter. Thus $^3P''$, though it has the J properties of a P term, combines with 3S , 3D , 1S , 1D , so that $L=1$ and $l_2=1$. $^3P''$ is therefore identical with $d\ p\ ^3P^\circ$ and may be written in the latter form. Similarly, $^3D''$ combines with 3P , 1P and therefore must be identical with d^2^3D . Combinations of $^3F''$ are known only with 3D and 1D , for combinations with 3S lie too far on the infra-red and 3G is wholly unknown, but the energy of the term argues strongly in favour of $d\ p\ ^3F$ and against $d\ f\ ^3F$, though the evidence here adduced allows the latter.

Russell and Saunders in their analysis accounted for practically all the known lines of the three alkaline earths, though they hesitated to assign electronic states to all the terms which were suggested by the term differences, and whose energies were thus known. Wentzel,* however, has filled even this small gap, and the analysis of the visible and near ultra-violet region of calcium may be considered complete.

2 Beryllium and magnesium

In the spectra of beryllium and magnesium, and in the iso-

* Wentzel, *ZP*, 1925, 34 730

electronic spark spectra B II to O V and Al II to Cl VI, there occur a striking group of five lines, the separations are nearly equal, and four of the five lines are of equal intensity, the central line of the group, however, is noticeably stronger, and in some spectra it may be resolved into two close components

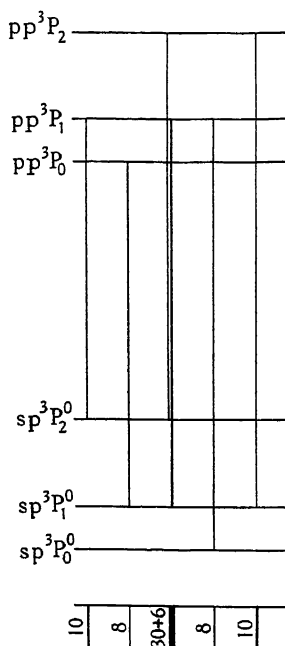


Fig 12 3 Structure of a displaced triplet of magnesium, this arises in a $3p^2\ ^3P \rightarrow 3s\ 3p\ ^3P^0$ transition

In 1925 Bowen and Millikan showed that these arise by the combination of the $s\ p^3P^0$ term with a 3P term, having so nearly the same intervals that the lines $^3P_2 \rightarrow ^3P_2^0$ and $^3P_1 \rightarrow ^3P_1^0$ often coincide. According to Sommerfeld's intensity rule, this central line is due to chief lines having $\Delta J = \Delta L$, while the other four lines are satellites of the first order. A more accurate theory would predict the intensities shown in Fig 12 3. Both are in good agreement with observation.

The wave-lengths and wave-numbers of the central line in spectra isoelectronic with magnesium are shown in Fig 12 4 *

* Bowen and Millikan, *PR*, 1925, 26 150

The differences show how accurately linear is the progression of frequency with atomic number, and this means that the group of lines follows the irregular doublet law, which they can do only if the transitions take place between levels having the same chief quantum number

This argument applies as well to Be as to Mg, though figures are here adduced only for the latter. But in Be the jump ends in a $2s\ 2p$ orbit, so that it must start in a $2p^2$ orbit, no other possibility exists, for there are no $2d$ orbits. The close similarity of

Spectrum	λ	ν	Diff
Mg I	2780 64	35962 96	20727 9
Al II	1763 95	56690 9	20295 5
Si III	1298 93	76986 4	20050 7
P IV	1030 53	97037 1	19948 5
S V	854 81	116985 6	19942 6
Cl VI	730 31	136928 2	

Fig 12.4 The central line of displaced triplets in spectra isoelectronic with magnesium

the lines in Mg and Be suggests that in Mg too the 3P term arises from the $3p^2$ configuration, and this hypothesis is confirmed, when the frequency of the arc line $p^2\ ^3P \rightarrow s\ p\ ^3P^\circ$ is compared with the spark line $p\ ^2P \rightarrow s\ ^2S$, the former is 35960 cm^{-1} and the latter 35760 cm^{-1} , so that both must surely arise in the same electron transition $2p \rightarrow 2s$

A comparison of these displaced terms with those found in the alkaline earths shows that in Be and Mg the lines are produced by one electron jumping, whereas in Ca, Sr and Ba two electrons jump simultaneously. But in both alike the displaced terms arise from the second lowest term of the spark spectrum

3. Zinc, cadmium and mercury

Displaced terms have also been identified in Cd by Ruark* and in Zn and Hg by Sawyer†, but instead of the six lines to be expected four only have been found, these being interpreted as

* Ruark, *JOSA*, 1925, 11 199

† Sawyer, *JOSA*, 1926, 13 431

the combination of the low $s\ p^3P^{\circ}_{0,1,2}$ terms with the displaced terms $p^2\ ^3P_{0,1}$ (Fig. 12 5)

Whether the $p^2\ ^3P_2$ term is really missing or whether, as Foote, Takamine and Chenault* have suggested, it coincides with $p^2\ ^3P_1$,

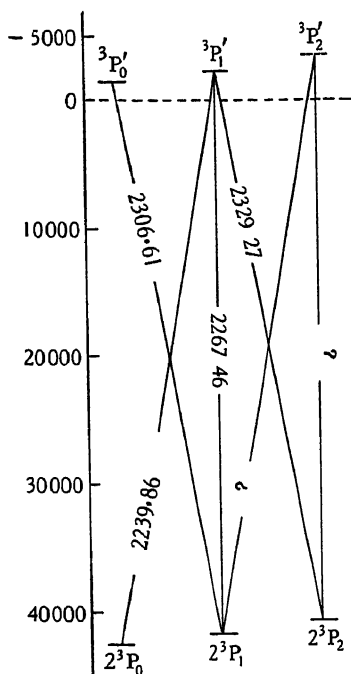


Fig 12 5 Level diagram showing a PP' triplet of cadmium, this arises as $5p^2\ ^3P \rightarrow 5s\ 5p\ ^3P^{\circ}$

so that in Cd the 2329 Å and 2268 Å lines are really narrow doublets, remains a question still undecided. Certainly the intensities are somewhat irregular and the 2329 Å line is much stronger than the other lines

BIBLIOGRAPHY

These displaced spectra are fully discussed by Grottrian, *Graphische Darstellung der Spektren*, 1928, 1 188-210

* Foote, Takamine and Chenault, *PR*, 1925, 26 174

CHAPTER XIII

COMBINATION OF SEVERAL ELECTRONS

1 Combination of unlike electrons

The coupling of orbital and spin vectors, found to explain the displaced terms of the alkaline earths, gives also an adequate account of the terms of many more complex spectra

In this coupling the orbital and spin vectors of the electrons, commonly written l_1, l_2 , s_1, s_2 , first combine to resultants **L** and **S** respectively, and then **L** and **S** combine to form **J**. The problem therefore is to determine all the values of *L* and *S* to which a given electron configuration can give rise, to solve this

Electron added	State of ion			
	S	P	D	F
s	S	P	D	F
p	P	SPD	PDF	DFG
d	D	PDF	SPDFG	PDFGH

Fig 13 1 Atomic states resulting from the addition of an electron to an ion

problem, consider the orbital and spin vectors separately. When a p and a d electron combine the *l* vectors are 1 and 2, so that their resultant must be 1, 2 or 3, values which correspond to P, D or F terms, further, each electron has a spin vector of $\frac{1}{2}$, so that the atomic spin vector due to two electrons may be 0 or 1, values which correspond to multiplicities of 1 and 3. Accordingly, a p and a d electron combine to form six terms $^1P, ^1D, ^1F, ^3P, ^3D$ and 3F .

A similar train of reasoning will determine the terms produced when an electron combines with an ion, for if an s electron is to be added to an ion in the 4F state, the orbital vectors to be combined are 0 and 3, so that the resultant must be 3, while the *s* vectors are $\frac{1}{2}$ and $1\frac{1}{2}$, so that the resultant may be 1 or 2, accordingly the terms produced are 3F and 5F . Fig 13 1 summarises the results obtained in this way, for it shows the terms arising when an s, p

or d electron is added to an ion in the S, P, D or F state, the multiplicity being omitted since it always increases and decreases by unity

When three or more electrons have to be combined together, the work must proceed by steps, two electrons being combined and then the third added to each of the terms produced by the two. Thus an spd configuration would give rise to the nine terms $^2(\text{PDF})$, $^3(\text{PDF})$ and $^4(\text{PDF})$

In the above discussion the word 'term' is used to denote a multiplet term, but in analysis each component of the multiplet appears as a separate empirical level or 'term'. Theory thus states precisely how many levels will arise from a given configuration,

M_L m_l	-2	-1	0	1	2
1	-1	0	1	2	3
0	-2	-1	0	1	2
-1	-3	-2	-1	0	1
Term of atom	F	D	P		

Fig 13.2 Addition of a p electron to an ion in the D state, showing how P, D and F states result

and what their J values will be, and it is an outstanding achievement of the theory that the empirical terms occur in just the number predicted and have the J values assigned. Sometimes, as might be expected, some of the levels are missing, but they are nearly always levels which would produce only lines of low intensity. In the great mass of spectra so far analysed, there is only a single level which seems to be adequately substantiated and is surplus to the theory, it is found in Pd I.

Though the above method of combining two electrons is the simplest, another method due to Russell* is not without interest. Instead of combining the l_1 and l_2 vectors of ion and electron, combine the m_{l_1} and m_{l_2} values, all combinations are permitted,

* Russell, *PR*, 1927, 29 782.

so that a matrix of $(2l_1 + 1)(2l_2 + 1)$ values results, and this can be split into a number of sequences by Breit's alleys, exactly as in the theory of the Paschen-Back effect Fig 13 2 shows the calculation when a p electron is added to an ion in a D state

The same method can be used to combine the m_{s_1} and m_{s_2} of ion and electron to obtain the multiplicity of the atomic terms

2 Combination of equivalent electrons

Two electrons having the same values of n and l are said to be 'equivalent'

When two equivalent electrons are to be combined, the above argument gives all the terms which might be produced, but it gives more than are produced, for the combination is restricted by Pauli's exclusion principle Of this complication the simplest example is the combination of two electrons in the alkaline earths, thus in magnesium the two valency electrons must have $n \geq 3$, since the first two groups are full, and so the lowest terms may be expected to arise from the combination of two 3s electrons s electrons, however, always have $m_l = 0$, so that if Pauli's exclusion principle is to be satisfied and the two electrons are not to occupy the same orbit, m_s must be $+\frac{1}{2}$ for one and $-\frac{1}{2}$ for the other Consequently $M_s = 0$ and $S = 0$, showing that the lowest term is 1S_0 and that a 3S term can appear only when the two s electrons producing it have different chief quantum numbers Thus Pauli's exclusion principle clearly explains why the 1^3S_1 term is missing in all the alkaline earth spectra and the 3S_1 sequence begins with a 2^3S_1 term

l_1	l_2	m_{l_1}	m_{l_2}	m_{s_1}	m_{s_2}	M_L	M_S	Term
0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	1S

Fig 13 3 Combination of two equivalent s electrons, showing that only a 1S term results

Fig 13 3 shows the argument summarised in a table A similar table for two equivalent p electrons appears in Fig 13 4

In this table all possible combinations of m_{l_1} and m_{l_2} are included, but the two electrons are to be considered interchange-

able, so that $m_{l_1}=1$ and $m_{l_2}=0$ is identical with $m_{l_1}=0$ and $m_{l_2}=1$. Further, the derivation of the terms, from the sums M_L and M_S needs to be explained. A 1D term has $L=2$ and $S=0$, so that it will give rise to a series of values of M_L from 2 to -2 all

l_1	l_2	m_{l_1}	m_{l_2}	m_{s_1}	m_{s_2}	M_L	M_S	Terms
1	1	1	1	$\frac{1}{2}$	$-\frac{1}{2}$	2	0	1D
			0	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	1	1 0 0 -1	
			-1	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	0	1 0 0 -1	3P
		0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	
			-1	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	-1	1 0 0 -1	
		-1	-1	$\frac{1}{2}$	$-\frac{1}{2}$	-2	0	

Fig 13 4 Combination of two equivalent p electrons, showing how the resulting terms are calculated

having $M_S=0$. To work from M_L and M_S to L and S therefore, start with the highest value of M_L available, namely 2 in the above figure, and cross out both a series of values of M_L from 2 to -2 and the corresponding values of M_S , namely 0. When these values have been deleted, the table of Fig 13 4 reduces to that shown in Fig. 13 5

M_L	M_S	Terms
1	1 0 -1	3P
0	1 0 -1	
0	0	1S
-1	1 0 -1	

Fig 13 5 A step in the elucidation of the previous figure, the M_L and M_S values which remain when the 1D term has been removed

Now repeat the operation. The highest value of M_L is 1, and the greatest corresponding values of M_S also 1, so that the next term to be written in the margin must have $L=1$ and $S=1$, and the values to be deleted are $M_L=1, 0, -1$ and $M_S=1, 0, -1$. Afterwards there remains only $M_L=0$ when $M_S=0$, representing a 1S term.

The calculation of terms arising from three equivalent p electrons introduces no new principles, the work, summarised in Fig 13 6, shows that the terms to be expected are 2D , 2P and 4S .

The combination of four equivalent p electrons is worked out

in Fig 13 7. The terms arising, be it noted, are those arising from two equivalent p electrons, and indeed this could have been predicted For six p electrons form a closed group, in which every orbit permitted by the exclusion principle is occupied, and consequently $M_L = M_S = 0$ Therefore when there are four p electrons, the orbits left unoccupied will be precisely those

l_1	l_2	l_3	m_{l_1}	m_{l_2}	m_{l_3}	m_{s_1}	m_{s_2}	m_{s_3}	M_L	M_S	Terms
1	1	1	1	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\pm\frac{1}{2}$	2	$\pm\frac{1}{2}$	2D
					-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\pm\frac{1}{2}$	1	$\pm\frac{1}{2}$	
			0	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\pm\frac{1}{2}$	1	$\pm\frac{1}{2}$	2P
					-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\pm\frac{1}{2}$	-1	$\pm\frac{1}{2}$	
			-1	-1	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\pm\frac{1}{2}$	-1	$\pm\frac{1}{2}$	
					0	$\frac{1}{2}$	$-\frac{1}{2}$	$\pm\frac{1}{2}$	-2	$\pm\frac{1}{2}$	
			1	0	-1	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	0	$\pm\frac{1}{2}$	4S

Fig 13 6 Combination of three equivalent p electrons

l_1	l_2	l_3	l_4	m_{l_1}	m_{l_2}	m_{l_3}	m_{l_4}	m_{s_1}	m_{s_2}	m_{s_3}	m_{s_4}	M_L	M_S	Terms
1	1	1	1	1	1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	2	0	1D 3P 1S
						0	-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	1	1 0 0 -1	
						-1	-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	
				0	0	1	-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	0	1 0 0 -1	
						-1	-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-2	0	
				-1	-1	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	-1	1 0 0 -1	

Fig 13 7 Combination of four equivalent p electrons

l_1	l_2	l_3	l_4	l_5	l_6	m_{l_1}	m_{l_2}	m_{l_3}	m_{l_4}	m_{l_5}	m_{l_6}	m_{s_1}	m_{s_2}	m_{s_3}	m_{s_4}	m_{s_5}	m_{s_6}	M_L	M_S	Term
1	1	1	1	1	1	1	1	0	0	-1	-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	1S

Fig 13 8. Combination of six equivalent p electrons

occupied by two electrons, and the values of M_L and M_S will be unaltered in magnitude The change in sign will not affect the values of L and S or consequently the terms. For the same reasons five p electrons give rise to a 2P term just as one electron does

The terms resulting from various numbers of p electrons are collected together in Fig 13 9, while below are similar tables for d and f electrons Of these, the first two were given by Hund and

the last by Gibbs, Wilber and White * To save space terms of the same multiplicity are sometimes bracketed together, thus $4P\ 4F$ is abbreviated to $4(PF)$, and the number of times a particular term occurs is indicated by a figure written directly below the letter

Number of p electrons	Terms resulting			
0 or 6	$1S$			
1 or 5		$3P$		
2 or 4	$1S\ 1D$		$3P$	
3		$3P\ 3D$		$4S$

Fig 13 9 Terms resulting from the combination of equivalent p electrons

No of d electrons	Terms			
0 or 10	$1S$			
1 or 9	$3D$			
2 or 8	$1(SDG)$	$3(PF)$		
3 or 7	$1(PDFGH)$	$4(PF)$		
4 or 6	$1(SDFGI)$	$3(PDFGH)$	$5D$	
5	$1(SPDFGHI)$	$4(PDFG)$	$6S$	

Fig 13 10 Terms resulting from the combination of equivalent d electrons

No of f electrons	Terms				
0 or 14	$1S$				
1 or 13	$3F$				
2 or 12	$1(SDGI)$	$3(PFH)$			
3 or 11	$1(PDFGHIKL)$	$4(SDFGI)$			
4 or 10	$1(SDFGHIKLN)$	$3(PDFGHIKLM)$	$5(SDFGI)$		
5 or 9	$1(PDFGHIKLMNO)$	$4(SPDFGHIKLM)$	$6(PFH)$		
6 or 8	$1(SPDFGHIKLMNQ)$	$3(PDFGHIKLMNO)$	$5(SPDFGHIKL)$	$7F$	
7	$1(SPDFGHIKLMNOQ)$	$4(SPDFGHIKLMN)$	$6(PDFGHI)$	$8S$	

Fig 13 11 Terms resulting from the combination of equivalent f electrons

3 Deep terms of the short periods

If the terms of low energy arise from electrons of low energy then the deep terms of the short periods must be derived from

* Gibbs, Wilber and White, *P.R.*, 1927, 29 790

equivalent s and p electrons. Moreover, they can be read off from Fig 13 9, for a closed shell contributes nothing to L or S , so that the terms which arise from an $s^2 p^n$ configuration are precisely those which would arise from p^n .

The elucidation of these deep terms is important, because a few terms often give the key to the whole spectrum, and transitions ending in deep terms yield the brightest lines.

Outer electrons			Ground term	Other deep terms	Some spectra in which these terms have been found
Total	s	p			
1	1	—	2S	—	Na I, Cs I, Ca II, Al III
2	2	—	1S	—	Mg I, Ca I, Al II, C III
	2	1	2P	—	Al I, Tl I, C II, N III
4	2	2	3P	1D 1S	Cr I, Pb I, N II, O III
5	2	3	4S	2D 2P	N I, Sb I, O II, Cl III
6	2	4	3P	1D 1S	O I, Se I, Cl II, A III
7	2	5	2P	—	F I, Cl I, A II, Na III
8	2	6	1S	—	Ne I, Kr I, Na II, Ca III

Fig 13 12 Low terms of the short periods

4 Two energy rules

In the spectra of the short periods the deep terms are few in number, and so those predicted by theory are easily matched with those found empirically. But of the higher terms very many will arise from a single configuration, and to know which of these normally lie low and which high is a great help to the accurate labelling of the empirical terms. This problem was first studied by Hund, who solved it in two empirical energy rules.

The first rule states that those terms lie deepest in which the electronic spin vectors are parallel to one another, and in which therefore S is a maximum. Thus in the alkaline earths, the triplet term arising from any electronic configuration lies deeper than the corresponding singlet term, while in the short periods Fig 13 12 shows that of the terms arising from the simplest electron configuration that of highest multiplicity is the ground term.

The second rule adds that of terms which have the same multiplicity, those lie deepest in which the electronic orbital vectors are also parallel, and in which therefore L is a maximum.

Accordingly, in carbon 1D lies lower than 1S , and in nitrogen 2D lies lower than 2P

Thus of two terms arising from a set of equivalent electrons, that which lies furthest to the right in Figs 13 9–13 11 lies deepest in the spectrum. These rules never fail to predict the ground term of a spectrum correctly, but among higher configurations many exceptions occur

5. Inverted terms

When a group of electrons is more than half full the deep terms arising from it are inverted. Thus the 2P ground term of AlI is erect, but the 2P ground term of ClI is inverted, the 3P ground term of silicon is erect, but that of oxygen is inverted

This fact is best examined in the light of a third energy rule, which asserts that the deepest components of a term arise when the orbital and spin vectors of the electrons are anti-parallel. When there are no restrictions this rule ensures that in the lowest component J will be a minimum and the terms will therefore be erect, but when the group is more than half full the restrictions imposed by the exclusion principle interfere

The ground terms of the elements of a short period may be built up graphically by adding one electron at a time. The first electron, having its l and s vectors anti-parallel, as shown in Fig 13 13, gives rise to a $^2P_{\frac{1}{2}}$ term. On adding the second electron the first energy rule makes s_2 parallel to s_1 , and would make l_2 parallel to l_1 if the exclusion principle did not interfere, but equivalent electrons may not have the same values of m_l if they already have the same values of m_s , so if m_{l_1} was -1 , m_{l_2} must be 0 or 1, and of these 0 will give the lower component since it will make the vectors more nearly anti-parallel, accordingly, in Fig 13 13 the l_2 vector is drawn horizontal

The addition of a third electron emphasises no new principle, but the fourth, if it is to produce the empirical term, must set its orbital vector l anti-parallel not to the resultant S but to the electronic vector s_4 . The lowest component then appears as 3P_2 , and as the other components of the ground term are 3P_1 and 3P_0 the term appears inverted.

This theory, which can of course be applied also to d and f electrons (Fig 13 14), explains well enough why the ground terms

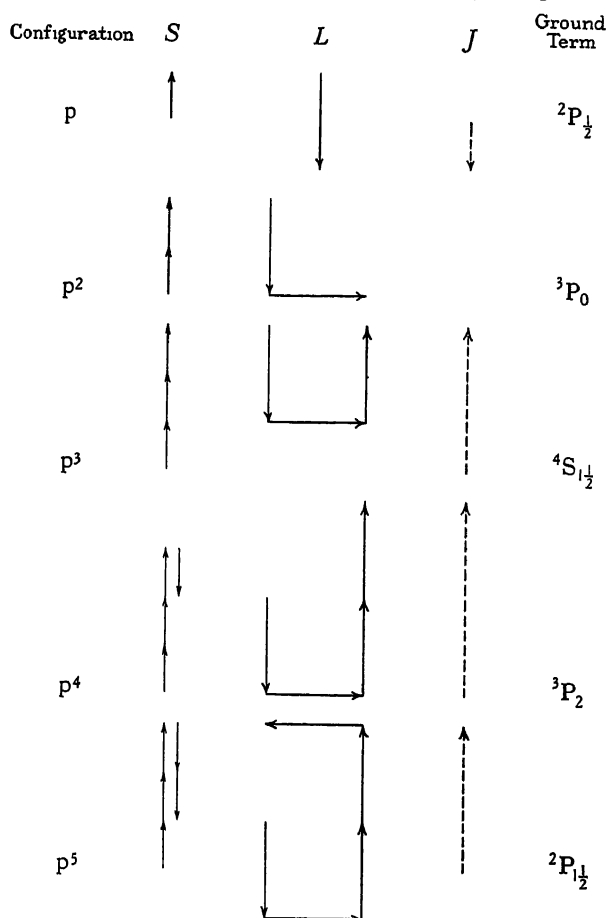


Fig 13 13 Vector diagram showing how the ground terms of a p shell may be derived with the help of the Pauli exclusion principle

of certain spectra are inverted, but it would not necessarily lead one to expect that many of the higher terms would also be inverted as in fact they are. A theory developed by Goudsmit,* serves however to bring out this point.

* Goudsmit, *PR*, 1928, 31 946 Ruark and Urey, *Atoms, molecules and quanta*, 1930, 332

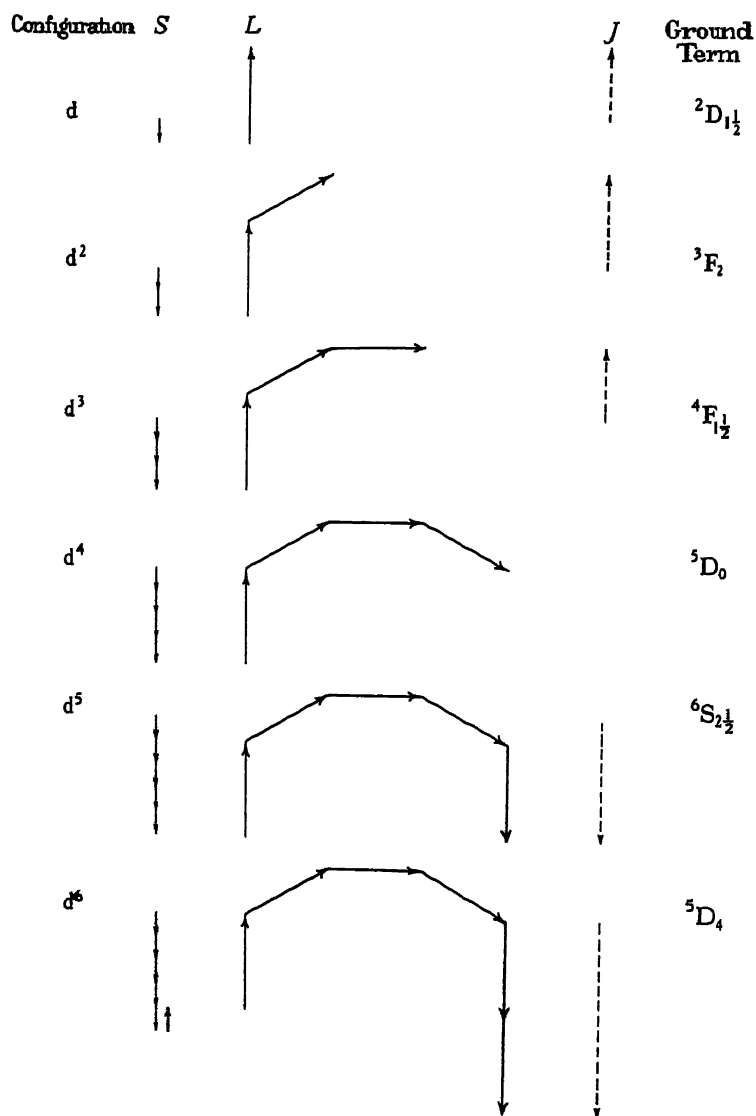


Fig 13 14 Vector diagram showing how the ground terms of a d shell may be derived.

Previously, the wave-number of a component of a multiplet term has been written

$$E/hc = \nu_G + \Gamma, \quad (4\ 1)$$

where ν_G is the wave-number of the centroid. Let us assume that when several electrons are active the atomic displacement Γ is itself the sum of electronic displacements $\gamma_1, \gamma_2, \dots, \gamma_n$.

In the vector model the energy of an electron having orbital and spin vectors $\mathbf{l}_1, \mathbf{s}_1$ is assumed proportional to them and to the cosine of the angle between them, so that

$$\gamma_1 = a_1 l_1 s_1 \cos(\mathbf{l}_1 \mathbf{s}_1) \quad (13\ 1)$$

Now \mathbf{l}_1 precesses round \mathbf{L} and \mathbf{s}_1 round \mathbf{S} , so that

$$\gamma_1 = a_1 l_1 \cos(\mathbf{l}_1 \mathbf{L}) s_1 \cos(\mathbf{s}_1 \mathbf{S}) \cos(\mathbf{L} \mathbf{S}),$$

or summing for all the electrons,

$$\Gamma = \Sigma \gamma_1 = \cos(\mathbf{L} \mathbf{S}) \Sigma a_1 l_1 \cos(\mathbf{l}_1 \mathbf{L}) s_1 \cos(\mathbf{s}_1 \mathbf{S}) \quad (13\ 2)$$

In order to advance beyond this equation, Slater* had to make a further postulate, the electrons are to be divided into two groups according as their spin vectors are parallel or anti-parallel to the atomic spin vector, the resultants L' and L'' of the corresponding orbital vectors are then 'action variables', which interpreted in terms of the vector model means that they must both be integral

Consider now the 3P ground term of an element, such as oxygen, with four p electrons. Three of these will have \mathbf{s} parallel to \mathbf{S} , while one will be anti-parallel. Clearly then $L'' = 1$, while L' might assume any value less than 3 did the exclusion principle not intervene, requiring that if all three electrons have the same value of m_s then they can none of them have the same value of m_l , consequently, when S' is $1\frac{1}{2}$, the only possible value of L' is zero. Equation (13 1) now reduces to

$$\Gamma = as \cos(\mathbf{L} \mathbf{S}) (L' - L''), \quad (13\ 3)$$

for if all electrons are equivalent $a_1 = a_2 = \dots = a_n$ and all may be written a .

This formula accounts for the empirical facts. So long as a shell is less than half full Γ will be positive and the terms erect, but

* Slater, *PR*, 1926, 28 291

when there are present three p, five d, or seven f electrons, $L' = L'' = 0$, and the ground term will be an S term of multiplicity one greater than the number of electrons present. When a shell is more than half full L' , but not L'' , is zero, so that Γ is negative and the terms are inverted.

A very pretty confirmation of this theory is found in nitrogen, most of whose terms arise from the $2p^2 3P$ configuration of NII and are consequently erect, two deep terms, however, a $4P$ and a $2P$, are inverted, and it seems clear that these have the electronic structure $2s \ 2p^4$ *.

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* Compton and Boyce, *PR*, 1929, **33** 147

CHAPTER XIV

ELEMENTS OF THE SHORT PERIODS

1 Elements to be considered

The seventh from last element of each period has a 1S ground term, and this may be taken to mean that the configuration of each of these elements consists only of complete shells, the last six elements of each period shown in Fig 14.1, are therefore

Column	III	IV	V	VI	VII	VIII
Configuration	p	p^2	p^3	p^4	p^5	p^6
Elements	B Al Ga In Tl	C Si Ge Sn Pb	N P As Sb Bi	O S Se Te Po	F Cl Br I	Ne Ar Kr Xe Rn
Ground term	2P	3P	4S	3P	2P	1S
Prominent multiplicities	2	1, 3	2, 4	3, 5	2, 4	1, 3

Fig 14.1 Elements arising from configurations of p electrons

formed by the entrance of six electrons into a new shell, the sixth electron completing the shell and forming the inert gas. As this shell must consist of p electrons in the first short period, it presumably consists of p electrons in the other periods.

If this is true, then the partly filled shell of p electrons should produce the ground term of each spectrum, the terms being 2P , 3P , 4S , 3P , 2P , 1S in successive columns, and in fact these are the terms found. Of the higher terms the great majority are formed by adding an electron to the ground term of the ion. Consider, for example, the spectrum of carbon in which there are only two p electrons, the ground term of C II is 2P , and accordingly C I should consist of singlets and triplets, the terms produced being 1P and 3P , $^1(SPD)$ and $^3(SPD)$, or $^1(PDF)$ and $^3(PDF)$, according as the second electron moves in an s, p or d orbit. And similarly in column VI the ground term of the spark spectrum is 4S , so that the arc spectrum should consist chiefly of the resulting

triplets and quintets, though some less prominent terms may arise by the addition of an electron to the metastable states $p^3\ ^2P$ and $p^3\ ^2D$ of the ion

2 Irregularities and their cause

In the early days of spectrum analysis, spectroscopists were able to order only those spectra in which series are prominent, for a series was the only regularity which they had recognised. To-day, the spectroscopist who has analysed a spectrum into a complex of terms can name those terms only if they exhibit some regularity, and the regularities on which he chiefly relies are the selection rules, which serve to determine J , the magnetic splitting factor, and the intensity and interval rules. The three last are linked together, for when a spectrum fails to obey one rule, it often fails to obey all three, and in terms of the vector model this is taken to mean that the coupling of the vectors ceases to be that postulated by Russell and Saunders.

What then are the influences which make a term irregular? Briefly they may be summarised as three, first is an increase in atomic number, carbon is more regular than lead, and neon than krypton, secondly, the column of the periodic table is significant, for the spectra of the elements on the left-hand side are more regular than those on the right, beryllium is more regular than nitrogen and nitrogen than neon, while third stands the height or energy of a term, for the higher a term lies the less likely is it to obey the simple rules, and particularly the interval rule. These influences are here stated as empirical rules, their explanation will be attempted only in a later chapter.

3. The earth metals, s^2p configuration*

The spark spectra of the earth metals, like the spark spectra of the alkalis, have a 1S ground term, so that the arc spectra should consist of doublets, and in fact a system of doublets has been found and analysed into principal, sharp, diffuse and fundamental series. Where the terms have been resolved they have been shown

* Grotrian, *Graphische Darstellung der Spektren*, 1928, 1 122f and 2 80, 96

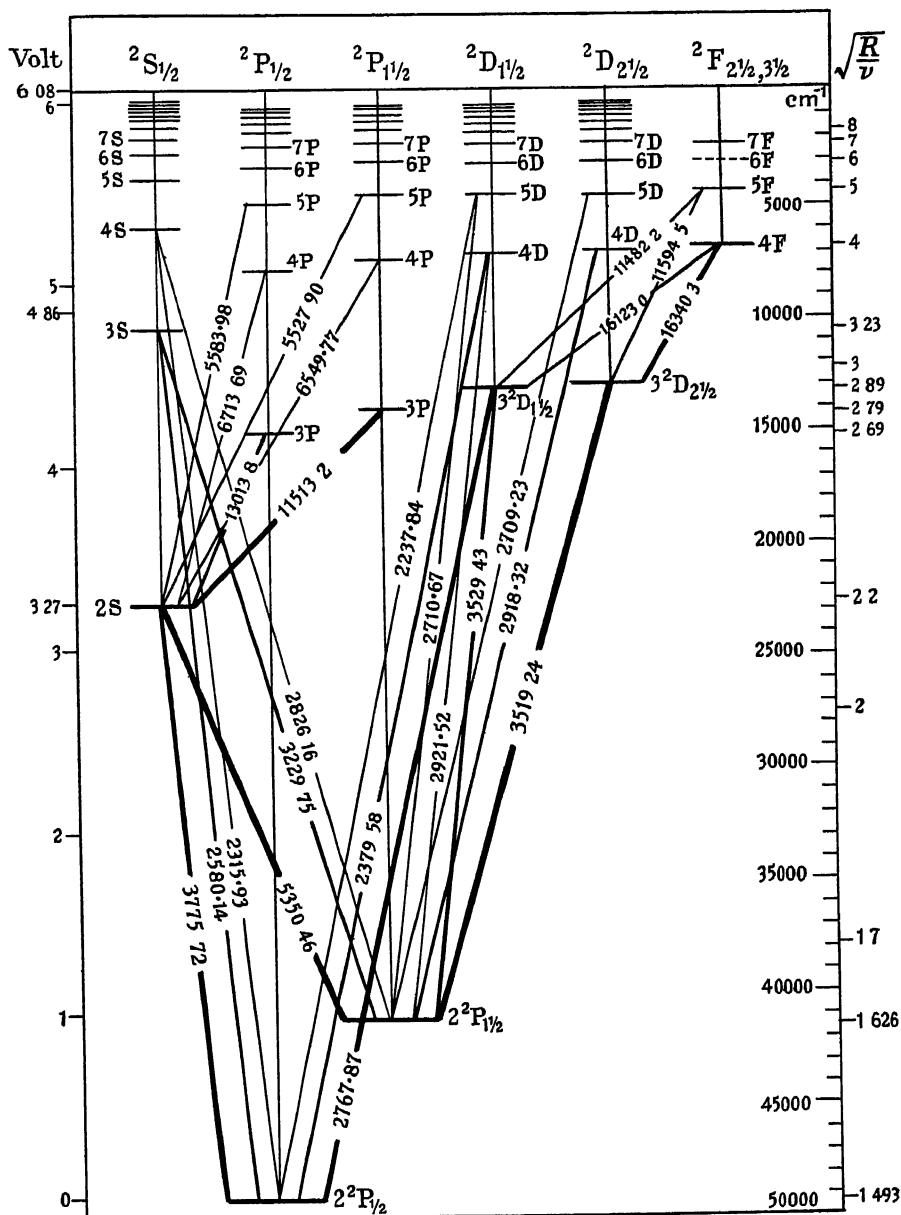


Fig 14.2 Level diagram of thallium (After Grotrian, *Graphische Darstellung der Spektren*)

to be usually erect, in Pb II, however, the 2D and 2F terms are inverted

The ground term is clearly a P term, for the principal series extends into the infra-red and has a limit of much greater wavelength than the limits of the sharp and diffuse series (Fig 14 2), in this the doublets of the earth metals may be compared to the triplets of the alkaline earths. Moreover, the absorption spectra confirm the ground term, for when the metallic vapour is maintained at a low temperature, and illuminated with white light, the sharp and diffuse series are absorbed, but not the principal series

Al II		Al I		
$l_1 l_2$	Term	l_3	Terms	$l_1 l_2 l_3$
$3s^2$	1S	3p	$^2P^\circ$	$s^2 p$
		4s	2S	$s^2 s$
		4p	$^2P^\circ$	$s^2 p$
		3d	2D	$s^2 d$
		4f	$^2F^\circ$	$s^2 f$
$3s 3p$	$^3P^\circ$	3p	4P $^2(SP D)$	sp^2
		4s	$^4P^\circ$ $^2P^\circ$	$sp s$
		4p	$^4(SP D)$ $^2(SP D)$	$sp p$
		3d	$^4(P D F)^\circ$ $^2(P D F)^\circ$	$sp d$
$3p^2$	3P	3p	$^4S^\circ$ $^2P^\circ$ $^2D^\circ$	p^3

Fig 14 3 Terms predicted in column III

Above the ground term of Al I lies first an S and then a D term, and theory makes it clear that these arise by the addition of 4s and 3d electrons to the 1S ground term of Al II. Terms of the arc spectrum may, it is true, arise from spark terms other than 1S , but they do not belong to the simple doublet system, in practice these terms do not produce bright lines in the elements of column III A, but they appear lying quite low in all the spark and higher spark spectra, which are isoelectronic with this column, such as Si II and Sb III. The structure of these terms is shown in Fig 14 3

4. Column IV, $s^2 p^2$ configuration

In the fourth column Hund's scheme predicts that the $s^2 p^2$ configuration will produce five low terms $^3P_{0,1,2}$, 1D_2 and 1S_0 in

that order proceeding from the ground term up, and empirically in carbon, silicon, germanium, tin and lead five low terms have been found

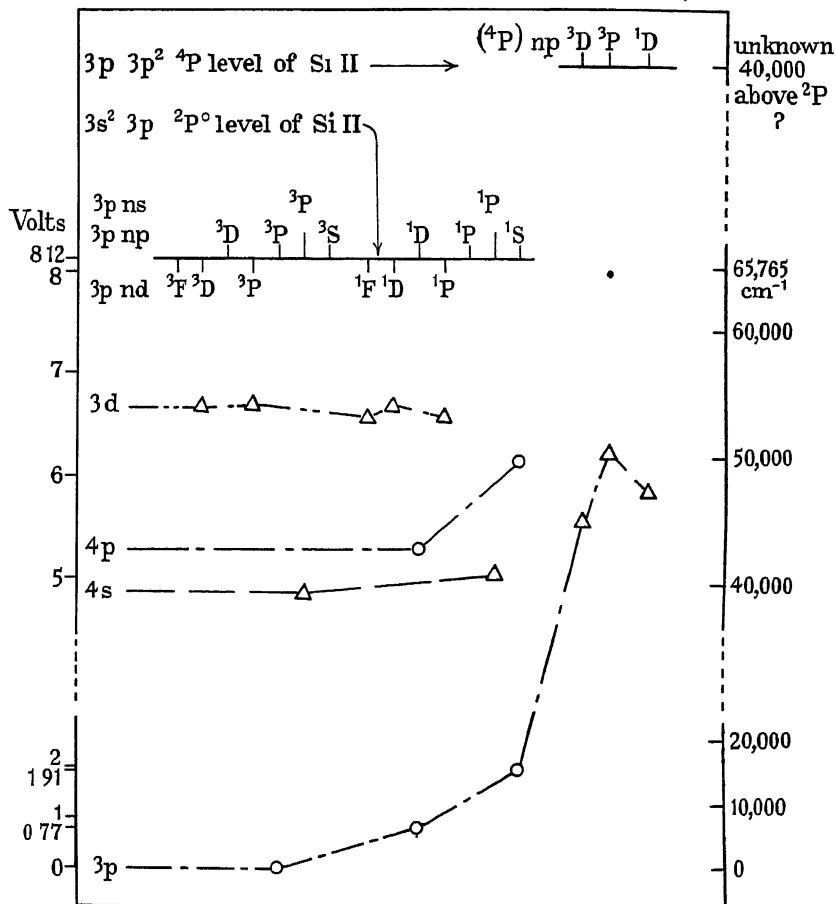


Fig 14.4 Level diagram of silicon In this and the following diagrams, even terms are shown by a circle and odd terms by a triangle Terms produced by the same electron are joined by a broken line, when the electron is an s electron the line is drawn —, when a p electron — — and when a d electron — . . .

In the region extending from the red end of the spectrum down to 2000 Å the carbon arc produces only one line, 2478 Å, but when a trace of carbon dioxide is added to helium at a pressure

of 20 to 30 mm many lines occur which have been attributed to the neutral carbon atom. Analysis of the spectrum shows that the reason why the visible lines are difficult to excite is that they arise between high terms, the low terms produce lines in the ultra-violet, and these occur quite readily both in the arc and in vacuum tubes containing carbon compounds

C II		C I				
$l_1 l_2 l_3$	Term	l_4	Terms			$l_1 l_4$
$2s^2 2p$	$^2P^\circ$	2p	3P	1D	1S	$s^2 p^2$
		3s	$^3P^\circ$		$^1P^\circ$	$s^2 p s$
		3p	$^3(SPD)$		$^1(SP D)$	$s^2 p p$
		3d	$^3(PDF)^\circ$		$^1(PDF)^\circ$	$s^2 p d$
$2s 2p^2$	4P $^2(SP D)$	2p	$^5S^\circ$	$^3(SP D)^\circ$	$^1(SD)^\circ$	sp^3
		3s	5P		3P	$sp^2 s$
	4P	3p	$^5(SP D)^\circ$		$^3(SP D)^\circ$	$sp^2 p$
		3d	$^5(PDF)$		$^3(PDF)$	$sp^2 d$
	2D	3s	3D		1D	$sp^2 s$
		3p	$^3(PDF)^\circ$		$^1(PDF)^\circ$	$sp^2 p$
	2P	3s	3P		1P	$sp^2 s$
		3p	$^3(SP D)^\circ$		$^1(SP D)^\circ$	$sp^2 p$
	2S	3s	3S		1S	$sp^2 s$
		3p	$^3P^\circ$		$^1P^\circ$	$sp^2 p$

Fig 14 5 Terms predicted in column IV

The C I spectrum arises from the 2P ground term of C II and has been analysed quite regularly into a number of triplets and singlets, in contrast Pb I has been analysed only into series. To understand this better, consider the changes which occur in the $s^2 p^2$ and $s^2 p s$ configurations with increasing atomic number. Thus in C I the 3P ground term has nearly the ideal interval ratio of the Russell-Saunders coupling, while the extreme interval is small compared with the distance which separates the 3P from the 1D term. From this ideal the low terms of Si I, Ge I and Sn I fall further and further away until in lead Δ^3P_{01} is greater than Δ^3P_{12} , while the extreme triplet interval Δ^3P_{02} is as great as the separation of the 3P_2 and 1D_2 terms (Fig 14 6)

Again examine the intervals of the s^2p s configuration which produces a 3P and 1P term, in carbon the extreme interval of the 3P term is only 60 cm^{-1} , while 1500 cm^{-1} separates 3P_2 from 1P_1 .

	C	Si	Ge	Sn	Pb
$^3P_0-^3P_1$	14.8	77.1	557.1	1692	7,817
$^3P_1-^3P_2$	27.5	146.1	852.8	1736	2,831
$^3P_2-^1D_2$	10,150	6075.5	5716.0	5185	10,818
$^1D_2-^1S_0$	11,452	9095.4	9241.2	8550	8,000
$\Delta^3P_{12}/\Delta^3P_{01}$	1.86	1.90	1.53	1.02	0.36

Fig 14.6 Column IV Intervals in the ground configuration, s^2p^2

but in tin the triplet and singlet are transformed into two diads, having intervals of only 300 and 600 cm^{-1} , but separated from one another by 4000 cm^{-1} (Fig 14.7)

	C	Si	Ge	Sn	Pb
$^3P_0-^3P_1$	20	77	251	274	327
$^3P_1-^3P_2$	40	195	1415	3714	—
$^3P_2-^1P_1$	1589	1037	903	628	—
Δ^3P_{02}	60	272	1666	3988	—
Δ^3P_{12} of ion	64	287	1768	4253	14,071

Fig 14.7 Column IV Intervals in the lowest s^2p s configuration

These changes run parallel to the increasing interval of the 3P ground term of the spark spectrum, and the relation between the two may be explained in terms of the vector model, provided that the separation of two terms is assumed roughly proportional to the strength of the coupling producing it. This assumption is indeed implicit in the description of regular terms already given, for if, to fix ideas, the two p electrons of Cr are considered, the triplet and singlet terms are found widely separated, a fact which the vector model translates to read that the spin coupling ($\mathbf{s}_1\mathbf{s}_2$) is strong, or again the two singlet terms are also widely separated, and so the orbital coupling ($\mathbf{l}_1\mathbf{l}_2$) must also be supposed strong, but the intervals of the 3P term are small and so the coupling responsible, namely (\mathbf{LS}), must be weak.

Consider then the addition of an electron to an ion specified by ${}_iL$, ${}_iS$ and ${}_iJ$, if the coupling is Russell-Saunders, the coupling of

1L and 1S must be split when the electron is added, and only after 1L has combined with l and 1S with s may their resultants L and S combine together to form an atomic resultant J . In symbols, the coupling then appears as

$$\{({}^1Ll)({}^1Ss)\} = \{LS\} = J$$

And this theory may be expected to yield a satisfactory account of the facts, so long as the $({}^1L, {}^1S)$ coupling is weak, that is, so long as the interval of the spark ground term is small, a condition satisfied in carbon but not in tin. In contrast, when $({}^1L, {}^1S)$ is strong, we obtain a more satisfactory account of the term scheme by considering the ionic and electronic vectors permanently coupled to 1J and j respectively, then the splitting is produced by the strong coupling $({}^1L, {}^1S)$ and the weak coupling $({}^1Jj)$, so that in Sn I the s^2p s configuration produces not a triplet and a singlet but two diads

This new coupling is commonly referred to as (jj) coupling, being written symbolically as

$$({}^1L, {}^1S)(ls) = ({}^1Jj) = J$$

It is characterised by strong inter-system lines, and in fact these are easily discovered in lead and can be traced greatly weakened in intensity back to silicon, but there is still a little doubt about the three lines found by Jog* in Cl I , for if the lines found are those sought, the singlet term values assigned by Fowler must be changed by no less than 667 cm^{-1}

5 Column V, s^2p^3 configuration

Hund's theory predicts that in the elements of column V the ground term should be ${}^4S_{1\frac{1}{2}}$ and that above this should lie four metastable states ${}^2D_{2\frac{1}{2}, 1\frac{1}{2}}$ and ${}^2P_{1\frac{1}{2}, \frac{1}{2}}$. And empirically these five terms have been found in the arc spectra of nitrogen, phosphorus, arsenic, antimony and bismuth

Of these spectra only N I and P I have been fully analysed. N I was not easily observed, for many of the lines lie inconveniently far out in the infra-red or ultra-violet, moreover, when excited by an arc strong bands appear which are apt to mask the

* Jog, *N*, 1929, 123 318

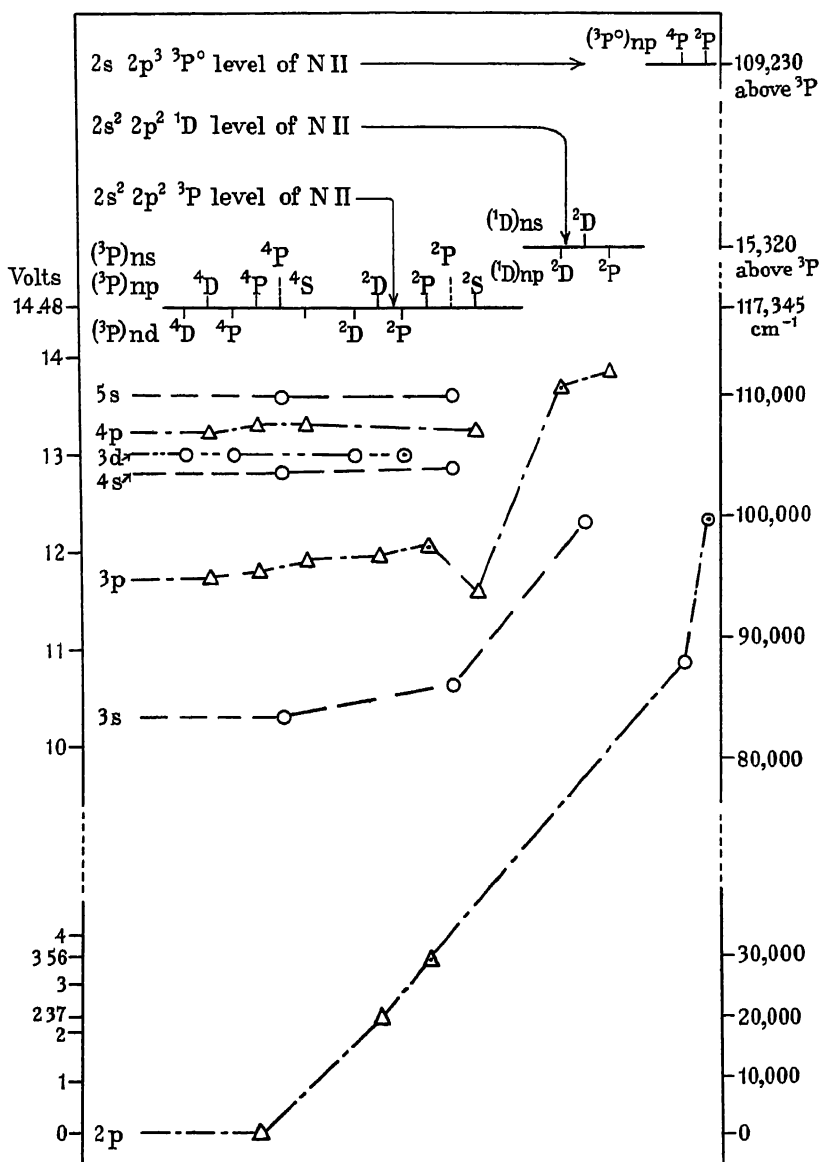


Fig 148 Level diagram of nitrogen

ultra-violet lines, while more vigorous excitation only brings out the spark lines. Compton and Boyce have, however, excited the ultra-violet lines with single electron impacts, a convenient technique because work on neon and argon shows that a single impact will not disturb more than two electrons, no lines of the

N II				N I				
l_1	l_2	l_4	Term	l_5	Term		l_1	l_5
$2s^2\ 2p^2$			3P	$2p$	$^4S^\circ$	$^2D^\circ\ ^2P^\circ$	$s^2\ p^3$	
				ns	4P	2P	$s^2\ p^2\ s$	
				np	$^4(SP D)^\circ$	$^2(SP D)^\circ$	$s^2\ p^2\ p$	
			nd	$^4(PDF)$	$^2(PDF)$	$s^2\ p^2\ d$		
			1D	ns	2D		$s^2\ p^2\ s$	
				np	$^2(PDF)^\circ$		$s^2\ p^2\ p$	
			1S	ns	2S		$s^2\ p^2\ s$	
				np	$^2P^\circ$		$s^2\ p^2\ p$	
$2s\ 2p^3$			$^3P^\circ$	$2p$	4P	$^2(SP D)$	sp^4	

Fig 14 9 Terms predicted in column V

second spark spectrum being produced even when the potential is more than adequate

In P I many lines lie even further out in the infra-red than in N I, so far out in fact that the usual sensitiser neo-cyanine would not serve, recently, however, at the Bureau of Standards a new sensitiser has extended the range which can be photographed and a thorough analysis of P I has been made

In the three remaining elements few terms outside the ground configuration have been named, though the energies of many are known, the intervals of these five terms, however, still show them as a singlet and two doublets in Sb I (Fig 14 10), while in Bi I the

Interval	N	P	As	Sb	Bi
$^4S_{1/2} - ^3D_{1/2}$	19,202	11,366	10,591	8512	11,418
$^3D_{1/2} - ^3D_{3/2}$	-8	15	322	1342	4,019
$^3D_{3/2} - ^3P_{1/2}$	9,606	7,346	7,272	6541	6,223
$^3P_{1/2} - ^3P_{3/2}$	0	25	461	2069	11,505

Fig 14 10 Column V Intervals of the ground configuration, $s^2 p^3$. Note the change from LS coupling in nitrogen to jj coupling in bismuth

sequence of J values is still unchanged, so that the terms may be named by analogy, in an atom of high atomic weight (LS) coupling is not to be expected

Of the higher terms little need be said, in N I the lowest are 2P and 4P from the $2p^2\ 3s$ configuration, while very little higher lies the $2s\ 2p^4\ ^4P$ term. The energy relations are shown in Fig 14 8

Most of the term series of N I approach the 3P ground term of N II as limit, but experiment shows that different series approach different components of the limit, for if two components of a multiplet tend to the same limit the interval decreases very rapidly, roughly in fact as $1/n^{*3}$, but in N I the intervals of some terms decrease only slowly or actually increase, this being true in particular of the $2p^2\ ns\ ^4P$ and $2p^2\ nd\ ^2D$ series

N I					N II	
n	3	4	5	6		
$^4P_{\frac{1}{2}}-^4P_{\frac{3}{2}}$	33 8	50 0	44 3	44 7	$^3P_0-^3P_1$	50
$^4P_{\frac{3}{2}}-^4P_{\frac{5}{2}}$	46 7	68 7	70 1	72 0	$^3P_1-^3P_2$	84
$^4P_{\frac{5}{2}}-^4P_{\frac{7}{2}}$	80 5	118 7	114 4	116 7	$^3P_0-^3P_2$	134

Fig 14 11 Intervals of the $2p^2\ ns\ ^4P$ series in N I

With large values of n the intervals of the $2p^2\ ns\ ^4P$ series may reasonably be supposed to approach the values of 50 and 84 cm^{-1} , which are the intervals of the ground term of N II. And if this is true then $^4P_{\frac{1}{2}}$ must tend to the lowest or 3P_0 limit, while $^4P_{\frac{7}{2}}$ must approach the highest or 3P_2 limit. In Fig 14 12 these facts are shown graphically, the separations being measured horizontally away from the middle terms, $^4P_{\frac{3}{2}}$ in N I and 3P_1 in N II. Unfortunately in most series only two or three terms are known, and though the intervals of these may be roughly constant, they do not establish the limits beyond doubt as the above series do. Even in the $2p^2\ nd\ ^2D$ series, of which five terms are known, one would hesitate to say whether the intervals tend to a limiting value of 50 or 84 cm^{-1} , though evidently they do not tend to zero (Fig 14 13)

The passage of the different components of a multiplet term to

different limits often leads to wide departures from the interval rule, and these, like the departures due to increasing atomic number, may be described with the vector model. For consider

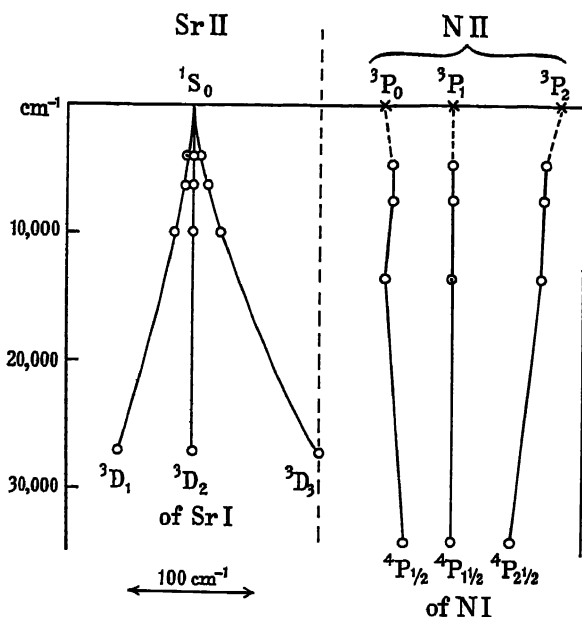


Fig 14.12 Intervals of the $5s\ nd\ ^3D$ series of Sr I contrasted with those of the $2p^2\ ns\ ^4P$ series of N I, in the first the different series converge to the same limit, in the second they converge to three different limits

n	3	4	5	6	7
$^3D_{1\frac{1}{2}} - ^3D_{3\frac{1}{2}}$	23 5	22 2	18 3	58 3	42 6

Fig 14.13 Intervals of the $2p^2\ nd\ ^3D$ series of N I

the addition of an electron defined by n, l, s to an ion defined by ${}_iL, {}_iS$, then in the low terms the couplings $({}_iLl)$ and $({}_iSs)$ are strong, while (LS) and (ls) suffice only to produce the splitting of the multiplet term into its components, but (Li) and (Ss) decrease as the chief quantum number of the electron increases and actually vanish when n tends to infinity, while (LS) remains constant, so that when n is sufficiently large the latter becomes

the more important and the splitting becomes that of the ionic ground term

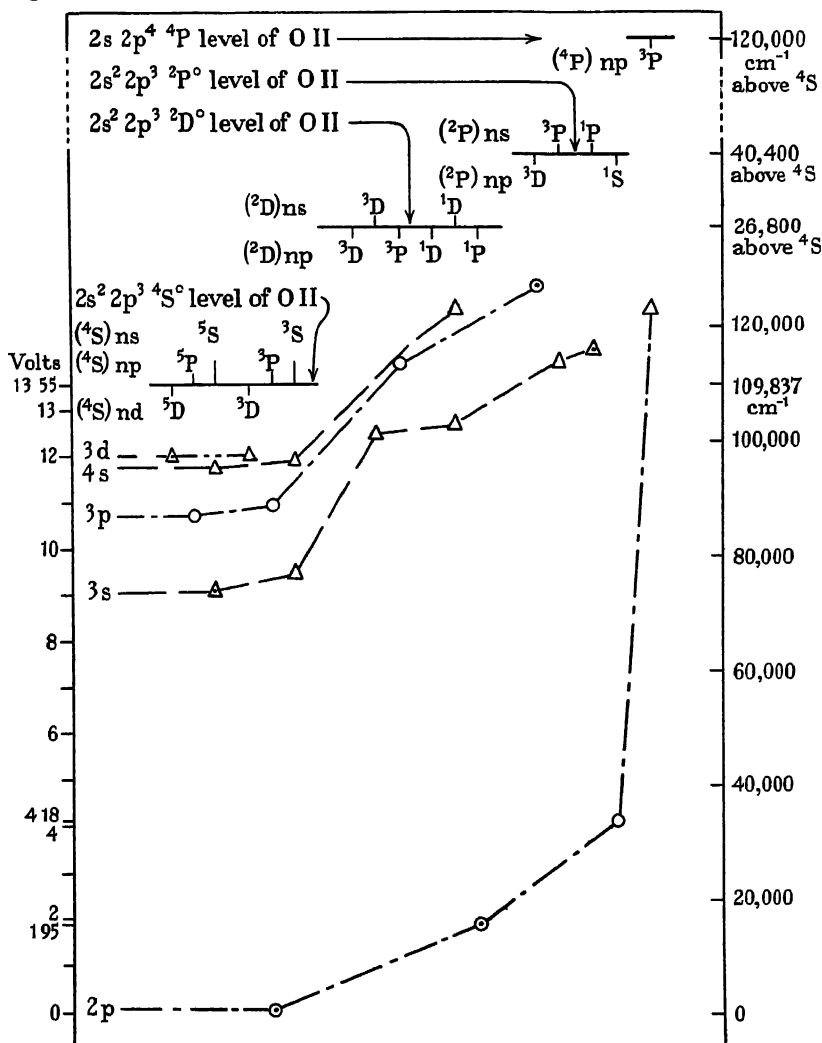


Fig 14 14 Level diagram of oxygen.

6 Column VI, s^2p^4 configuration

Hund's theory predicts in column VI five low terms, $^3P_{2,1,0}$, 1D_2 and 1S_0 , and empirically the 3P term has been identified in

the spectra of oxygen, sulphur, selenium and tellurium, while the two singlet terms are still missing only in sulphur (Fig 14 14). The most notable changes in the energies of these low terms with increasing atomic number are first the closing together of the five terms, and secondly the movement of the 3P_1 term from its normal position one-third of the distance between 3P_0 and 3P_2 up to and past 3P_0 , so that in tellurium the 3P term is partially inverted (Fig 14 15)

Interval	O	S	Se	Te
$^3P_2-^3P_1$	158	398	1,989	4,751
$^3P_1-^3P_0$	67	174	545	-44
$^3P_0-^1D_2$	15,641	—	7,042	5,852
$^1D_2-^1S_0$	17,925	—	13,794	12,640

Fig 14 15 Column VI Intervals of the ground configuration, $s^2 p^4$

Beyond these low terms very little is known of Te I, while in Se I only a few quintet terms have been identified, but a list of the higher terms predicted for all spectra of column VI is given in Fig 14 16

O II			O I			
$l_1 l_2 \quad l_5$	Term	l_6	Terms		$l_1 \quad l_6$	
$2s^2 \ 2p^3$	$^4S^\circ$	$2p$	$^5S^\circ$	3P	$^1D^1S$	$s^2 p^4$
		ns	5P	$^3S^\circ$		$s^2 p^3 \ s$
		np	5P	3P		$s^2 p^3 \ p$
		nd	$^5D^\circ$	$^3D^\circ$		$s^2 p^3 \ d$
	$^2D^\circ$	ns	$^3D^\circ$	$^1D^\circ$		$s^2 p^3 \ s$
		np	$^3(PDF)^\circ$	$^1(PDF)^\circ$		$s^2 p^3 \ p$
		nd	$^3(SPDFG)^\circ$	$^1(SPDFG)^\circ$		$s^2 p^3 \ d$
	$^2P^\circ$	ns	$^3P^\circ$	$^1P^\circ$		$s^2 p^3 \ s$
		np	$^3(SPD)^\circ$	$^1(SPD)^\circ$		$s^2 p^3 \ p$
		nd	$^3(PDF)^\circ$	$^1(PDF)^\circ$		$s^2 p^3 \ d$
$2s \ 2p^4$		$2p$	$^3P^\circ$	$^1P^\circ$	sp^5	

Fig 14 16 Terms predicted in column VI

In O I and S I series were identified by Paschen and Runge before the end of last century, and these are listed in Fowler's *Report* as singlets and triplets. Theoretically these series should be prominent, because the normal state of the ion is an S state,

but a 4S ground term should produce systems of triplets and quintets, not of singlets and triplets. This discrepancy between experiment and modern theory was only resolved in 1923 when Hopfield identified series of lines arising from the combination of Fowler's 'singlet' S and D series with a 3P ground term. The numerical values of the three components of the term are 109,833, 109,674 and 109,607,* showing intervals of 159 and 67 respectively and an interval ratio of 2:4:1. In combinations of this term the line of shortest wave-length is the most intense, so that the 3P term must be inverted.

As inter-system lines are seldom strong, the combination of this triplet term with a 'singlet' series suggested that the supposed singlets were really triplets too narrow to be resolved. True, the 3P ground term also combines with Fowler's 1s 'triplet' term, but the lines are much weaker, and though the $1s \rightarrow ^3P_2$ and $1s \rightarrow ^3P_1$ lines are quite clear, the $1s \rightarrow ^3P_0$ line is missing in both O I and S I. If, however, the supposed triplets are really quintets the absence of this particular line is readily explained, for the 1s term is then 5S_2 , and the combination with 3P_0 is forbidden, since J would have to change by two units. This evidence alone seems conclusive, but in 1923 the argument from theory was of little weight and so Paschen and Landé† felt bound to settle the question by measuring the Zeeman splitting of two quintet combinations 7771 Å, $3p\ ^5P_{123} \rightarrow 3s\ ^5S_2$ and 3974 Å, $4p\ ^5P_{123} \rightarrow 3s\ ^5S_2$.

Above the 3P term thus identified the metastable states 1D_2 and 1S_0 should lie, and recently combinations of these states with various higher terms have been identified in O I, but history records that these lines were found only after McLennan‡ had shown in a brilliant research that the green auroral line 5577 Å arises in a jump between these two states. As the transition is forbidden by two selection rules the evidence requires careful scrutiny, it will be considered in a later chapter on quadrupole radiation.

* Hopfield, *AJ*, 1924, 59 114.

† Laporte, *Nw*, 1924, 12 598, attributes this work to Paschen and Landé, but the reference given is inaccurate and the paper does not appear under either name in *Science Abstracts*, 1922-24.

‡ McLennan, *PRS*, 1928, 120 327.

7 The halogens, s^2p^5 configuration

According to Hund's scheme the p^5 configuration produces only an inverted 2P term, and in fact this is the normal state of all halogen atoms

The higher terms may be divided into three groups according as they arise from the 3P , 1D or 1S term of the spark spectrum. Of the predicted terms shown in Fig 14 17, a large number have been

F II				F I			
l_1	l_2	l_3	Term	l_7	Terms	l_1	l_7
$2s^2 2p^4$				$2p$	$^2p^\circ$	$s^2 p^5$	
			3P	ns np nd	4P $^4(SPD)^\circ$ $^4(PDF)$	2P $^2(SP\bar{D})^\circ$ $^2(PDF)$	$s^2 p^4 s$ $s^2 p^4 p$ $s^2 p^4 d$
			1D	ns np nd	— — —	2D $^2(PDF)^\circ$ $^2(SP\bar{D}FG)$	$s^2 p^4 s$ $s^2 p^4 p$ $s^2 p^4 d$
			1S	ns np nd	— — —	3S $^2P^\circ$ 2D	$s^2 p^4 s$ $s^2 p^4 p$ $s^2 p^4 d$

Fig 14 17 Terms predicted in the halogens

found in the arc spectra of all the halogens, and in the spark spectra of neon, argon and krypton. In particular, Bakker, De Bruin and Zeeman* have made an extensive magnetic analysis of A II and have studied the spectrum very thoroughly, it provides much material for the study of irregular g values and of series limits, and will be referred to again in that context, the terms obey the normal multiplet laws badly, but the Rydberg formulae rather well†. In the lighter elements the empirical terms are easily named, for they approximate to the Russell-Saunders laws, but only the J values of the higher terms of F I are known.

Directly above the ground term come even terms from the $p^4 s$ and $p^4 d$ configurations. Of these the $p^4 (^3P)s$ configuration should produce five terms, named according to the Russell-Saunders scheme $^4P_{2\frac{1}{2}, 1\frac{1}{2}, 1}$ and $^2P_{1\frac{1}{2}, \frac{1}{2}}$, and in fact in F I the extreme

* Bakker, De Bruin and Zeeman, *K Akad Amsterdam, Proc*, 1928, **31** 780.

† Rosenthal, *AP*, 1930, 4 80

us in saying that the coupling in F I is Russell-Saunders, but in I I is (jj)

A comparison of the isoelectronic spectra Cl I and A II brings out one common effect of increasing the nuclear charge, in Cl I the succession of the configurations is apparently $4s, 4p, 5s, 5p, 3d,$

	F	Cl	Br	I
$^4\text{P}_{3/2} - ^4\text{P}_{1/2}$	275	530	1471	1459
$^4\text{P}_{1/2} - ^4\text{P}_{1/2}$	160	338	1977	4803
$^4\text{P}_{1/2} - ^2\text{P}_{1/2}$	1892	1398	300	924
$^2\text{P}_{1/2} - ^2\text{P}_{1/2}$	325	640	1787	4530
$\Delta^3\text{P}_{02}$ of $10n$	491	991	—	—

Fig 14 20 Intervals of the $p^4 (^3\text{P}) s$ configuration, showing the change from LS to jj coupling

terms arising from all the configurations except $5s$ having been identified, but in A II the succession is $4s, 3d, 4p, 5s, 4d,$ the increased nuclear charge having thus moved the d terms down relative to the s and p terms Of this movement much more will be heard in the long periods

8 The inert gases, s^2p^6 configuration

The p^6 group, being a complete shell, produces only a ^1S term, and empirically this always lies so much deeper than any of the odd terms that the lines which result lie in the far ultra-violet The ground term and the lowest configuration of odd terms have been identified in the arc spectra of neon, argon, krypton and xenon and also in the spark spectra $\text{Na II}, \text{K II}, \text{Rb II}$ and $\text{Cs II},$ of these neon was the subject of such a thorough study at a time when the structure of complex spectra was very little understood that it still deserves pride of place (Figs 14 21, 22)

The spectrum of neon consists of two parts, one in the visible analysed into series by Paschen in 1918,* and a few ultra-violet lines unknown to Paschen, but discovered by Lyman and Saunders in 1925† and attributed at once to the deep ^1S ground term The large number of series discovered by Paschen may be

* Paschen, *AP*, 1919, 60 405, 1920, 63 201

† Lyman and Saunders, *PR*, 1925, 25 886a

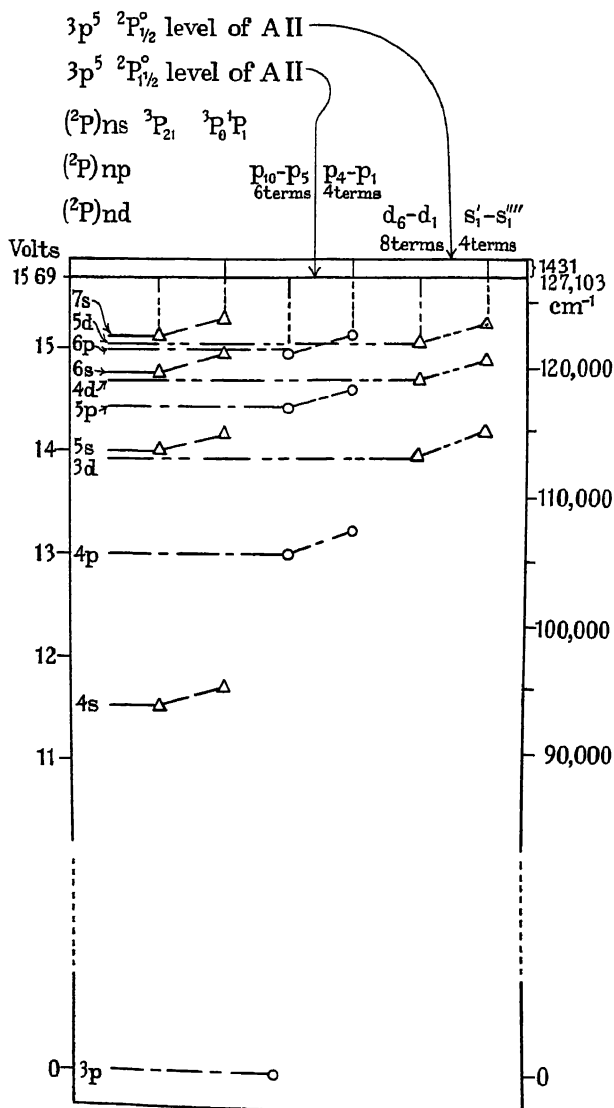


Fig 14 21 Level diagram of argon, this differs from the level diagram of neon only in the energies of the terms

divided by the energies of their lowest terms into three groups, of these the lowest contains four terms, named from below up s_5 , s_4 , s_3 and s_2 , and identified in modern theory with the $2p^5(^2P)3s$ configuration. Above these lie ten terms called by Paschen p terms, and written from below up p_{10} to p_1 , these combine with

Ne II				Ne I			
l_1	l_2	l_7	Terms	l_8	Terms	Paschen's terms	l_1 l_8
$2s^2$	$2p^5$		$^2P^\circ$	$2p$	1S	—	s^2p^6
			ns		$^3P^\circ$ $^1P^\circ$	s_2, s_3, s_4, s_5	$s^2p^5 s$
			np		$^3(SPD)$ $^1(SPD)$	p_1 p_{10}	$s^2p^5 p$
			nd		$^3(PDF)^\circ$ $^1(PDF)^\circ$	s_1 and d	$s^2p^5 d$

Fig 14 22 Terms predicted in the inert gases

the s terms and clearly arise from the $p^5(^2P)3p$ configuration. Higher still appear twelve terms written some of them d and some s_1 , these arise from the $p^5(^2P)3d$ configuration. All these terms were assigned J values by Landé* as the result of a magnetic analysis (Fig 14 23), and these J values agree precisely with those required by theory

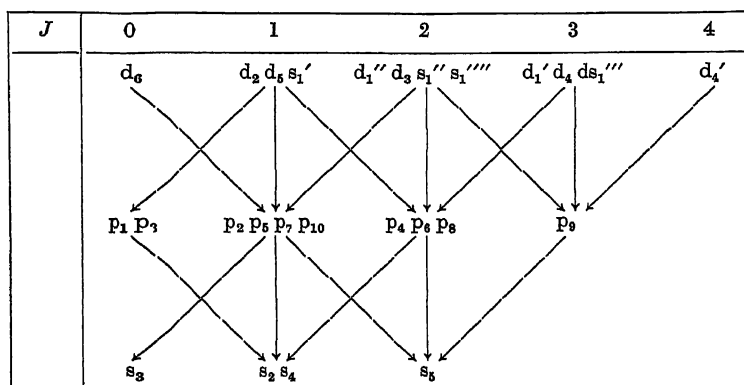


Fig 14 23 Permitted combinations and J values of the empirical terms of neon

Though thus far agreement is so satisfactory the s terms alone come near to obeying the multiplet laws. By Hund's energy rules

* Landé, *PZ*, 1921, 22 417

these four terms should be ${}^3P_{2,1,0}$ and 1P_1 in order of increasing energy (Fig 14 23), the empirical J values establish the 3P_2 and 3P_0 terms in the positions indicated, while for s_4 and s_2 Back*

Terms		g factor		Limit	
Emp	Theor	Emp	Theor	Emp	Theor
s_2	1P_1	1 04	1	Upper	${}^2P_{1\frac{1}{2}}$
s_3	3P_0	$\frac{0}{0}$	$\frac{0}{0}$	Upper	${}^2P_{\frac{1}{2}}$
s_4	3P_1	1 46	1 5	Lower	${}^2P_{1\frac{1}{2}}$
s_5	3P_2	1 50	1 5	Lower	${}^2P_{1\frac{1}{2}}$

Fig 14 24 The $2p^5$ ns terms of neon, the empirical g values are those of the first or $3s$ term of the series

obtained g values of 1.46 and 1.03, in satisfactory agreement with the theoretical values of 1.5 for 3P_1 and 1.0 for 1P_1 . Moreover, this allotment is confirmed by the ultra-violet combinations with the 1S_0 ground term, for the line of lower frequency $s_4 \rightarrow {}^1S_0$ is weaker than the line of higher frequency $s_2 \rightarrow {}^1S_0$, and this agrees with the general rule that inter-system lines are weaker than combinations between terms of the same system.

The p terms present much greater difficulties, a very brief examination shows that the interval rule is quite useless, while the Landé g formula is none too well obeyed. Accordingly, the matching proposed by Hund† (Fig 14 24) must be regarded at best as the reasoned guess of an expert, for a thorough study of the transition from Russell-Saunders to (jj) coupling is necessary before a satisfactory solution can be reached. Judging from H. N. Russell's success with the complex spectra of the iron row, a satisfactory solution might be expected from a careful study of intensities, but this does not seem to have been attempted yet.

In ordering the high terms to series based on these low terms Paschen found clear evidence that the series might be divided into two groups, some tending to a lower limit and others to a limit 780 cm^{-1} higher, these two limits appear in modern theory

* Back, *AP*, 1925, 76 317

† Shenstone, *N*, 1928, 121 619

‡ Hund, *ZP*, 1929, 52 601

§ Pogánv, *ZP*, 1935, 93 376 and chapter XVIII, § 7

as the two components of the 2P ground term of Ne II, now known to have an interval of 782 cm^{-1} . Of the s terms s_2 and s_3 tend to the upper limit, s_4 and s_5 to the lower limit, the limits of the p series are shown in Fig 14 25

Empirical				Theoretical	
Term	J	g	Limit	Term ^a	g
P_1	0	$\frac{9}{8}$	Upper	3P_0	$\frac{9}{8}$
P_3	0	$\frac{9}{8}$	Lower	1S_0	$\frac{9}{8}$
P_2	1	1 340	Upper	3P_1	1 5
P_5	1	0 999	Upper	1P_1	1 0
P_7	1	0 699	Lower	3D_1	0 5
P_{10}	1	1 984	Lower	3S_1	2 0
P_4	2	1 301	Upper	3P_2	1 5
P_6	2	1 229	Lower	3D_2	1 167
P_8	2	1 137	Lower	1D_2	1 0
P_9	3	1 329	Lower	3D_3	1 333

Fig 14 25 The $2p^5 \ 3p$ terms of neon

^a The naming of the terms is based on the most exiguous evidence

Interval	Ne	A	Kr	Xe
$^3P_2 - ^3P_1$	417	607	945	978
$^3P_1 - ^3P_0$	359	803	4275	8152
$^3P_0 - ^1P_1$	1070	846	655	988

Fig 14 26 Intervals of the $p^5 (^2P)$ s configuration in the inert gases

The spectra of the heavier inert gases are very similar to neon, but as the interval of the ground term of the spark spectrum increases, its influence on the arc spectrum becomes increasingly evident, it appears, for instance, in the intervals of the four (2P)s terms, shown in Fig 14 26, while in Kr I the p terms divide themselves into two groups with an interval of no less than 5200 cm^{-1} between them, the upper group consists of four terms and the lower group of six just as theory will be shown to require, but the chance of fitting the Russell-Saunders notation to these terms is more remote even than in neon

Indeed, the heavy inert gases centre in themselves three influences which all conspire to break up the simple Russell-

Saunders coupling, first, increasing atomic number makes for loss of regularity, this was discussed particularly in columns IV and V, but it is visible in every column, secondly, the increasing separation of the ground term of the ion tends to divide the terms by limits rather than by multiplets, and lastly spectra grow less regular as one passes from left to right across the periodic table, neon is less regular than carbon and carbon less regular than sodium

BIBLIOGRAPHY

The first systematic account was by Hund in *Linienspektren und periodisches System der Elemente*, 1927, and this is still standard

Term values are largely taken from Bacher and Goudsmit, *Atomic Energy States*, 1932. References to particular elements can be traced in the select bibliography of Appendix vi

CHAPTER XV

LONG PERIODS

1 The ground terms

In the short periods the s electron carries appreciably less energy than the p electron, and very little study suffices to show that the first two electrons enter an s shell and the last six a p. In the long periods however, while an s electron still carries less energy than a p, the s and d electrons carry roughly the same energy, so that a first glance shows one group of twelve elements instead of two groups of two and ten

1	2	3	4	5	6	7	8	9	10	11	12
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn
Rb	Sr	Y	Zr	Cb	Mo	Ma	Ru	Rh	Pd	Ag	Cd
Cs	Ba	La-Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg

Bohr's work on the periodic system showed that in the first two of these twelve elements the electron enters an s orbit, but that thereafter as the nuclear charge increases the d orbits grow more stable and the s less, until in the spark spectrum of a system of ten electrons all ten occupy d orbits, and produce the 1S_0 ground term characteristic of a complete shell. The eleventh and twelfth electrons then re-enter the s shell producing copper, zinc and their homologues. Thus in only eight of the twelve elements are the configurations still in doubt, these eight were enclosed by Bohr in a frame, and will be described in successive periods as the elements of the iron frame, the palladium frame and the platinum frame

Since only two electrons can enter an s shell, an atom with n outer electrons has the choice of only three configurations, d^n , $d^{n-1} s$ and $d^{n-2} s^2$, the chief quantum numbers will usually be omitted, in order to simplify the discussion, for the orbits are always 3d and 4s in the iron frame, 4d and 5s in the palladium frame and 5d and 6s in the platinum frame

The terms, which any configuration produces, can be calculated, and if the lowest term of each configuration is that with

largest spin and orbital vectors, a list of possible configurations and the ground terms which they produce can be compiled, as Fig 15 1 shows But the ground term is the easiest of all terms to determine empirically, so that the argument which produced this table may be reversed, and the ground term used to decide which configuration produces the lowest term (Figs 15 2-4) When

No of electrons	Configuration		
	d^n	$d^{n-1} s$	$d^{n-2} s^2$
0	1S		
1	2D	2S	
2	3F	3D	1S
3	4F	4F	3D
4	5D	5F	3F
5	6S	6D	4F
6	5D	7S	5D
7	4F	6D	6S
8	3F	5F	5D
9	2D	4F	4F
10	1S	3D	3F
11		2S	2D
12			1S

Fig 15 1 Ground terms of three configurations

No of electrons	Arc spectra			Spark spectra			No of electrons
	Atom	Ground term	Con-figuration	Atom	Ground term	Con-figuration	
1	K	2S	s	Ca^+	2S	s	1
2	Ca	1S	s^2	Sc^+	3D	$d s$	2
3	Sc	3D	$d s^2$	Ti^+	4F	$(d^2 s)$	3
4	Ti	3F	$d^2 s^2$	V^+	5D	d^4	4
5	V	4F	$d^3 s^2$	Cr^+	6S	d^5	5
6	Cr	7S	$d^5 s$	Mn^+	7S	$d^5 s$	6
7	Mn	6S	$d^5 s^2$	Fe^+	6D	$d^6 s$	7
8	Fe	5D	$d^6 s^2$	Co^+	3F	$d^6 s$	8
9	Co	4F	$(d^7 s^2)$	Ni^+	3D	d^7	9
10	Ni	3F	$d^8 s^2$	Cu^+	1S	d^{10}	10
11	Cu	2S	$d^{10} s$	Zn^+	2S	$d^{10} s$	11
12	Zn	1S	$d^{10} s^2$	Ga^+	1S	$d^{10} s^2$	12

Fig 15 2 Ground terms of the iron frame elements

In this and the two following figures, the ground term is given only when it is known empirically; a ? following the term shows that it is still open to doubt A configuration enclosed in brackets cannot be deduced from the ground term alone, but is known from further study of the spectrum, where a configuration is given in the absence of the ground term, reliance has been placed on the argument of §4 of this chapter

there are 3, 6 or 9 electrons, two configurations produce the same ground term, and then further study is necessary to distinguish between them, where a configuration has been obtained in this way, it is enclosed in brackets

No of elec trons	Arc spectra			Spark spectra			No of elec trons
	Atom	Ground term	Con-figuration	Atom	Ground term	Con-figuration	
1	Rb	2S	s	Sr ⁺	2S	s	1
2	Sr	1S	s ²	Y	1S	s ²	2
3	Y	2D	d s ²	Zr ⁺	4F	d ² s	3
4	Zr	3F	d ² s ²	Cb ⁺	5D	d ⁴	4
5	Cb	6D	d ⁴ s	Mo ⁺	$^6S^?$	d ⁵	5
6	Mo	7S	d ⁵ s	Ma ⁺	—	d ⁵ s	6
7	Ma	—	d ⁵ s ²	Ru ⁺	4F	d ⁷	7
8	Ru	5F	d ⁷ s	Rh ⁺	—	d ⁸	8
9	Rh	4F	(d ⁸ s)	Pd ⁺	2D	d ⁹	9
10	Pd	1S	d ¹⁰	Ag ⁺	1S	d ¹⁰	10
11	Ag	2S	d ¹⁰ s	Cd ⁺	2S	d ¹⁰ s	11
12	Cd	1S	d ¹⁰ s ²	In ⁺	1S	d ¹⁰ s ²	12

Fig 15 3 Ground terms of the palladium frame elements

No of elec trons	Arc spectra			Spark spectra			No of elec trons
	Atom	Ground term	Con-figuration	Atom	Ground term	Con-figuration	
1	Cs	2S	s	Ba ⁺	2S	s	1
2	Ba	1S	s ²	La ⁺	3F	d ²	2
3	La	2D	d s ²	Lu ⁺	1S	s ²	3
	Lu	2D	d s ²	Hf ⁺	2D	d s ²	
4	Hf	3F	d ² s ²	Ta ⁺	—	—	4
5	Ta	—	d ³ s ²	W ⁺	—	—	5
6	W	5D	(d ⁴ s ²)	Re ⁺	—	—	6
7	Re	6S	d ⁵ s ²	Os ⁺	—	—	7
8	Os	—	d ⁶ s ²	Ir ⁺	—	—	8
9	Ir	$^2D^?$	d ⁹ ?	Pt ⁺	—	—	9
10	Pt	3D	d ⁹ s	Au ⁺	1S	d ¹⁰	10
11	Au	2S	d ¹⁰ s	Hg ⁺	2S	d ¹⁰ s	11
12	Hg	1S	d ¹⁰ s ²	Tl ⁺	1S	d ¹⁰ s ²	12

Fig 15 4 Ground terms of the platinum frame elements

With only one exception the arc spectra of the iron frame (Fig 15 2) have $d^{n-2} s^2$ as their ground configuration, while the spark spectra have $d^{n-1} s$ or d^n , a contrast which shows how general is the tendency of the d electrons to sink relative to the s electrons as the nuclear charge increases, of this tendency an oft-

quoted example is the descent of the s electron of $K I$ and $Ca II$ into the d orbit of $Sc III$. In the palladium frame (Fig 15 3) the tendency is again in evidence, the d^n configuration produces the ground terms of six spark spectra, but only one are. Comparison of the iron and palladium frames shows that the d orbits are more stable in the latter, of the platinum frame (Fig 15 4) not much is yet known, but there is some reason to think that in the early elements the d orbits are more stable still, but that after the intrusion of the f shell between lanthanum and lutetium, the d shell is less stable even than in the iron frame. Thus $La II$ has a $d^2 3F$ ground term in contrast to the $ds^3 D$ of $Sc II$ and the $s^2 1S$ of $Y II$, but the ground states of $Lu II$, $Lu I$, $Hf I$ and $Re I$ all seem to contain two s electrons, besides varying numbers of d electrons.

2. Configurations and analysis

In all three frames the s and d electrons have roughly the same energy, and this determines the general form of the spectra. A configuration which contains only s and d electrons outside the last closed shell will consist of even terms, and accordingly the low terms of all elements are even, they will usually arise from two different configurations, and as no combinations are permitted, the higher of the two is metastable.

Above these low terms come a group of odd terms containing a p electron, and above those again a second group of even terms containing only s and d electrons. Theory allows many higher terms, but in practice the lines involving these terms are weak or absent, so that series consist of only two or at most three terms.

This means that the method of analysis differs from that applied to simple spectra*, the spectroscopist tries to find not series, but multiplets. As in other spectra constant differences between the frequencies of pairs of lines indicate terms, and these may be separated into odd and even, because an odd term combines only with an even. The ground term can usually be picked out because it gives rise to the *rare ultimes* and most of the absorption lines, the spark produced under water is also useful,

* For an account of methods of analysis, see Russell, *AJ*, 1927, 66 348

for the spectrum consists of a few lines all ending in the ground term

The approximate energy of a term can be determined by examining the temperature class of the lines to which it gives rise. If it is of low energy content, then it will appear bright at a low temperature and in King's classification will belong to a low class. Thus and the division into odd and even terms will often determine the configuration to which a term belongs.

Finally, the terms of a configuration may be divided into multiplets by examining the intensities of the lines to which they give rise, for when the coupling is not normal the intensities are far less seriously disturbed than either Landé's interval ratios or the magnetic splitting factors.

To suit these new methods of analysis, new ways of specifying the empirical terms have had to be developed, thus the analysis may succeed in finding terms, but be unable to divide them into multiplets, the terms are then numbered 1, 2, 3, beginning from the term of lowest energy, or if the J value is also known then this may be added as a suffix $1_{2\frac{1}{2}}$. Or again the terms may have been worked out, and then each term is specified by a small letter placed before its term symbol, in this notation the letters a to e are reserved for the low terms, z , y , x , for the middle or odd terms, and f , g , h , for the high terms, in each group the terms are lettered from low to high energy.

3 Individual spectra

The spectra of the long periods are most naturally classified by the number of electrons outside the last inert gas shell, and in this order they will be reviewed.

The energies of all the low terms and a varying number of odd terms are tabulated, the energy given for any multiplet is that of the component with greatest J , for this will produce the strongest lines, the energies are measured up from the ground term, and the ground term itself is in clarendon type. To avoid any misunderstanding the chief quantum number is added for spectra of the iron frame, to apply to the palladium and platinum frames it should be increased by one and two respectively.

One electron K I, Rb I, Cs I, Ca II, Sr II, Ba II, Ra II, Sc III, Y III, La III, Lu III, Ti IV, Zr IV, Ce IV

In the arc spectra of the alkalis and the spark spectra of the alkaline earths the ground term is always 2S , the single electron occupying a 4s, 5s or 6s orbit. With further increase in the nuclear charge, however, the electron falls into a d orbit, for Sc III, Y III, Zr IV, and Ce IV are all known to have a 2D ground term.

The introduction of a group of f electrons produces, however, a surprising change, Lu III, which differs from La III only in having this group, has a 2S ground term, so that the 6s orbit must have grown more stable than the 5d.

Low terms

	K	Rb	Cs	Ca II	Sr II	Ba II	Ra II
4s $^2S_{\frac{1}{2}}$	0	0	0	0	0	0	0
3d $^2D_{\frac{3}{2}}$	21,539	19,355	14,597	13,711	16,837	5,675	26,209

	Sc III	Y III	La III	Lu III	Ti IV	Zr IV	Ce IV
4s $^2S_{\frac{1}{2}}$	25,537	7466	13,590	0	80,379	38,258	5152
3d $^2D_{\frac{3}{2}}$	198	725	1,604	8648	348	1,250	3305

Fig 15.5 One electron. Energies of the low terms

Two electrons. Ca I, Sr I, Ba I, Sc II, Y II, La II, Lu II, Ti III, Zr III.

The ground terms of the arc spectra of the alkaline earths and the second spark spectra of column IV are respectively 1S_0 and 3F_2 , the first arising from the s^2 configuration and the second from the d^2 , and this is natural enough, since the d orbit normally sinks relative to the s as the nuclear charge increases. Between these two extremes the singly ionized earth metals should provide a natural transition, and in some measure they do, for the ground term of Sc II is d s 3D and of La II d $^2^3F$, while if the f shell screens the outer electrons from the nucleus the s^2^1S ground term of Lu II does not go unexplained. But that the ground term of Y II should be s^2^1S is a striking anomaly, for the above scheme would predict it as arising either from d s or d^2 , moreover, the ground term of nearly every spectrum is derived by the addition of an

electron to the ground term of the next higher spark spectrum, but Y II forms an exception to this rule also, for the single electron of Y III occupies a d orbit

The other low terms which theory dictates are shown in Fig 15 7, and just these terms are actually found in the spark

Low terms

	Ca	Sr	Ba	Sc II	Y II	La II	Lu II	Th III	Zr III
4s ² ¹ S ₀	0	0	0	11,736	0	7,395	0	—	—
3d 4s ³ D ₃	20,371	18,320	9,596	178	1,450	3,250	14,199	38,425	19,533
¹ D ₂	21,849	20,149	11,395	2,541	3,296	1,395	17,332	41,704	16,122
3d ² ³ F ₄	—	—	—	4,988	8,743	1,971	32,504	422	1,487
³ P ₂	48,564	32,625	23,919	12,154	14,098	6,227	—	10,721	8,840
¹ G ₄	—	—	—	14,261	15,683	7,473	—	14,398	2,534
¹ D ₂	—	—	—	10,945	14,833	10,095	—	8,473	3,392
¹ S ₀	—	—	—	—	—	—	—	14,053	3,835

Fig 15 6 Two electrons Energies of the low terms

Low terms

Configuration	Terms	
	Triplets	Singlets
4s ²	—	S
3d 4s	D	D
3d ²	PF	SDG

Middle terms

Ion		Atom		
<i>l</i> ₁	Term	<i>l</i> ₂	Terms	<i>l</i> ₁ <i>l</i> ₂
3d	² D	4p	³ (PDF) ^o ¹ (PDF) ^o	dp
4s	² S	4p	³ P ^o ¹ P ^o	sp

High terms

Ion		Atom		
<i>l</i> ₁	Term	<i>l</i> ₂	Terms	<i>l</i> ₁ <i>l</i> ₂
3d	² D	<i>n</i> s <i>nd</i>	³ D ¹ D ³ , ¹ (SPDFG)	d s d d
4s	² S	<i>n</i> s <i>nd</i>	³ S ¹ S ³ D ¹ D	s s s d

Fig 15 7 Terms predicted from two electrons

spectra, $d^2 1S$ has been a little difficult to locate, but it has been found all right in $Ti III$ and $Zr III$. These low terms are in satisfactory agreement with Hund's energy rules, in the $d s$ configuration of $Sc II$, for example, the $3D$ term lies below $1D$, while in the d^2 configuration $3F$ lies below $3P$, among the singlets, however, b^1D is lower than a^1G . Deviations such as this are common in all spectra.

The middle and high terms have also been collected in Fig. 15.7, and the general relations of a typical spectrum are shown in

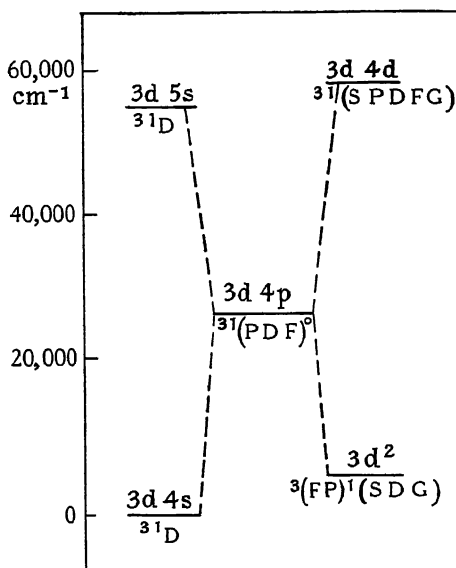


Fig. 15.8 Simplified level diagram of $Sc II$

Fig. 15.8, where the four groups of even terms, arising from the configurations $3d^2$, $3d 4s$, $3d 4d$ and $3d 5s$, all combine with the central group of odd terms, but not at all with each other. This means, of course, that all terms of the $3d^2$ configuration are metastable, for no direct return to the ground state is possible.

In the high $3d 4d$ configuration of $Sc II$ the agreement with Hund's energy rules is much less satisfactory than among the low terms, among the triplets $3F$ and $3P$ lie close together, but some $4000 cm^{-1}$ above the $3G$, $3D$ and $3S$ levels, while among the

singlets, on the other hand, 1F and 1P lie some 4000 cm^{-1} below the 1G , 1D and 1S levels, each group being fairly compact. This curious alternating arrangement of energy levels finds a parallel in pentads of similar origin in other spectra, but it is quite contrary to Hund's energy rules and has only recently received any theoretical explanation.

All the triplet terms of Sc II are erect, and most of them obey the interval rule well (Fig. 15.9).

Configuration	Interval	Interval quotient
$3d\ 4s$	$\Delta^3D_{1\ 2}$ 67.7	33.8
	$\Delta^3D_{2\ 3}$ 109.9	36.6
$3d^2$	$\Delta^3F_{2\ 3}$ 80.7	26.9
	$\Delta^3F_{3\ 4}$ 104.2	26.0
$3d^2$	$\Delta^3P_{0\ 1}$ 27.4	27.4
	$\Delta^3P_{1\ 2}$ 52.9	26.4

Fig. 15.9. Interval rule in the low terms of Sc II

Three electrons Sc I , Y I , La I , Lu I , Ti II , Zr II , Ce II , Hf II

The ground terms of the arc spectra of scandium, yttrium and lutetium are $d\ s^2\ ^2D$, while the usual subsidence of the d relative to the s orbit produces $d^2\ s\ ^3F$ in the spark spectra of titanium and

Low terms

	Sc	Y	La	Lu	Ti II	Zr II	Ce II	Hf II
$3d\ 4s^2\ ^2D_{1\frac{1}{2}}$	168	530	1053	1994	25,193	18,397	420	—
$3d^2\ 4s$	$^4F_{1\frac{1}{2}}$ 11,677	11,532	4122	—	893	1,323	1923	8362
	$^4P_{2\frac{1}{2}}$ —	15,477	—	—	10,025	8,058	1870	—
	$^2G_{4\frac{1}{2}}$ 20,237	18,499	—	—	15,258	14,190	—	—
	$^2F_{3\frac{1}{2}}$ 15,042	15,864	—	—	4,898	6,468	—	—
	$^2D_{3\frac{1}{2}}$ 17,013	16,159	—	—	8,744	4,505	—	73051
	$^2P_{1\frac{1}{2}}$ —	19,406	—	—	16,625	6,112	—	—
	$^2S_{\frac{1}{2}}$ —	—	—	—	21,338	25,202	—	—
	$^4F_{4\frac{1}{2}}$ 33,906	29,843	—	—	1,216	3,758	—	—
	$^4P_{2\frac{1}{2}}$ 36,573	32,366	—	—	9,518	9,969	—	—
	$^2H_{4\frac{1}{2}}$ —	—	—	—	12,775	12,360	—	—
$3d^3$	$^2G_{4\frac{1}{2}}$ —	—	—	—	9,118	8,153	—	—
	$^2F_{3\frac{1}{2}}$ —	—	—	—	20,892	19,433	—	—
	$^2D_{3\frac{1}{2}}$ 36,330	—	—	—	12,758	14,733	—	—
	$^2D_{2\frac{1}{2}}$ —	—	—	—	—	14,163	—	—
	$^2P_{2\frac{1}{2}}$ —	—	—	—	9,976	20,080	—	—
	$^2P_{1\frac{1}{2}}$ —	—	—	—	—	—	—	—
	$^2P_{1\frac{1}{2}}$ —	—	—	—	—	—	—	—

Fig. 15.10. Three electrons. Energies of the low terms

zirconium It has been suggested that the ground term of Hf II is $d^2 s^2 D$, but this irregularity still awaits confirmation

The low terms predicted by theory are collected in Fig 15 10, every one of these terms has been found in Zr II, while all the odd terms have been found in Y I, thus once again the great power of Hund's theory is demonstrated The greatly increased complexity of Sc I and Y I compared with Sc II and Y II is perhaps so obvious that it hardly deserves mention

Low terms

Configuration	Terms	
	Quartets	Doublets
3d 4s ² 3d ³ 4s 3d ³	— PF PF	D SPDGF PDFGH 2

Middle terms

Ion		Atom		
$l_1 l_2$	Term	l_3	Terms	$l_1 l_2 l_3$
4s ²	¹ S	4p	² P	s ² p
3d 4s	³ D ¹ D	4p 4p	⁴ (PDF) ² (PDF) ² (PDF) ² (PDF)	d sp
3d ³	⁴ F ⁴ P ¹ G ¹ D ¹ S	4p 4p 4p 4p 4p	⁴ (DFG) ² (DFG) ⁴ (SPD) ² (SPD) ² (FGH) ² (PDF) ² P	d ³ p

Fig 15 11 Terms predicted from three electrons

The lines due to transitions from high to middle terms are, in the spectrum of Y I, so much weaker when the terms are doublet than when quartet, that very few high doublet terms have been identified.

Four electrons. Ti I, Zr I, Hf I, V II, Cb II

The arc spectra of titanium, zirconium and hafnium all have as ground term ³F₂ of the $d^2 s^2$ configuration, but the d orbit evidently falls abruptly relative to the s at this point, for the spark spectra seem to have $d^4 {}^5D$ as ground term

The spectra of titanium and zirconium have been very thoroughly analysed at the Bureau of Standards by Russell and Kiess, and almost all the low terms from the $d^2 s^2$ and $d^3 s$ configurations have been identified. In their analysis of hafnium

Low terms

	Ti	Zr	Hf	V II	Cb II
$3d^2 4s^2 {}^3F_4$	387	1,241	4568	—	—
3P_2	8,602	4,186	8984	—	—
1G_4	12,118	8,057	—	—	—
1D_2	7,255	5,101	5639	—	—
1S_0	15,167	13,142	—	—	—
$3d^3 4s {}^5F_5$	6,843	5,889	—	3,163	4147
3F_4	11,777	12,342	—	9,097	—
5P_3	14,106	11,258	—	13,741	—
3P_2	18,912	15,932	—	—	—
3H_6	18,193	15,120	—	20,363	—
1H_5	20,796	18,739	—	—	—
3G_5	15,220	12,773	—	14,655	—
1G_4	18,288	17,753	—	—	—
3F_4	—	15,700	—	—	—
1F_3	29,818	—	—	—	—
3D_3	17,540	14,697	—	—	—
1D_2	20,210	17,228	—	—	—
3D_3	—	—	—	—	—
1D_2	—	—	—	—	—
3P_2	18,145	17,143	—	—	—
1P_1	20,063	—	—	—	—
$3d^4 {}^5D_4$	—	22,398	—	339	1225
3H_6	—	—	—	—	—
3G_5	36,201	—	—	16,532	—
3F_4	—	—	—	13,608	—
3F_4	—	—	—	—	—
3D_3	—	—	—	—	—
3P_2	—	—	—	11,908	—
3P_2	—	—	—	—	—

Fig 15 12 Four electrons Energies of the low terms.

Meggers and Scribner had to rely on wave-lengths and furnace intensities only, but this has not prevented them from identifying seven even and 63 odd terms. From a study of the observed combinations, J values have been assigned and the suggestion put forward that the lowest three terms are $d^2 s^2 {}^3F$. The J values would allow the next terms to be 1D_2 and 3P , and this allotment would make the low terms very similar to those of Ti II, but further discussion must wait on the promised measurements of the Zeeman effect.

Among the odd terms those due to the configuration $d^3 sp$ lie lowest in the arc spectra, but terms from the two other configurations have been recognised

Numerous combinations have been found between both singlet and triplet terms, and between triplet and quintet terms, but very

Low terms

Configuration	Terms		
	Quintet	Triplet	Singlet
$3d^3 4s^2$	—	PF	SDG
$3d^3 4s$	PF	PDFGH 2 2 2	PDFGH 2
$3d^4$	D	PDFGH 2 2	SDFGJ 2 2 2

Middle terms

Ion			Atom		
l_1	l_2	Term	l_1	Terms	l_1 l_2
$3d^3$	$4s$	4F	$4p$	$^5,3(DFG)$	$d^3 sp$
		4P		$^5,3(SPD)$	
		2G		$^3,1(FGH)$	
		2F		$^3,1(DFG)$	
		2D		$^3,1(PDF)$	
		2P		$^3,1(SPD)$	
		2S		3,1P	
$3d^3$		4F	$4p$	$^5,3(DFG)$	$d^3 p$
		4P		$^5,3(SPD)$	
		2H		$^3,1(GHI)$	
		2G		$^3,1(FGH)$	
		2F		$^3,1(DFG)$	
		2D		$^3,1(PDF)$	
		2D		$^3,1(PDF)$	
		2P		$^3,1(SPD)$	
$3d 4s^2$		2D	$4p$	$^3,1(PDF)$	$d s^2 p$

Fig 15 13. Terms predicted from four electrons.

few between singlets and quintets, and even these few are all susceptible of the same explanation. When two terms of the same configuration and the same J have nearly the same energy, they share both their intensities and their g values. Thus $d^3 p^1 D_2^0$ combines with the low even $^5 D_3$ term because the former lies very near to the $^5 P_2^0$ term of the same configuration, the empirical g values of the $4d^3 5p^1 D_2^0$ and $^5 P_2^0$ terms of Zr I are both 1.42,

whereas the Landé values are 1 000 and 1 833 respectively, their respective energies are 34,850 and 34,761 cm^{-1}

Five electrons V I, Cb I, Cr II, Mo II

The ground term of these spectra starts as ^4F of the $d^3 s^2$ configuration in the iron row, changes to ^6D from $d^4 s$ in the palladium row, and settles down as $d^5 {}^6\text{S}$ in both the spark spectra, these changes clearly conform to the general type

Low terms

	V I	Cb I	Cr II	Mo II
$3d^3 4s^2 {}^4\text{F}_{4\frac{1}{2}}$	553	—	—	—
${}^4\text{P}_{2\frac{1}{2}}$	9,825	—	—	—
$3d^4 4s {}^6\text{D}_{4\frac{1}{2}}$	2,425	1050	12,498	known
${}^4\text{H}_{6\frac{1}{2}}$	15,063	—	30,393	—
${}^4\text{G}_{8\frac{1}{2}}$	17,242	—	33,696	—
${}^4\text{F}_{4\frac{1}{2}}$	15,770	—	31,221	—
${}^4\text{D}_{3\frac{1}{2}}$	8,716	—	20,025	known
${}^4\text{P}_{2\frac{1}{2}}$	15,572	—	—	—
${}^2\text{P}_{1\frac{1}{2}}$	—	—	17,593	—
$3d^5 {}^6\text{S}_{5\frac{1}{2}}$	—	—	0	ground?
${}^4\text{G}_{9\frac{1}{2}}$	—	—	20,514	—
${}^4\text{F}_{4\frac{1}{2}}$	—	—	32,856	—
${}^4\text{D}_{3\frac{1}{2}}$	—	—	25,035	—
${}^4\text{P}_{2\frac{1}{2}}$	—	—	—	—
${}^2\text{D}_{3\frac{1}{2}}$	—	—	21,824	—

Fig 15 14 Five electrons Energies of the low terms

Of the other low terms little is known outside the Cr II spectrum, in which many quartet but only two doublet terms have been found, these doublets are conspicuous in breaking Hund's energy rules, for they lie among the lower of the quartet terms

The odd terms are best developed by the addition of a p electron to the lowest terms of the parent spectrum, in this way a number of triads are formed, and these agree very satisfactorily with the terms found empirically Thus in V I there occur a triad of sextet terms at 18,000 cm^{-1} and a triad of quartet terms at about 22,000 cm^{-1} , the J values show that in fact these are the two triads ${}^6(\text{DFG})^\circ$ and ${}^4(\text{DFG})^\circ$ which should arise by the addition of a p electron to the low $d^3 s {}^5\text{F}$ term of V II

Of all these five electron spectra Cr II has been the most thoroughly analysed, and in it nearly all the terms both obey the

interval rule reasonably well and have normal g values. The ${}^6D^\circ$ and ${}^4P^\circ$ terms of the $d^4 p$ configuration are irregular judged by either criterion, but a term which deviates from one rule, often deviates from the second as well.

Low terms

Configuration	Terms		
	Sextet	Quartet	Doublet
$3d^3 4s^2$	—	PF	PDFGH
$3d^4 4s$	D	PDFGH 2 2 2	² SDFGJ 2 2 2
$3d^5$	S	PDFG	SPDFGHJ 3 2 2

Middle terms

Ion			Atom		
l_1	l_2	Term	l_3	Terms	l_1 l_3
d^4		5D	p	${}^{6,4}(PDF)$	$d^4 p$
$d^3 s$		5F	p	${}^{6,4}(DFG)$	$d^3 sp$

Fig 15 15 Terms predicted from five electrons

Six electrons Cr I, Mo I, W I, Mn II

The analysis of these spectra has hardly extended beyond the septet and quintet systems, only a few triplets having been found, and no singlets though these undoubtedly occur. The arc spectra of chromium and molybdenum and the spark spectrum of manganese all have $d^5 s {}^7S$ as ground term, in contrast the ground term of tungsten is probably 5D , but the analysis is so little advanced

Low terms

	Cr I	Mo I	W I	Mn II
$3d^4 4s^2 {}^5D_4$	8,307	12,346	6219	—
$3d^5 4s {}^7S_3$	0	0	2951	0
5S_2	7,593	10,768	—	9,473
5G_4	20,519	16,828	—	—
5F_5	—	—	—	—
5D_4	24,282	—	—	—
5P_3	21,841	18,229	—	—
$3d^6 {}^5D_4$	—	—	—	14,324

Fig 15 16 Six electrons Energies of the low terms

that this cannot be surely assigned to the $d^4 s^2$ as against the d^6 configuration

Low terms

Con figuration	Terms			
	Septet	Quintet	Triplet	Singlet
$3d^4 4s^2$	D	D	P D F G H 2 2	S D F G J 2 2 2
$3d^5 4s$	S	S P D F G	S P D F G H J 2 4 3 3	S P D F G H J 3 2 2
$3d^6$	—	D	P D F G H 2 2	S D F G J 2 2 2

Middle terms

Ion			Atom			
l_1	l_2	Term	l_3	Terms	l_1	l_2
$3d^3$	$4s^2$	4F	4p	$5,^3(DFG)$	d^3	s^2p
$3d^4$	$4s$	6D	4p	$7,^5(PDF)$	d^4	sp
$3d^5$		6S	4p	$7P$	d^5	p

Fig 15 17 Terms predicted from six electrons

The list of low terms is complete, but the middle terms are developed from the low terms of the spark spectrum, a complete list would be so long as to give no guidance at all

Seven electrons Mn I, Re I, Fe II, Ru II

The ground term of the arc spectra of manganese and rhenium is $d^5 s^2 ^6S$, but it changes first to $d^6 s ^6D$ and then to $d^7 ^4F$ in the spark spectra of iron and ruthenium respectively, thus once again the d orbit is seen to sink relative to the s orbit, when we pass from one period to the next or from an arc to a spark spectrum

Low terms

	Mn	Re	Fe II	Ru II
$3d^5 4s^2 ^6S$	0	0	23,318	—
$3d^6 4s ^6D_{4+}$	17,052	11,754	0	9,151
$^6G_{5+}$	—	—	25,429	—
$^4F_{4+}$	—	—	22,637	—
$^4D_{3+}$	23,297	—	7,955	19,379
$^4P_{2+}$	—	—	20,830	—
$3d^7 ^4F_{4+}$	—	—	1,873	0
$^4P_{2+}$	—	—	13,474	8,257
$^2F_{5+}$	—	—	—	22,289

Fig 15 18 Seven electrons Energies of the low terms

The ground term of Mn I has been recently confirmed by showing that all the resonance lines arise from jumps ending in the 6S state *

Ion			Atom			
l_1	l_6	Term	l_7	Terms	l_1	l_7
$d^5 s$		7S	s d	6S 6D	$d^5 s^2$ $d^6 s$	
		5S	s d	6S (as above) 6D (as above) 4D	$d^5 s^2$ $d^6 s$	
$d^4 s^2$		5D	d	6S (as above) $^4(PDFG)$	$d^5 s^2$	

Fig 15 19 Low terms of Mn I developed from the low terms of Mn II

Configuration	Terms		
	Sextets	Quartets	Doublets
$3d^5 4s^2$	S	PDFG	SPDFGHJ 3 3 2
$3d^6 4s$	D	PDFGH 2 2 2	SDFGJ 2 2 2
$3d^7$	—	PF	PDFGH 2

Middle terms

Ion			Atom			
l_1	l_6	Term	l_7	Terms	l_1	l_7
$d^5 s$		7S	p	6P 6P	$d^5 s p$	
		5S	p	6P 4P		
$d^4 s^2$		5D	p	$^6 4(PDF)$	$d^4 s^2 p$	
d^6		5D	p	$^6, 4(PDF)$	$d^6 p$	

Fig 15 20 Terms predicted from seven electrons

With the increasing number of electrons, even the low terms become so complex that they are not easily identified, to meet this difficulty the low terms may be developed from the lowest terms of the spark spectrum, just as the odd terms have been developed. Thus in Fig 15 19 there are only a few terms where in Fig 15 20 there is a huge mass

* Friedrichson, ZP, 1930, 64 43

Since the ground term of the spark spectra of Mn and Re is 7S , series are to be expected in the arc spectra, and in fact Catalan* did first analyse Mn I into a number of series. The lowest terms of the odd series are shown in Fig 15 20

The terms of these spectra are nearly all inverted, while those of Ru II show very regular g values

Eight electrons Fe I, Ru I, Co II

The ground term of the arc spectrum of iron is $d^6 s^2 {}^5D$, but the ground term changes in the normal way to $d^7 s {}^5F$ and then to $d^8 {}^3F$ in the arc spectrum of ruthenium and the spark spectrum of cobalt respectively.

Low terms

	Fe I	Ru I	Co II
$3d^6 4s^2 {}^5D_4$	0	? 7,483	—
$3d^7 4s {}^5F_5$	6,928	0	3,350
3P_3	17,550	? 8,771	17,771
3F_4	11,976	6,545	9,813
3P_2	22,838	—	—
$3d^8 {}^3F_4$	—	? 9,120	0
3P_2	—	? 10,623	—

Fig 15 21 Eight electrons Energies of the low terms

Ion			Atom				
l_1	l_7	Term	l_8	Terms		l_1	l_8
d_6	s	6D	s d	5D ${}^5(PF)$		d^6	s^2 d^7
		4D	s d	5D (as above) ${}^3(PDFG)$ ${}^5(PF)$ (as above) ${}^3(PDFG)$		d^6	s^2 d^7
d^7	1F	s d	5F 3F ${}^3(PF)$		d^7	s d^8	
		4P	s	5P 3P		d^7	s
		d	${}^3(PF)$ (as above)		d^8		

Fig 15 22 Low terms of Fe I developed from the low terms of Fe II

Hund's theory predicts for the $d^6 s^2$ configuration one quintet and seven triplet levels besides a number of singlets. In the iron

* Catalan, *Phil Trans R S*, 1922, 223 127.

are the 5D term is inverted and follows the interval rule fairly well, as the following table shows (Fig 15 24)

Low terms

Configuration	Terms		
	Quartet	Triplet	Singlet
$3d^8 4s^2$	D	PDFGH _{2 2}	SDFGJ _{2 2 2}
$3d^7 4s$	PF	PDFGH _{2 2 2}	PDFGH ₂
$3d^8$	—	PF	SDG

Middle terms

Ion			Atom		
l_1	l_2	Term	l_3	Terms	l_1 l_2 l_3
d^6	s	6D	p	${}^7, {}^6(PDF)$	d^6 s p
		4D	p	${}^5, {}^3(PDF)$	d^6 s p
d^7		4F	p	${}^5, {}^3(DFG)$	d^7 p
		4P	p	${}^5, {}^3(SPD)$	—

Fig 15 23 Terms predicted from eight electrons

Term	Interval $\Delta E/c\hbar$	A
5D_0	89 9	89 9
5D_1	184 1	92 1
5D_2	288 1	96 0
5D_3	415 9	104 0
5D_4		

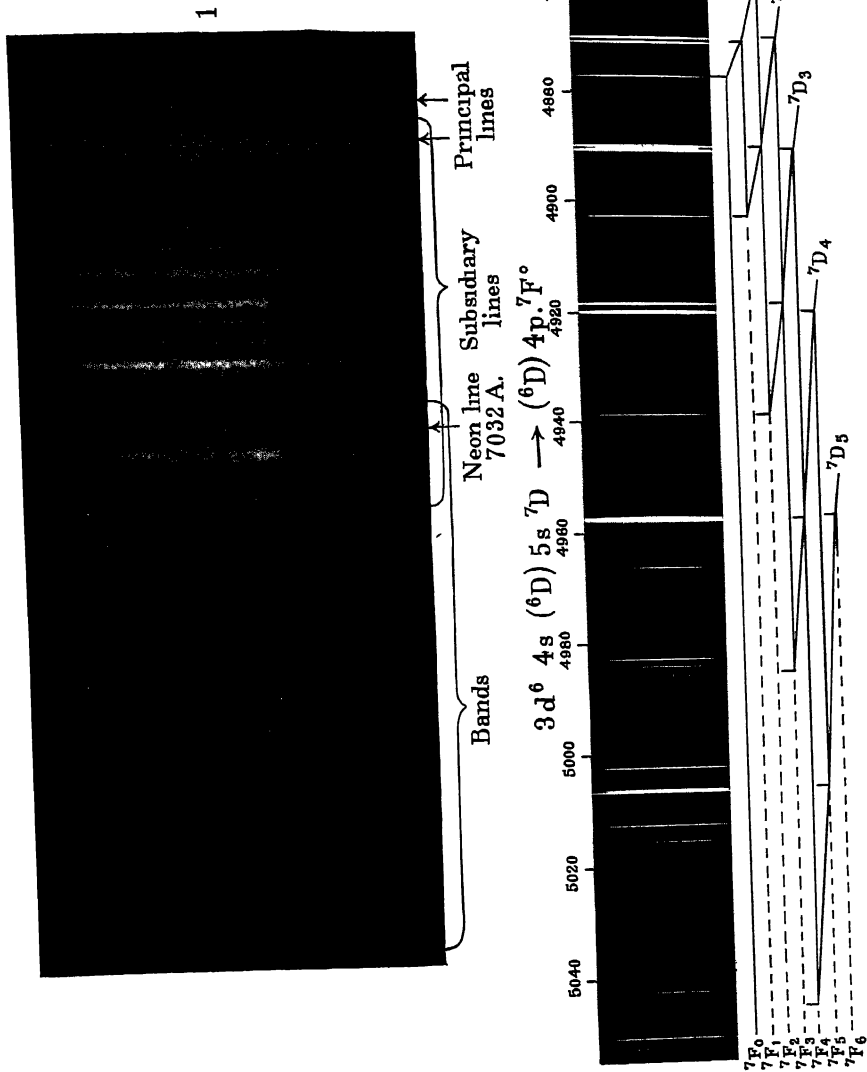
Fig 15 24 Intervals of the $d^6 s^2 {}^5D$ term of Fe I

Of the triplet levels, only four have been found, these are all inverted and obey the interval rule about as well as the 5D term Goudsmit's theory of the displacement sum predicts that the 3D term will be erect, but the term is still unidentified.

The other low configuration of the iron arc is $d^7.s$, the four terms so far identified being clearly produced by an s electron adding itself to the $d^7 {}^4F$ and 4P terms, for the derivation is in-

PLATE V

- 1 Fluorescent spectrum of chromium in Al_2O_3 at -186°C . At the right-hand or blue end of the spectrum a strong principal doublet appears, a thousand times over-exposed. Next to these are the subsidiary lines, weaker but sharp, while on the left are still weaker bands. The line near the centre is the 7032 Å of neon. (After Deutschbein, *PZ*, 1932, **33** 875.)
- 2 DF septet from the iron arc. This multiplet arises as $3d^6\ 4s(^6D)\ 5s\ ^7D \rightarrow (^6D)\ 4p\ ^7F^o$. (Lent by Prof. H. Dingle.)



licated both by the energies and the intervals of the terms. In the spark 4F lies 11,000 cm^{-1} below 4P , and in the arc 5F and 3F lie about the same distance below the corresponding 5P and 3P terms. Again, 4F obeys the interval rule well, while the derived 5F and 3F terms do not deviate much (Fig. 15.25). But the ionic 4P term is quite irregular and so are the derived terms of the arc spectrum, even the irregularities are similar (Fig. 15.26).

Term	$\Delta E/h$	I	Term	$\Delta E/h$	A	Term	$\Delta E/h$	A
6F_1	168.9	81.5	3F_2	407.6	135.9	${}^4F_{11}$	279.6	111.8
6F_2	257.7	85.9	3F_3	584.7	146.2	${}^4F_{21}$	407.7	116.5
6F_3	351.3	87.8	3F_4			${}^4F_{31}$	557.6	123.9
6F_4	488.5	97.7				${}^4F_{41}$		

Fig. 15.25 Intervals of the $d^7 ({}^4F)$ s ${}^5, {}^3F$ terms of Fe I and the $d^7 {}^4F$ term of Fe II

Term	$\Delta E/h$	I	Term	$\Delta E/h$	A	Term	$\Delta E/h$	A
6P_1	200.4	100.2	3P_0	104.9	104.9	4P_1	231.7	154.5
6P_2	176.8	58.9	3P_1	108.5	54.2	${}^4P_{11}$	198.7	79.5
6P_3			3P_2			${}^4P_{21}$		

Fig. 15.26 Intervals of the $d^7 ({}^4P)$ s ${}^5, {}^3P$ terms of Fe I and the $d^7 {}^4P$ term of Fe II

A very large number of odd terms have been identified and named in the spectra of Fe I and Co II, for the terms satisfy the simple laws, their intervals are regular, and almost without exception they are inverted. But the terms of Ru I are much less regular, so that a thorough Zeeman analysis was necessary before the terms could be named, and even now only a J value has been assigned to very many, the interval law is only roughly satisfied.

Nine electrons. Co I, Rh I, Ni II, Pd II

The ground term starts as $d^7 s^2 {}^4F$ in the arc spectrum of cobalt, changes to $d^8 s^2 {}^4F$ in the arc spectrum of rhodium and finishes as $d^9 {}^2D$ in the spark spectra of nickel and palladium,

Low terms

	Co I	Rh I	Ni II	Pd II
$3d^7 4s^2 {}^4F_{4\frac{1}{2}}$	0	12,733	—	—
${}^4P_{2\frac{1}{2}}$	15,184	—	—	—
${}^2F_{5\frac{1}{2}}$	—	—	—	—
${}^2P_{1\frac{1}{2}}$	20,501	—	—	—
$3d^8 ({}^3F) 4s {}^4F_{4\frac{1}{2}}$	3,483	0	8,393	25,081
${}^2F_{3\frac{1}{2}}$	7,442	5,691	13,549	32,278
$({}^3P) 4s {}^4P_{2\frac{1}{2}}$	13,795	9,221	25,035	36,281
${}^2P_{1\frac{1}{2}}$	18,390	11,968	29,069	43,648
$({}^1G) 4s {}^2G_{4\frac{1}{2}}$	16,468	16,018	32,498	44,506
$({}^1D) 4s {}^2D_{2\frac{1}{2}}$	16,778	13,521	23,107	41,198
$({}^1S) 4s {}^2S_{\frac{1}{2}}$	—	—	24,825	—
$3d^9 {}^2D_{2\frac{1}{2}}$	21,920	3,310	0	0

Middle terms

	Co I	Rh I	Ni II	Pd II
$3d^7 4s ({}^5F) 4p {}^6G_{6\frac{1}{2}}$	25,138	—	—	—
${}^6F_{5\frac{1}{2}}$	23,612	—	—	—
${}^6D_{4\frac{1}{2}}$	24,628	—	—	—
${}^4G_{6\frac{1}{2}}$	28,845	—	—	—
${}^4F_{4\frac{1}{2}}$	28,346	—	—	—
${}^4D_{3\frac{1}{2}}$	29,294	—	—	—
$({}^3F) 4p {}^4G_{6\frac{1}{2}}$	41,528	—	—	—
${}^4F_{4\frac{1}{2}}$	41,225	—	91,797	—
${}^4D_{3\frac{1}{2}}$	39,649	—	—	—
${}^2G_{4\frac{1}{2}}$	31,700	—	—	—
${}^2F_{3\frac{1}{2}}$	31,871	—	93,525	—
${}^2D_{2\frac{1}{2}}$	33,463	—	—	—
$3d^8 ({}^3F) 4p {}^4G_{6\frac{1}{2}}$	32,431	29,105	53,495	68,611
${}^4F_{4\frac{1}{2}}$	32,842	29,431	54,556	69,878
${}^4D_{3\frac{1}{2}}$	32,027	27,076	51,557	65,247
${}^2G_{4\frac{1}{2}}$	33,440	31,614	55,299	72,285
${}^2F_{3\frac{1}{2}}$	35,451	32,004	57,079	73,327
${}^2D_{2\frac{1}{2}}$	36,092	32,046	57,419	72,733
$({}^3P) 4p {}^4D_{3\frac{1}{2}}$	—	36,787	70,776	83,056
${}^4P_{2\frac{1}{2}}$	—	35,334	66,569	76,767
${}^4S_{1\frac{1}{2}}$	—	—	74,299	86,280
${}^2D_{2\frac{1}{2}}$	—	—	71,770	83,802
${}^2P_{1\frac{1}{2}}$	—	—	72,984	85,151
${}^2S_{\frac{1}{2}}$	—	—	74,282	85,071
$({}^1G) 4p {}^2H_{5\frac{1}{2}}$	—	—	75,719	85,593
${}^2G_{4\frac{1}{2}}$	—	—	79,923	89,982
${}^2F_{3\frac{1}{2}}$	—	—	775,916	86,043
$({}^1D) 4p {}^2F_{3\frac{1}{2}}$	—	—	67,693	79,708
${}^2D_{2\frac{1}{2}}$	—	—	68,634	82,057
${}^2P_{1\frac{1}{2}}$	—	—	68,864	80,956
$({}^1S) 4p {}^2P_{1\frac{1}{2}}$	—	—	60,502	—

Fig. 15 27 Nine electrons Energies of the low and middle terms

moreover, whereas the ground 2D term is only 8000 cm^{-1} below the next lowest term in Ni II it is $25,000\text{ cm}^{-1}$ in Pd II . These changes are in the usual order.

The energies of the low and middle terms are always related, but no group of electrons shows these relations more clearly than the group of nine here considered. In Co I the ground term is of the $3d^7 4s^2$ configuration and the lowest odd terms are derived

Low terms

Configuration	Terms	
	Quartet	Doublet
$3d^7 4s^2$	PF	PDFGH
$3d^8 4s$	PF	$^2\text{SPDFG}$
$3d^9$	—	D

Middle terms

Ion			Atom		
l_1	l_2	Term	l_3	Terms	l_1 l_2
d^8		3F	p	$^4\text{ }^2(\text{DFG})$	d^8 p
		3P	p	$^4\text{ }^2(\text{SPD})$	
d^7 s		5F	p	$^6\text{ }^4(\text{DFG})$	d^8 sp
		5P	p	$^6\text{ }^4(\text{SPD})$	

Fig. 15 28 Terms predicted from nine electrons.

from $3d^7 4s 4p$, on the other hand, when the ground term is from $d^8 s$ as it is in Rh I or from d^9 as in Ni II and Pd II , practically all the known odd terms are from $d^8 p$. Again, in Ni II the energies of the even and odd terms run strikingly parallel, of the $3d^8 4s$ configuration 2D and 2S lie exceptionally low, and this is matched among the odd terms by the anomalous positions of $^2(\text{PDF})$ and 2P from $3d^8 p$. Presumably these two irregularities have a common cause in the $d^8\text{}^1D$ and $d^8\text{}^1S$ terms of Ni III , but unfortunately the latter spectrum has not yet been analysed.

In Ni II all but one of the low even terms are inverted, but of the odd $d^8 p$ configuration eight terms are erect or only partially inverted. In Pd II the intervals are rather irregular.

Ten electrons Ni I, Pd I, Pt I, Cu II, Ag II, Au II, Cd III, Hg III, In IV, Tl IV

The ground term of all the spark and higher spark spectra is $d^{10}1S$, but the arc spectra exhibit two irregularities. Thus the ground term of nickel is $3d^8 4s^2^3F$, although the ground term of Ni II is $3d^8^2D$ and the ground term of an arc spectrum is usually obtained by adding an electron to the ground term of the spark, the only other exception to this rule seems to be Y II. Again, if the ground terms of nickel and platinum are respectively $d^8 s^2^4F$ and $d^9 s^4F$, the ground term of palladium might be expected to be one or the other of these, whereas in fact it is $d^{10}1S$.

Low terms

	Ni I	Pd I	Pt I	Cu II	Ag II	Au II
$3d^8 4s^2$ 3F_4	0	25,101	824	—	—	—
3P_2	15,610	37,952	6,567	—	—	—
1G_4	22,102	—	21,967	—	—	—
1D_2	13,521	—	26,639	—	—	—
1S_0	—	—	—	—	—	—
$3d^9 4s$ $^2D_{3/2}$	204	6,564	0	21,925	39,164	15,036
1D_2	3,410	11,722	13,496	26,261	46,045	29,618
$3d^{10}$ 1S_0	14,729	0	6,140	0	0	0

Fig 15 29 Ten electrons Energies of the low terms

Low terms

Configuration	Terms	
	Triplet	Singlet
$d^8 s^2$	PF	SDG
$d^9 s$	D	D
d^{10}	—	S

Middle terms

Ion			Atom		
l_1	l_2	Term	l_{10}	Terms	l_1 l_{10}
d^9		2D	p	$^3 1(PDF)$	d^9 p
$d^8 s$		4F 2F	p p	$^5, 3(DFG)$ $^3 1(DFG)$	d^8 sp —

Fig 15 30 Terms predicted from ten electrons

The changes in the arc spectra in passing from row to row are also instructive, in Ni I the term separations are wide and the multiplets overlap, but, though complex, the spectrum is admirably regular, all the even terms and nearly all the odd terms are inverted, and the intervals conform to Landé's rule, so simple was the analysis indeed that the Zeeman effect has not been studied. In the Pd I spectrum the wide interval, 3512 cm^{-1} , of the 2D ground term of Pd II begins to exert an effect, as some triplet series tend to the lower and some to the higher limit, the interval ratios change rapidly with the serial number. And the terms of Pt I are even less regular than those of Pd I, in analysis the interval rule is useless, and the intensity rules can be treated only as approximations, the measurements of g , too, depart from Landé's values, but they suffice with the J selection rule to determine the J value of all the empirical terms. The determination of the orbital and spin vectors is, however, far more difficult, for the failure of the simple rules is a sure sign that the coupling is no longer Russell-Saunders. However, all the low terms predicted from $d^8 s^2$, $d^9 s$ and d^{10} seem to have been identified, save only $d^8 s^2 {}^4S$, which has not yet been found in any spectrum. Names have also been assigned to some of the odd terms arising from the $d^8 sp$ and $d^9 p$ configurations.

In Cu II and Ag II the terms are generally inverted, but the interval rule is very badly satisfied, in Ag II irregular g values are also indicated.

In the analysis of Pd I there is a surplus level, known as k_1 , which is of interest chiefly as being the only level which fails to fit into the Hund scheme. The level is determined by five exact combinations, but the lines due to k_1 are all listed as diffuse and differ in this from all other lines of the palladium spectrum, the level can hardly be a hyperfine component, for no other levels show a similar structure, and the interval separating it from the nearest normal level is over 3 cm^{-1} .

Eleven electrons Cu I, Ag I, Au I, Zn II, Cd II, Hg II

As the d level can hold only ten electrons, the elements copper, silver and gold, which have eleven electrons outside the last inert

gas shell, should exhibit the simple alkali spectra provided only that d^{10} is firmly bound. In fact all three elements do exhibit doublet series and they all have a 2S ground term. But besides

Low terms

	Cu I	Ag I	Au I	Zn II	Cd II	Hg II
$3d^9 4s^2 \ ^2D_{3/2}$	11,203	—	9161	62,721	69,259	35,514
$3d^{10} 4s \ ^1S_0$	0	0	0	0	0	0

Fig 15 31 Eleven electrons Energies of the low terms

this simple system, which was discovered in very early days, the last decade has revealed in copper and gold other quartet and doublet terms, in silver these terms have not been found, though all the lines of Ag I have been classified *

Cu ⁺			Cu		
l_1	l_{10}	Term	l_{11}	Terms	
$3d^{10}$		1S	s	2S	Alkali-like systems
			p	$^2P^o$	
			d	2D	
$3d^9 4s$		$^3D \ ^1D$	4s	2D	Deep even term
		3D	4p	$^4(PDF)$ $^2(PDF)$	Middle group of odd terms
		1D	4p	$^2(PDF)$	
		3D	ns	$^4D \ ^2D$	High even terms
			nd	$^4(SPDFG)$ $^2(SPDFG)$	
		1D	ns	2D	
			nd	$^2(SPDFG)$	

Fig 15 32 Terms predicted in copper

The lowest term of Cu II is $d^{10}1S$, and from it arise by the addition of s, p and d electrons the alkali-like system, from the 3D and higher 1D terms of Cu II arise all the other known terms, those predicted are shown in Fig 15 32, while those actually found are

* Blair, PR, 1930, 36 1531

The contrast between copper and gold on the one hand and silver on the other appears not only in their spectra but also in their chemical properties. Both copper and gold can be monovalent, but copper is commonly divalent and gold may be trivalent, showing that the lower electron group is not as firmly established as in the alkali metals, silver on the other hand is always monovalent.

4. The three rows compared

The structure of a spectrum is primarily a function of the number of electrons, but it is also affected by the relative positions of the low terms, for they will determine which multiplets appear bright and which faint. Usually the ground term changes from frame to frame, and when this happens in a spark spectrum, even the prominent multiplicities of the arc spectrum change. In the Fe I spectrum triplets, quintets and septets are known, but in the homologous Ru I only triplets and quintets, Co I has doublets, quartets and sextets, but Rh I only doublets and quartets, Ni I has singlets, triplets and quintets, but Pd I only singlets and triplets. And the reasons for these differences are not far to seek, the ground term of Fe II is $d^6 s^6 D$, but of Ru II $d^7 {}^4 F$, and if the ground terms of Ni II and Pd II are identical, as presumably are also the ground terms of Co II and Rh II, though Rh II has not yet been analysed, the $d^8 s^4 F$ term lies so much lower in Ni II than in Pd II that terms derived from it appear in one arc spectrum and not in the other.

Since the energies of the three low configurations are of such importance, they are worth detailed consideration. Relative energies alone are known, and we therefore elect to consider the energies of the $s^2 d^{n-2}$ and d^n configurations relative to $s d^{n-1}$, since the last has been identified in more spectra than either of the other two. Many figures are still missing from the tables, but a fair sequence of $s^2 d^{n-2}$ is available in the arc spectra of the iron and palladium rows, while rather less complete sequences for d^n are found in the spark spectra of the same rows, accordingly $(s^2 d^{n-2} - s d^{n-1})$ and $(s d^{n-1} - d^n)$ are plotted against n for the arc (Fig 15 34) and spark (Fig 15 35) spectra respectively. That

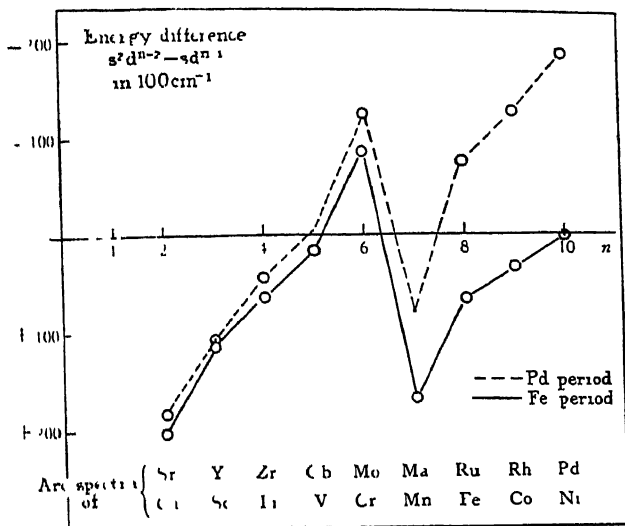


Fig. 15 34 The energy difference of the $s^2 d^{n-2}$ and $s^2 d^{n-1}$ configurations in the arc spectra of the iron and palladium frames n is the number of electrons outside the inert gas shell. Due to a slip in drafting the energy scales of this and the two succeeding figures read down instead of up.

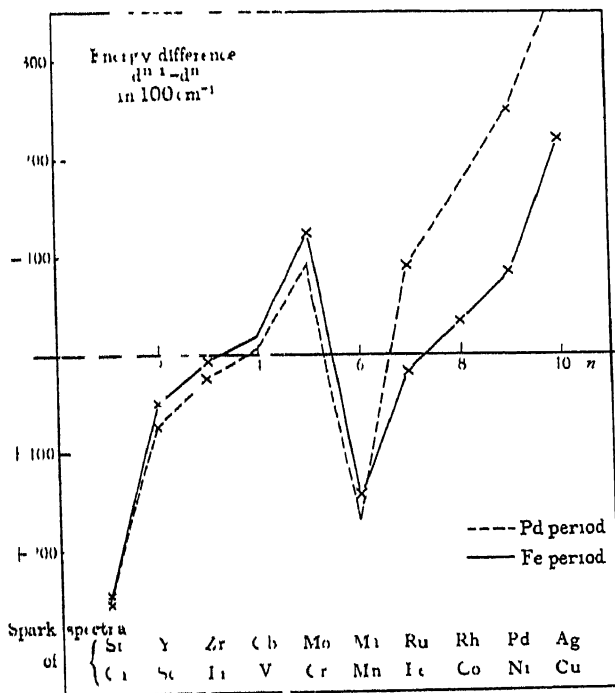


Fig. 15 35 The energy difference of the $s^2 d^{n-1}$ and d^n configurations in the spark spectra of the iron and palladium frames

the curves for the iron and palladium rows would resemble one another might have been anticipated, but the resemblance between the ($s^2 d^{n-2} - s d^{n-1}$) and ($s d^{n-1} - d^n$) differences is altogether surprising, true the first curve is displaced a step to the right relative to the second, but closer examination reveals that

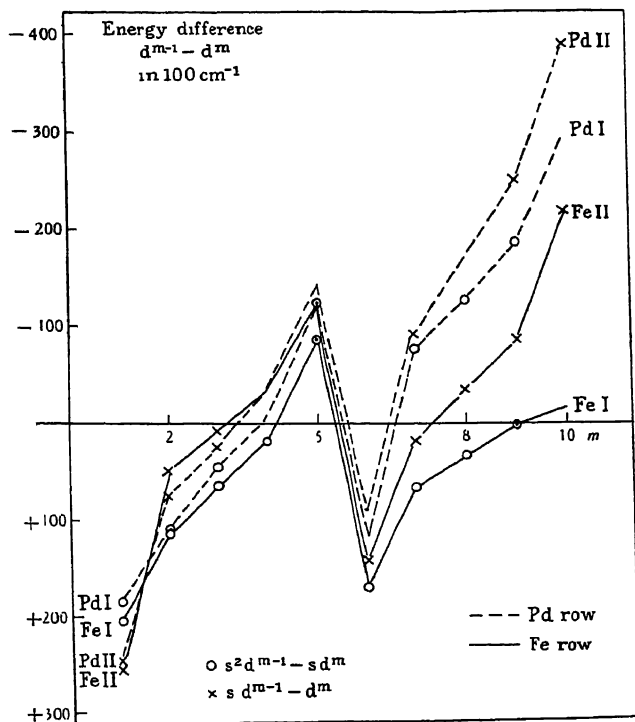


Fig 15 36 The energy difference of the d^{m-1} and d^m configurations m is here $n-1$ in the arc and n in the spark spectra, n is still the number of electrons outside the inert gas shell

the ($s^2 d^{n-2} - s d^{n-1}$) curve for the iron row arc actually resembles the ($s d^{n-1} - d^n$) curve of the iron row spark more closely than it resembles the ($s^2 d^{n-2} - s d^{n-1}$) curve of the palladium row arc. The fourfold magnification which occurs in all simple spectra does not affect these low terms at all, though it does still affect the ionisation potentials.

To make these facts stand out, write $m=(n-1)$ in the $(s^2 d^{n-2} - s d^{n-1})$ difference and $m=n$ in $(s d^{n-1} - d^n)$, and then plot both the differences $(s^2 d^{m-1} - s d^m)$ and $(s d^{m-1} - d^m)$ against m (Fig 15 36) That the four curves are so very similar can only mean that they all measure essentially the difference between the configurations d^{m-1} and d^m , the number of s electrons being of little moment

The regularity thus revealed enables us to fill up certain gaps in our knowledge, thus in the palladium row, there now seems little doubt that the ground terms of Ma I, Ma II and Rh II—the

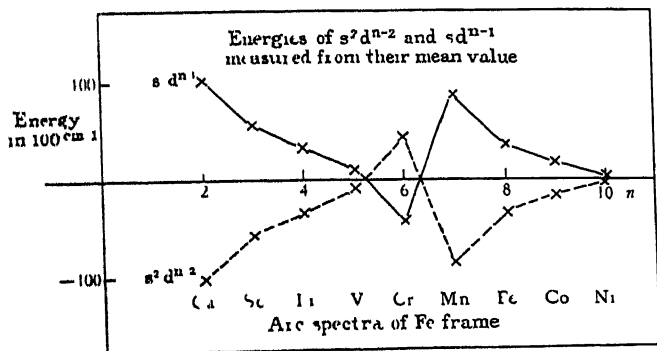


Fig 15 37 Arc spectra of the iron frame The energies of the $s^2 d^{n-2}$ and $s d^{n-1}$ configurations measured from their mean value

only spectra whose ground terms are not known—must be $d^5 s^2 {}^6S$, $d^5 s^2 {}^4S$ and $d^8 {}^1F$ respectively, while if in the platinum row the ground term of W I is 5D as has usually been supposed, then it must arise from the $d^4 s^2$ configuration and probably the ground configuration of every arc spectrum from Lu I to Os I is of the $d^n s^2$ type The supposed 2D ground term of iridium must be accepted with extreme reserve, for it would necessitate a d^9 configuration in the normal atom

The similarity of the four $(d^{m-1} - d^m)$ curves further excites a desire to explain at least their more striking features, and in fact Hund has already done this, for if the energies of the $s^2 d^{n-2}$ and $s d^{n-1}$ configurations are measured from their mean value (Fig 15 37), the sharp rise of the $(d^{m-1} - d^m)$ curves at $m=5$ and

the precipitate drop at $m = 6$ are revealed as both due to the low energy of the d^5 configuration compared with either d^4 or d^6

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As for the short periods, the only systematic account seems to be by Hund in *Linienspektren und periodisches System der Elemente*, 1927

The values of energy levels are taken largely from Bacher and Goudsmit, *Atomic Energy States*, 1932 Other statements about particular elements are based on the select bibliography appearing in Appendix vi

CHAPTER XVI

THE RARE EARTHS

1 In the periodic system*

Bohr's theory explains the intrusion of fourteen rare earths into the sixth period as due to the filling of a shell of 4f electrons. Now Bohr's theory also explains the increase in metallic properties which occurs in descending a column of the table as due to the valency electrons occupying orbits of successively higher quantum numbers, while the decrease in metallic properties which occurs in passing across the table from left to right he ascribed to the firmer binding of the electrons as the nuclear charge increases. If these predictions are general, they should be valid in the rare earth elements. Yttrium should be more metallic than scandium, and lanthanum than yttrium, but in passing through the rare earths from lanthanum to lutecium the elements should grow steadily less metallic.

The firmness with which the valency electron is bound is best measured by the ionisation potential, but for most of the rare earths the potential has only been estimated from the conductivities of the oxides in a flame,[†] and many have felt that the values are not so sure as those obtained by more direct methods. In the last two years this view has been confirmed by the analysis of the spectra of lanthanum and cerium, the ionisation potentials thus obtained are 5.59 and 6.54, which compare with the flame values of 5.49 and 6.91 volts.

For confirmation turn first to the molecular volume of homologous compounds, and then to two chemical reactions. The more firmly the valency electrons are bound, the smaller should be the volume of the compound, and in fact this prediction is fulfilled in the sesqui-oxides and sulphates. The sesqui-oxides of the rare earths exist in three crystalline forms, which Goldschmidt named *A*, *B* and *C*. *A* is stable at high temperatures, *C* at low, but the

* Von Hevesy, *Die seltenen Erden*, 1926, 21 f.

† Rolla and Piccardi, *PM*, 1929, 7.286, and Fig. 10.13 of Volume I.

transition temperature changes from element to element, rising from lanthanum to lutecium, so that at room temperature the form *A* is stable in lanthanum and the form *C* in lutecium. The hexagonal crystal *A* has been measured in four elements between lanthanum and neodymium, the pseudo-trigonal crystal *B* in four between neodymium and gadolinium, and the regular crystal *C* in scandium, yttrium and in all the elements which succeed samarium (Fig 16 1). These measurements show that in the sixth

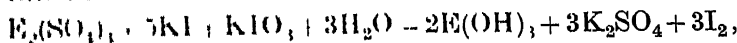
	Sc	Y	La	Ce	Pr	Nd	Il	Sm	Eu
Sesquioxide $\left\{\begin{array}{l} A \\ B \\ C \end{array}\right.$	— — 35 53	— — 45 13	50 28 — c 57	47 89 — —	46 65 — —	46 55 c 51' —	— — —	— 46 9 48 38	— 46 5 48 28
Sulphate octahydrate Element	— —	240 8 —	— 22 6	— 21 0	253 9 21 8	252 4 20 7	— —	247 9 —	247 3 —
	Gd	Tb	Dy	Ho	Er	Tu	Yb	Lu	
Sesquioxide $\left\{\begin{array}{l} A \\ B \\ C \end{array}\right.$	— c 43 47 58	— — 46 38	— — 45 49	— — 44 89	— — 44 38	— — 44 11	— — 42 5	— — 42 25	
Sulphate octahydrate Element	246 4 —	— —	242 8 —	241 1 —	239 3 —	— —	235 1 19 8	234 7 —	

Fig 16 1 Molecular volumes of some rare earth compounds

period the molecular volume of all three crystal forms decreases as the atomic number increases, while in travelling from scandium through yttrium to lanthanum there is a considerable expansion. These two variations are in the directions predicted by Bohr, together they bring yttrium out with very nearly the same atomic volume as holmium, an interesting coincidence, as a comparison of chemical properties would assign yttrium the same place in the rare earth sequence, the four elements which succeed holmium are thus actually less metallic than yttrium, though still much more metallic than scandium.

When the rare earth sulphates crystallise as the octahydrates, all except cerium are isomorphous. Their densities, which have been measured by Auer von Welsbach, show that once again the molecular volume decreases in passing from praseodymium to lutecium, and once again yttrium appears next to holmium.

There are also chemical methods by which the decreasing basicity may be demonstrated. If one measures the iodine liberated in the reaction



increase in the iodine is a sign of decreasing basicity. The order in which this reaction places the elements is identical with that obtained from the molecular volumes, while the difference between samarium and europium is abnormally small here as there

Another method is to warm a solution of the sulphate with an exactly equivalent weight of sodium carbonate, and to measure the rate at which carbonic acid is liberated. The order obtained is again the same as that of the molecular volumes. Pr, Nd, Sm, Eu, Gd, Tb, Dy, Y, Tu, Yb.

2. Valency

If the periodic system was based solely on chemical grounds, there would be no choice but to crowd all the fifteen elements between lanthanum and lutetium into column III, elsewhere in the periodic system the valency changes by one when the atomic number changes by one, but all fifteen rare earths are trivalent.

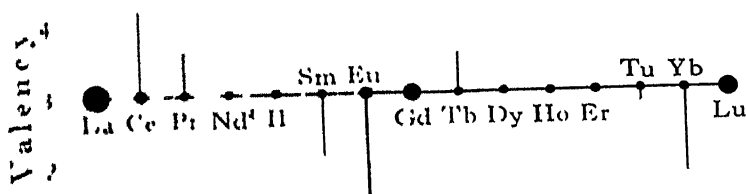


Fig. 16 2. Valencies of the rare earths, all are trivalent, and the size of the point shows the relative stability of the ion, Me^{III} , the appearance of quadrivalent compounds is shown by a line running up, and of divalent compounds by a line running down, the lengths of these lines give a rough measure of stability (After Jantsch and Klemm, *Z. f. anorg. u. allg. Chem.* 1933, **216** 80)

and so similar in other ways that chemists have been able to separate them from one another only by such laborious methods as fractional crystallisation of their salts and fractional decomposition of their nitrates. Six of the rare earths however form compounds, in which they exhibit a second valency. Cerium, praseodymium and terbium can all be quadrivalent, samarium, europium and ytterbium divalent (Fig. 16 2)

Five of these deviations can be linked up with the theories of atomic structure developed in earlier chapters. The more electrons a shell contains the less willing is it to part with one of them, sodium is more reactive than magnesium, and aluminium than silicon, if then the 4f shell obeys the same laws, it should be easier to remove an electron from cerium in which only one is present than from succeeding elements in which there are several. And in fact the first two elements are quadrivalent, while cerium becomes quadrivalent much more readily than praseodymium, for compounds of the latter readily oxidise cerous compounds to ceric, further, only one quadrivalent compound of praseodymium has been isolated pure, whereas a whole series of ceric compounds are known.

The divalency of europium and the quadrivalency of terbium are due to quite another cause. Both the p and d shells show that they are more stable when they are just half full than when they contain one electron more or less, nitrogen with three p electrons has a higher ionisation potential than either carbon or oxygen, while the examination of the low terms of the f elements, carried out in the last chapter, shows that the d^5 configuration is more stable than d^4 or d^6 , the difference is not so great as between d^9 and d^{10} , but the evidence is too clear to admit of doubt. Now the f shell is half full in the Gd^{3+} ion, which contains seven electrons, and one might reasonably expect the elements on either side of gadolinium to try to assume this configuration, europium which precedes gadolinium by keeping an extra electron and being divalent instead of trivalent, and terbium which succeeds gadolinium by parting with an extra electron and becoming quadrivalent.

The divalency of ytterbium exhibits the tendency to form the closed shell of fourteen f electrons, but the divalency of samarium remains unexplained, for there is no reason to think the f^6 configuration much more stable than the f^5 , in the d shell d^4 is more stable, but not much more stable, than d^3 . But we must not make too much of this failure, for $SmCl_2$ is definitely less stable than $EuCl_2$.

3. Arc and spark spectra

The paramagnetic susceptibilities of the rare earths have already been cited as strong evidence that in the trivalent ions, which occur in crystals and in solution, all electrons outside the xenon core occupy f orbits. But to obtain evidence of the structure of the elements themselves, appeal must be had to the arc and spark spectra. These exhibit an exceptional number of lines, in the spectrum of dysprosium, for example, over 3000 have been measured. This naturally makes the identification of small amounts of the rare earths very difficult, for if one finds the weak yttrium line 3468.0 \AA , one cannot distinguish it from four other lines, 3467.4 \AA of Gd, 3467.8 \AA of Cd, 3468.2 \AA of Tb or 3468.4 \AA of Th, unless one can measure the wave-length to a few tenths of an angstrom. To surmount this difficulty spectroscopists have been driven to use the 'residual lines', that is, the lines which are the last to fade when the proportion of the element in the mixture is steadily reduced. For example, if the line measured as 3468.0 \AA . is really an yttrium line, then the strongest yttrium line 3710.3 \AA . must appear much stronger than 3468.0 \AA . on the same plate; moreover, as 3710.3 \AA . is a residual line, it must be the last to vanish as the material examined is diluted.

As long ago as 1922 Bohr* stated that the atom in which an f electron first appears is cerium, and that the f shell is full in ytterbium, this statement he based on chemical properties and on a mathematical comparison of the stabilities of alternative orbits. Since then many spectra of elements lying just before and just after the rare earths have been analysed, while within the rare earth frame the low terms of Ce I, Sm I, Eu I and Gd I are now known †.

The fifty-fifth electron occupies the $6s$ shell in Cs I and Ba II, and sinks into a $5d$ orbit in La III, on Bohr's authority it was commonly expected to sink into a $4f$ orbit in Ce IV, but when the

* Bohr, *Theory of Spectra and Atomic Constitution*, 1922, 110

† Recent analyses of rare earth spectra are Ce I, Karlson, *ZP*, 1933, 85 482; Ce III, Kaha, *Indian Journ. Phys.*, 1933, 8 137, Eu II, Albertson, *PR*, 1934, 45 499a, Eu I, Russell and King, *PR*, 1934, 46 1023, Sm I and Gd I, Albertson, *PR*, 1935, 47 370

Ce IV spectrum was analysed the ground term was found to be 2D * Thereafter it seemed rather improbable that any of the three electrons required to produce neutral cerium would enter a 4f orbit, but apparently Bohr was correct after all, for the ground term of Ce I is 3H and this arises in the 4f 5d 6s² configuration

No of electrons	Atom	Ground term	Configuration
1	Cs I Ba II La III Ce IV	2S 2S 2D 2F	s s d f
2	Ba I La II Ce III	1S 3F 3F	s ² d ² d ²
3	La I	2D	d s ²
4	Ce I	3H	f d s ²
8	Sm I	7F_0	f ⁶ s ²
9	Eu II	8S_4	f ⁷ (8S) s
10	Eu I	$^8S_{3\frac{1}{2}}$	f ⁷ s ²
	Gd I	6D_2	f ⁷ d s ²
15	Tu I Yb II Lu III	2S	f ¹⁴ s
16	Yb I Lu II	1S	f ¹⁴ s ²
17	Lu I Hf II	2D 2D	f ¹⁴ d s ² f ¹⁴ d s ²

Fig 16.3 Ground terms of the arc and spark spectra with the configurations in which they arise

At the other end of the rare earth frame the ground terms of Lu III and Lu II are 2S and 1S respectively, showing that the 4f shell is complete when the nuclear charge is 71, for these terms must arise from the configurations f¹⁴ s and f¹⁴ s², but as the f shell is bound to grow more stable, when the nuclear charge increases, like the d shell of preceding periods, these results do not prove that the f shell is complete in ytterbium. Thus the spectra which have been analysed since 1922 are not inconsistent with Bohr's hypothesis, but they have not yet banished doubt

* Gibbs and White, *PR*, 1929, 33 157. Lang has since found a lower 2F term *PR*, 1936, 49 552a

4 Absorption spectra

As there are no specific chemical tests for any of the rare earths except cerium, the chemist relies on optical tests instead. Of these at least six are available, the arc and spark spectra, the absorption and phosphorescent spectra of a crystal or solution, and X-ray lines and absorption edges. Any of these can be used to identify an element, but of them all the absorption spectrum is often the simplest, for many of the trivalent ions are brightly

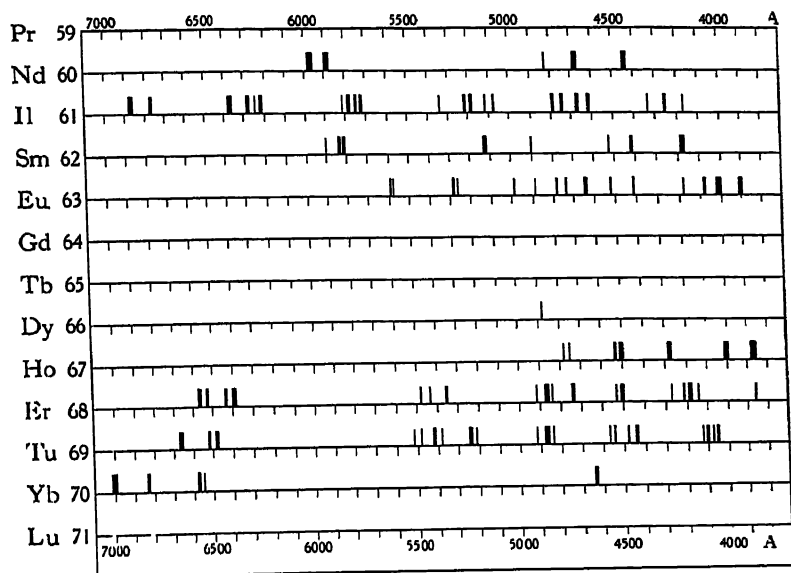


Fig 16.4 Absorption spectra of the rare earth ions (After Hevesy, *Die seltenen Erden*, 1926, 39)

coloured. Praseodymium is green, neodymium red-violet, samarium yellow and element 61 probably yellowish green, dysprosium and holmium are both yellow, erbium is rose and thulium green, of the rest, four, cerium, gadolinium, ytterbium and lutetium, are quite colourless, while europium and terbium show little trace (Fig 16.4). Thus the elements near the ends of the block, and those round gadolinium, show little or no colour, this can be explained as depending on the depth of the ground term. Thus the 8S ground term of the gadolinium ion, calculated by

Hund and confirmed by the paramagnetic susceptibility, should be peculiarly stable, since it arises in an f^7 configuration, and in fact the absorption bands of gadolinium lie in the ultra-violet

When the absorption spectrum is more carefully examined, it is found to consist of surprisingly narrow bands, many indeed are only 1 or 2 Å wide, whereas the absorption bands of the coloured ions of earlier periods often cover 100 Å or more, indeed the rare earth bands are better described as 'lines more or less diffuse' than as bands. Thus if one dilutes the solution of a rare earth salt until all trace of colour disappears, the stronger absorption lines still appear in the spectroscope, though if a solution of potassium permanganate is similarly diluted, the lines disappear with the colour, and this difference is to be attributed to the sharpness of the rare earth bands.

Do these bands arise in the molecule like the bands of a gas in a discharge tube, or are they atomic lines broadened by the varying fields of a crystal? In the band spectra of compounds, and especially of complex ions such as uranyl, homologous groups of lines recur at regular intervals, being due to atoms oscillating within the ion, but in the crystals of the rare earths no group of lines recurs. Again, the rare earth absorption lines split in a magnetic field, though the band spectra of uranyl compounds do not *. Moreover, at temperatures as low as 17° A. some are still bright, though a molecule could not vibrate at so low a temperature and could not therefore absorb radiation. True, the intensities of the lines change as the temperature falls, for while some remain bright, others fade, but the lines which fade are easily explained as arising from levels above the ground state, levels in which the Boltzmann distribution allows very few atoms at low temperatures †

The positions of the bands are largely independent of the anion and are the same in the solid as in solution. This alone is strong evidence that the lines are atomic in origin, and arise in a shell

* Becquerel, *le Radium*, 1907, 4, 328, *K. Akad. Amsterdam, Proc.* 1929, 32, 749.

† Becquerel, *Livre jubilaire de Kamerlingh Onnes*, 1922, 288, Ehrenfest, *ibid.* 362.

Freed and Spedding* worked first with the chloride, $\text{GdCl}_3 \cdot 6\text{H}_2\text{O}$, which crystallises in colourless plates. The spectra were taken along the principal axis, that is, perpendicular to the faces, but tests made perpendicular to the principal axis showed that within the limits of measurement the spectrum is independent of the direction in which the light passes. Moreover, in solutions of varying concentration, photographs show that the positions and general spacing of the multiplets are similar to that found in the crystal, except that the lines are rather more blurred and the multiplets are shifted slightly towards higher frequencies.

When the absorption spectrum of the chloride is compared with that of other gadolinium salts, the negative ion is seen to produce little change in the positions of the multiplets, though it changes the number of lines and their positions within the multiplet. Accordingly, the multiplets must arise from electronic transitions of the Gd^{3+} ion, but the splitting of these levels must depend on the forces exerted by the surrounding atoms. These forces seem to depend more on the crystal symmetry than on the negative radical, the spectra of monoclinic $\text{Gd}_2(\text{SO}_4)_3 \cdot 8\text{H}_2\text{O}$, $\text{GdCl}_3 \cdot 6\text{H}_2\text{O}$ † and $\text{GdBr}_3 \cdot 6\text{H}_2\text{O}$ ‡ are almost identical, but different from hexagonal $\text{Gd}(\text{C}_2\text{H}_5\text{SO}_4)_3 \cdot 9\text{H}_2\text{O}$ and $\text{Gd}(\text{BrO}_3)_3 \cdot 9\text{H}_2\text{O}$, while work on the triclinic acetate§ suggests still a third type, but only one band of the acetate has been examined, so it is perhaps unwise to generalise.

As the temperature is reduced little change appears in the spectrum, true the multiplet intervals increase slightly, and the whole spectrum shifts slightly towards the red, but this movement averages only some 4 cm^{-1} in a change from the laboratory to liquid hydrogen. These two changes are primarily due to the contraction of the crystal, which brings the ions closer together, and therefore makes the electric field more intense.

* Freed and Spedding, *PR*, 1929, **34** 945 $\text{GdCl}_3 \cdot 6\text{H}_2\text{O}$, $\text{Gd}_2(\text{SO}_4)_3 \cdot 8\text{H}_2\text{O}$

† Spedding and Nutting, *PR*, 1931, **38** 2294a

‡ Spedding and Nutting, *Am Chem Soc J* 1930, **52** 3747 $\text{GdBr}_3 \cdot 6\text{H}_2\text{O}$

§ Spedding and Nutting, *Am Chem Soc J* 1933, **55** 503 $\text{Gd}(\text{C}_2\text{H}_5\text{SO}_4)_3 \cdot 9\text{H}_2\text{O}$ and $\text{Gd}(\text{BrO}_3)_3 \cdot 9\text{H}_2\text{O}$

6 Samarium

At room temperature the absorption spectrum of samarium consists of diffuse lines and bands, lying chiefly between 3000 and 5000 Å. As the temperature is reduced the lines sharpen, until when the crystal is in liquid hydrogen the lines are very fine, the lines also change in intensity, and this change is much more striking in samarium than in gadolinium, at low temperatures some lines disappear, while new lines make their appearance*, and this is to be expected, since Hund's theory makes the ground term f^5H , and at low temperature lines arising in the higher components of this term must be very weak.

The Boltzmann distribution law indeed makes it certain that any absorption lines which appear below room temperature must arise in a level lying less than 500 cm^{-1} above the ground level. The visible spectra of crystals of chloride and bromate thus arise between half a dozen levels lying below 500 cm^{-1} and other levels lying between $17,000$ and $27,000\text{ cm}^{-1}$. In order to follow the changes in intensity more closely, the absorption spectrum of the chloride, $\text{SmCl}_3 \cdot 6\text{H}_2\text{O}$, was photographed at five temperatures between 293° and 20°A , first with a single crystal and then with powdered crystals†, the latter method brings out the weak lines, though it blurs the strong, as multiple internal reflection lengthens the path. Comparison of these photographs suggests the division of the lines into two groups, one consists of lines which increase in intensity as the temperature is lowered, many only appearing when the temperature has already fallen to -195°C , while in the other group the lines decrease in intensity as the temperature falls, many being absent at 20°A . These two groups are conveniently referred to as 'low temperature lines' and 'high temperature lines'. Many of the fainter low temperature lines appear on the violet side of a multiplet, while the high temperature lines seem to congregate on the red side.

The intensities of both groups of lines depend in part on the populations of the lower levels, and this in turn is governed by

* Freed and Spedding, *N*, 1929, 123 526

† Spedding and Bear, *PR*, 1932, 42 58, 76 $\text{SmCl}_3 \cdot 6\text{H}_2\text{O}$, single crystal and powdered crystal

Boltzmann's law, if three low levels exist, separated by intervals of 150 cm^{-1} , and all are of equal weight, the numbers of atoms in these levels at 20° and 78°A are those given in Fig 16 5 Thus in

Level Temp	0 cm^{-1}	150 cm^{-1}	300 cm^{-1}
20°A	1	$8.3 \cdot 10^{-6}$	$6.9 \cdot 10^{-11}$
78°A	1	$2.4 \cdot 10^{-2}$	$5.7 \cdot 10^{-4}$

Fig 16 5 Normal fraction of atoms existing at any time in three low levels

liquid hydrogen the number of ions lying in levels above the ground level is small, and there seems little doubt that the low temperature lines may be attributed to the ground level, while the high temperature lines arise in a group of levels lying between 100 and 300 cm^{-1}

These predictions are very satisfactorily confirmed by a search for constant intervals between lines of the low and high temperature groups (Fig 16 6) In the crystals of $\text{SmCl}_3 \cdot 6\text{H}_2\text{O}$

Low temperature line		High temperature line	$\Delta\nu\text{ cm}^{-1}$
$\lambda\text{ A}$	$\nu\text{ cm}^{-1}$	$\nu\text{ cm}^{-1}$	
5592.6	17875.8	17730.6	145.2
5582.5	17908.0	17763.1	144.9
4988.2	20042.5	19897.0	145.5
4899.6	20404.5	20259.0	145.5
4513.2	22151.8	22006.0	145.8
5592.6	17875.8	17716.3	159.5
5582.5	17908.0	17748.6	159.4
4988.2	20042.5	19882.7	159.8
4899.6	20404.5	20246.0	158.5
4866.0	20544.8	20386.0	158.8

Fig. 16 6 Doublet intervals of 145 and 159 cm^{-1} found in the absorption spectra of $\text{SmCl}_3 \cdot 6\text{H}_2\text{O}$ at low temperatures.

these differences reveal levels at $145, 160, 204, 217$ and 300 cm^{-1} above the ground level. Some of these levels are probably complex, the 300 cm^{-1} level in particular consisting perhaps of components at 295 and 315 cm^{-1} , for the spread varies with the

PLATE VI

1 Single crystal absorption spectrum of $\text{SmCl}_3 \cdot 6\text{H}_2\text{O}$ at four different temperatures. The photographs were taken at the temperatures shown on the left, these being the boiling points of the substances shown on the right. All the lines grow sharper as the temperature is reduced, but the intensity may increase or decrease.

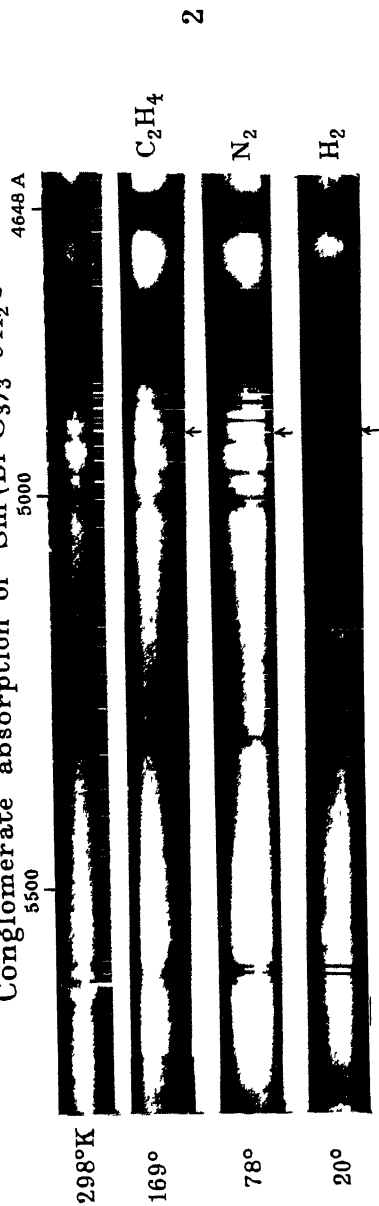
2 Conglomerate absorption spectrum of $\text{Sm}(\text{BrO}_3)_3 \cdot 9\text{H}_2\text{O}$ at four different temperatures. A conglomerate or mass of small crystals has a longer optical path than a single crystal, so that it brings out the weak lines, but it blurs the stronger multiplets.

(Photographs lent by Prof. F. H. Spedding.)

Single crystal absorption of $\text{SmCl}_3 \cdot 6\text{H}_2\text{O}$



Conglomerate absorption of $\text{Sm}(\text{BrO}_3)_3 \cdot 9\text{H}_2\text{O}$



height of the level from 2 to 30 cm^{-1} , and these involve errors rather greater than might be reasonably expected

Turning again to the photographs, the lines which originate in the 300 cm^{-1} level are found entirely absent at -195°C , while those arising in the 204 and 217 cm^{-1} levels fade rapidly as the temperature is further reduced

Examined by the same methods hexagonal crystals of the bromate, $\text{Sm}(\text{BrO}_3)_3 \cdot 9\text{H}_2\text{O}$, behave very much like the crystals of the chloride until the temperature falls to 78°A , the shift in position and widening of the multiplets may perhaps be slightly

Tem- perature	Low tempera- ture line	Satellite A		$\Delta\nu_A$	Satellite B		$\Delta\nu_B$
78° A	17847	17809	38	17783	64		
	18857	18817	40	18790	67		
	19987	19949	38	19920	67		
	20382	20342	40	20315	67		
	28125	28087	38	28055	70		

Tem- perature	Low tempera- ture line	A I	$\Delta\nu_{A I}$	A II	$\Delta\nu_{A II}$	B I	$\Delta\nu_{B I}$	B II	$\Delta\nu_{B II}$
20° A	17847 4 (II)	—	—	17802 5	44 9	—	—	17780	67
	17857 2 (II)	—	—	18812	45	—	—	—	—
	17949 (I)	17912	37	—	—	17867	82	—	—
	19985 6 (I II)	19949 7	35 9	19941	45	19904	82	19919	67
	20385 9 (I)	20348 9	37 0	—	—	—	—	—	—
	28125 2 (II)	—	—	28080	45	—	—	—	—

Fig 16 7 Low temperature lines and their high temperature satellites in the absorption spectrum of $\text{Sm}(\text{BrO}_3)_3 \cdot 9\text{H}_2\text{O}$, this shows how the energy levels split as the temperature is reduced from 78° to 20°A

greater, but only slightly, between 78° and 20°A however most of the lines split into two components, of which the red one is almost certainly complex (Fig 16 7) Thus above 78°A the temperature variation of the lines and a search for constant intervals reveals levels at 39 and 68 cm^{-1} above the ground level, with perhaps other levels between 100 and 230 cm^{-1} , but at a temperature of 20°A the levels revealed are at 0, 37, 45, 67 and 82 cm^{-1} (Fig 16 8)

Moreover, the transitions observed at 20°A suggest that these levels form two independent sets, 0, 37, 82 combine with one group of high levels, and 0, 45, 67 with another (Fig 16 8) That

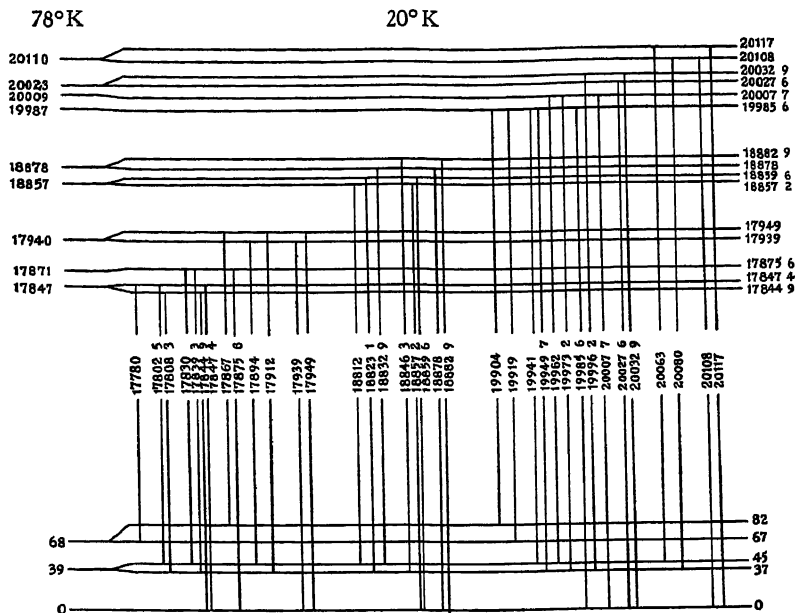
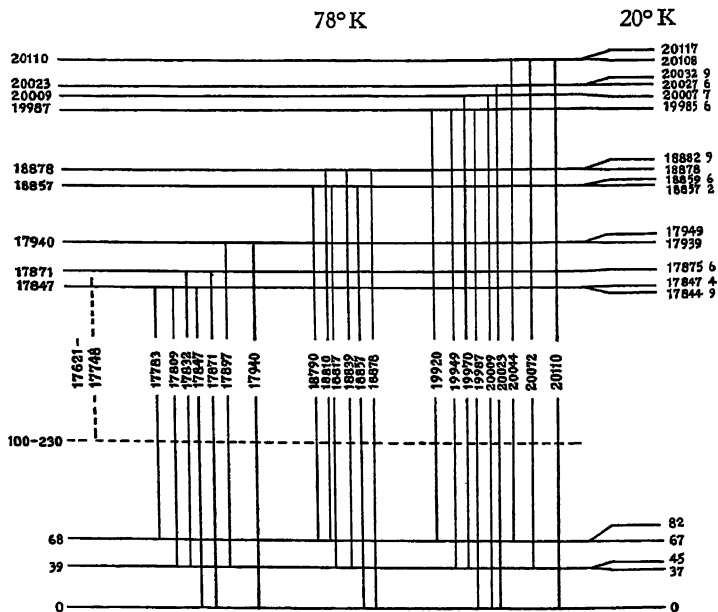


Fig 16 8 Energy levels of $\text{Sm}(\text{BrO}_3)_3 \cdot 9\text{H}_2\text{O}$ at 20°A , with the lines arising in transitions between them. The figures are wave numbers (After Spedding and Bear, *PR*, 1933, 44 290)



the ground term appears in both groups means nothing, for in fact the method, by which the splitting of the low levels is calculated, automatically reduces any real interval to zero, if all the low levels developed the same interval, one would be free to ascribe this splitting entirely to the high levels *

The existence of more than three low levels in the samarium salts makes it probable that in Sm^{3+} there is a second electronic level lying close to the ${}^6\text{H}_{5/2}$ term predicted by Hund, for in an electric field this term splits into only three components, and this fits in well with the magnetic susceptibility which does not agree with the value predicted, if ${}^6\text{H}_{5/2}$ is the only low term †

BIBLIOGRAPHY

The last three sections of this chapter should be read with chapter xxii Spencer, J F, *The metals of the rare earths*, 1919, is a very thorough study with full references, but it was written before Bohr had outlined the electronic structure of the elements Hevesy, *Die seltenen Erden*, 1926, makes full use of Bohr's theory

* Spedding and Bear, *PR*, 1933, 44 287 $\text{Sm}(\text{BrO}_3)_3 \cdot 9\text{H}_2\text{O}$

† Spedding, *Am Chem Soc J* 1932, 54 2593

CHAPTER XVII

INTENSITY RELATIONS

1 Experimental

For rough estimates of intensity the spectroscopist has often relied on his eye, but the eye is subjective and far from accurate, so that in recent years much attention has been paid to methods of estimating the density of a photographic plate. In general if the intensity of one component of a multiplet is expressed as a percentage of the brightest line, then these methods ensure that the percentage is correct to the nearest integer, but this statement is subject to a few restrictions, of which the most important is that the lines must not be too far apart, for no one knows quite how the sensitivity of a photographic plate varies with wavelength.

This is not the place to indulge in a description of experimental procedure, especially as it is fully described elsewhere,* but the wedge method may be briefly reviewed as illustrating the chief points of interest. As the density of a photographic image is not proportional to the length of exposure or the intensity of the incident light, but shows an initial lag (Fig 17 1), one is not justified in comparing two densities and then saying that the intensities must have been in the same ratio. Instead, one may only say that if the density at one point is equal to that at another, then the intensities were also equal. The word 'density' is here a technical term, being defined as

$$\log \left\{ \frac{\text{intensity of incident light}}{\text{intensity of transmitted light}} \right\}$$

This means that a scale ought to appear on each plate, and one way of obtaining this is to photograph a wedge of dull grey glass, the density will then be proportional to the distance from the edge of the wedge, and this may be measured by a micrometer fixed to the microscope. Thus one may say that the density of a

* Dobson, Griffith and Harrison, *Photographic photometry*, 1926

certain line is equal to that at a certain place on the wedge, and the latter may be measured on a scale which is a linear scale of intensity

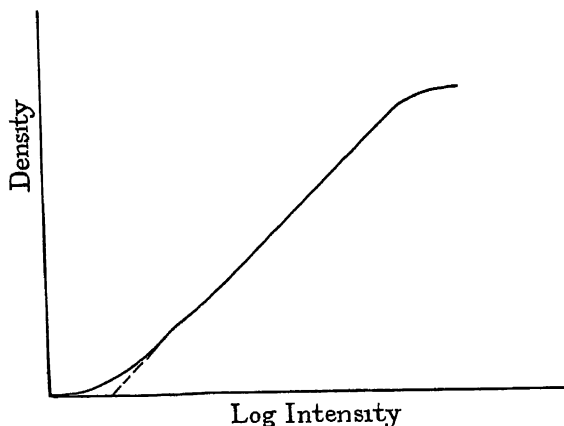


Fig 17.1 Density-intensity graph of a photographic plate

2 The normal multiplet

The Sommerfeld intensity rule has already been discussed, but in a form applicable only to multiplets in which $\Delta L = \pm 1$. Multiplets of the $P \rightarrow P^0$ type, in which $\Delta L = 0$, give their strongest lines when ΔJ is zero, while the two groups arising from $\Delta J = \pm 1$ are less intense than the chief lines but equal to one another. To include this type of transition the Sommerfeld rule may be restated *

The chief lines of any multiplet are due to those transitions in which $\Delta J = \Delta L$, a weaker group, technically known as satellites of the first order, arise when $\Delta J = \Delta L \pm 1$, while satellites of the second order occur when $\Delta J = \Delta L \pm 2$.

This rule is qualitative only, attempting to make it quantitative Sommerfeld in 1923 considered first those multiplets which arise by the combination of a single level with a multiplet level, the three SP lines in spectra of all multiplicities are of this type, and experiment† shows that the intensities due to the three transitions

* Sommerfeld and Heisenberg, *ZP*, 1922, 11 131

† Dorgelo, *ZP*, 1924, 22 170

vary as $(2J+1)$ for the three P terms, thus the intensities of the triplet lines ${}^3S_1 \rightarrow {}^3P_{2,1,0}$ of Mg are in the ratio of 5 3 1, while those of the octet lines ${}^8S_{3/2} \rightarrow {}^8P_{4,3,2}$ of Mn are in the ratio of 10 8 6

From this result three important rules emerge. First, the intensity ratio is independent of the serial number n . Secondly, the intensity is determined as much by the level to which an electron is going as the level which it is leaving, in calcium, for example, the intensity ratios of $2{}^3S \rightarrow 2{}^3P$ and $3{}^3P \rightarrow 2{}^3S$ are identical, though one triplet is of the sharp and the other of the principal series. The third is a point of theory, the intensities are in the

Term	P ₀	P ₁	P ₂	
D ₁	4425 43 25	4435 67 19	4456 61 1	45
D ₂	—	4434 95 54	4455 88 18	72
D ₃	—	—	4454 77 100	100
	25	73	119	Intensity sum

Fig. 17 2. Intensities in a diffuse triplet of calcium, the upper number is the wave-length, the lower the intensity measured by Burger and Dorgelo

ratios of the number of Zeeman components possessed by the level, and this fits in well with the practice of taking this number as the statistical weight of a level, a further significance will appear when the intensities of Zeeman multiplets are considered.

The next step was to examine the transitions between two multiplet levels. Working in this direction Burger and Dorgelo* first verified the validity of the above rules for the PD doublet of sodium where the D levels are not resolved, and then turned to the PD multiplet of calcium, in it they found that the sum of the intensities of the lines originating from one P level was to the sum of the intensities from another P level as the statistical weights of the levels. And the same held true for the three D levels.

Fig. 17 2 gives their measurements, the upper figure in each

* Burger and Dorgelo, *ZP*, 1924, 23 258

square being the wave-length and the lower the intensity adjusted to a scale in which 100 represents the brightest line. The sums of the intensities of the lines originating from the various P levels are given at the bottom, 25 73 119, and it will be observed that they are roughly in the ratio of 1 3 5. Similarly, the intensities, 45, 72, 100, arising from the three D levels are roughly in the ratio 3 5 7.

The summation rule used alone enables us to calculate the intensities resulting from the combination of two doublet levels (Fig. 17 3), but it will not suffice for more complicated multiplets.

Term	P_1	P_{1+}	
D_{1+}	5	1	6
D_{2+}	—	9	9
	5	10	ΣI

Term	D_{1+}	D_{2+}	
F_{2+}	14	1	15
F_{3+}	—	20	20
	14	21	ΣI

Fig. 17 3 Theoretical intensity ratios of diffuse and fundamental doublets

Thus, consider a PD triplet in which the intensities are those shown in Fig. 17 4. Burger and Dorgelo's summation rule shows that

$$\left. \begin{aligned} \frac{a_1}{1} = \frac{b_1 + b_2}{3} = \frac{c_1 + c_2 + c_3}{5} \\ \frac{a_1 + b_1 + c_1}{3} = \frac{b_2 + c_2}{5} = \frac{c_3}{7} \end{aligned} \right\}$$

But this gives only four equations to determine five ratios. To resolve the problem Russell,* among others, called in the corre-

	3P_0	3P_1	3P_2	
3D_1	a_1	b_1	c_1	
3D_2	—	b_2	c_2	
3D_3	—	—	c_3	

Fig. 17 4 Assumed intensities of a diffuse triplet

	3P_0	3P_1	3P_2	
3D_1	20	15	1	36
3D_2	—	45	15	60
3D_3	—	—	84	84
	20	60	100	

Fig. 17 5 Theoretical intensities of a diffuse triplet

spondence principle. His argument cannot be given here, but his result may be reviewed.

* Russell, *Nat Acad Sci Proc* 1925, 11 314, 322, Sommerfeld and Honl, *Preuss Akad Wiss Berlin*, 1925, 9 141, Honl, *AP*, 1926, 79 274, Kronig, *ZP*, 1925, 31 885, 33 261, Dirac, *PRS*, 1926, 111 281.

As the intensities of a multiplet are determined by the levels between which the electron jumps and not by the direction of the jump, it is not necessary to consider $\Delta L = \pm 1$ but only one of these, arbitrarily, then, we elect to consider only $L \rightarrow (L-1)$ and $L \rightarrow L$. For each of these transitions there are three values of ΔJ , so that six formulae may be expected, and these take their simplest form if J is defined as the larger of the two quantum numbers concerned.

In the transition $L \rightarrow (L-1)$ the correspondence principle gives

$$\text{for } J \rightarrow (J-1) \quad I_- = \frac{S}{4L} \frac{1}{J} P(J) P(J-1),$$

$$\text{for } J \rightarrow J \quad I_0 = \frac{S}{4L} \frac{(2J+1)}{J(J+1)} P(J) Q(J),$$

$$\text{for } (J-1) \rightarrow J \quad I_+ = \frac{S}{4L} \frac{1}{J} Q(J) Q(J-1).$$

In the transition $L \rightarrow L$ the summation rule shows that the two groups of satellites arising from $\Delta J = \pm 1$ must be identical, so that only two formulae are needed

$$\text{for } J \rightarrow J \quad I_0 = \frac{S(2L+1)}{4L(L+1)} \frac{(2J+1)}{J(J+1)} R^2(J),$$

$$\begin{array}{l} \text{for } J \rightarrow (J-1) \\ \text{or } (J-1) \rightarrow J \end{array} \quad I_{\pm} = \frac{S(2L+1)}{4L(L+1)} \frac{1}{J} P(J) Q(J-1)$$

In these equations $P(J)$, $Q(J)$ and $R(J)$ are convenient abbreviations defined as

$$P(J) = (J+L)(J+L+1) - S(S+1),$$

$$Q(J) = S(S+1) - (J-L)(J-L+1),$$

$$R(J) = J(J+1) + L(L+1) - S(S+1)$$

In theory these formulae should compare the intensities of any two lines arising in a transition from one configuration to another, but they are valid only when the coupling is Russell-Saunders and there are no inter-system lines. As very few spectra satisfy this condition the formulae are normally applied only to the intensities of components of a single multiplet, and for this purpose the first factor, which is a function of S and L only, may be dropped

INTENSITY TABLES

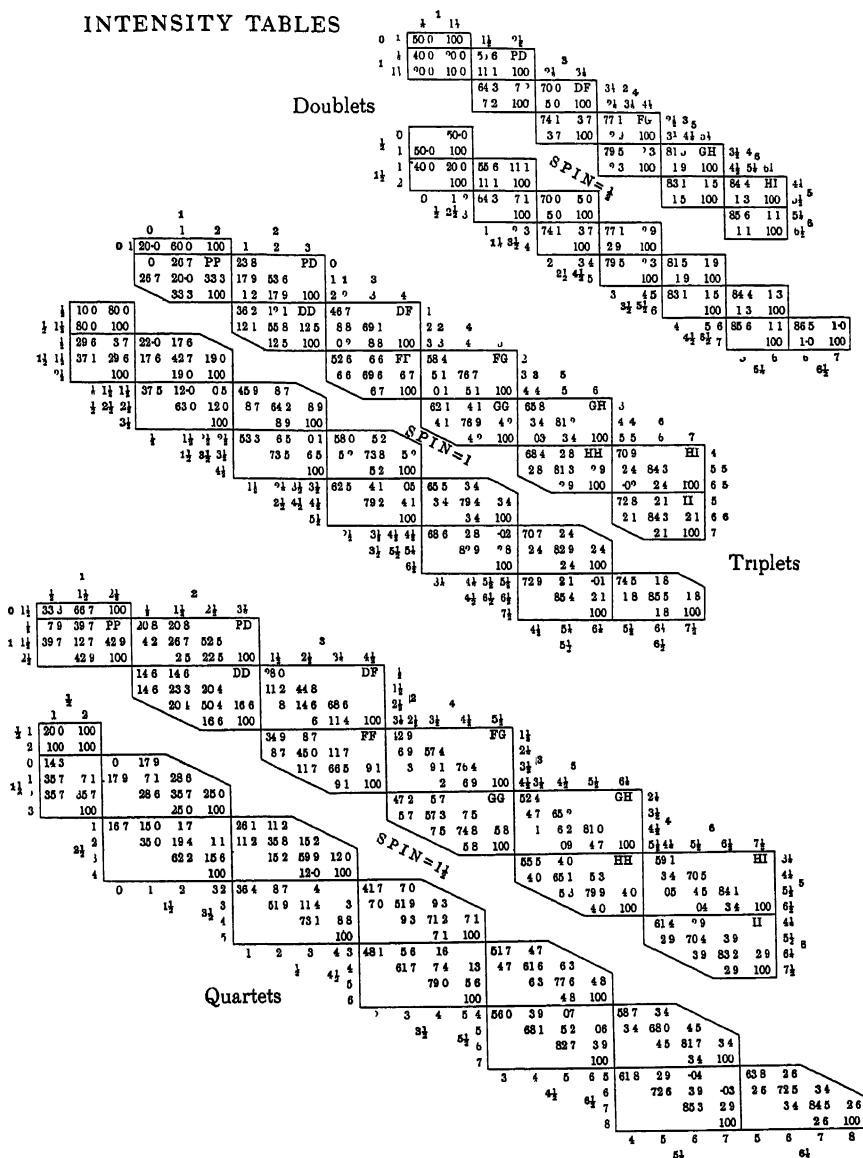
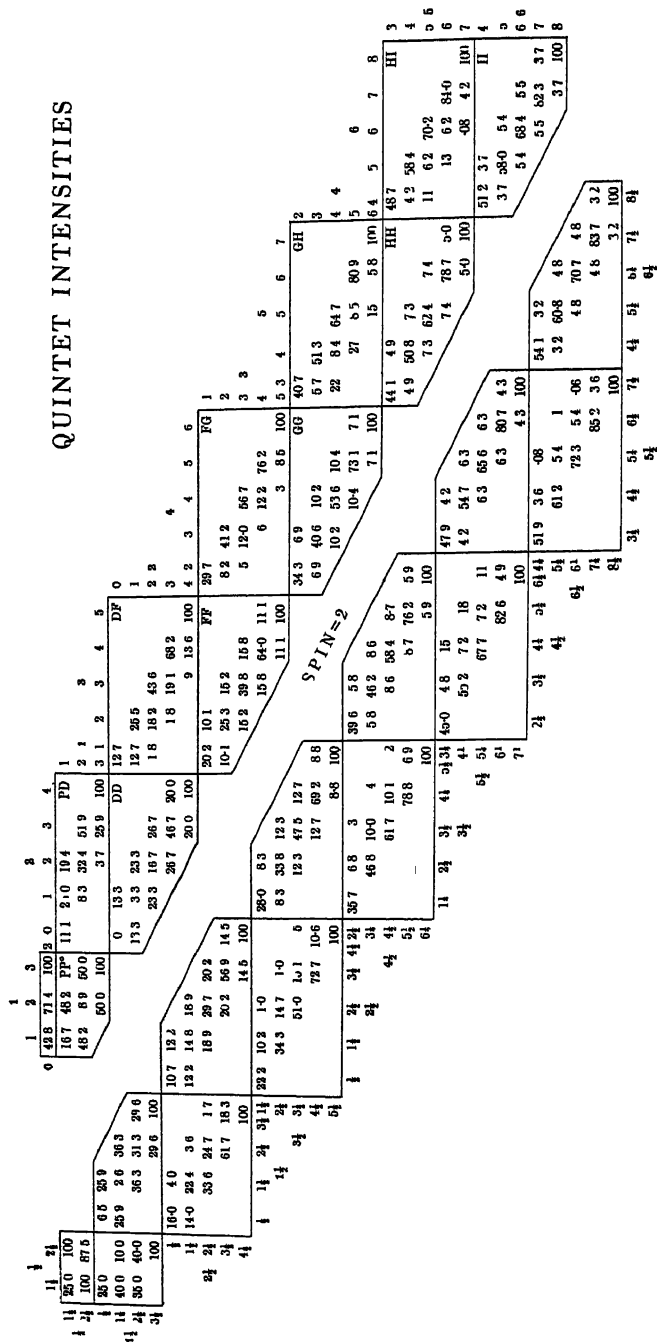


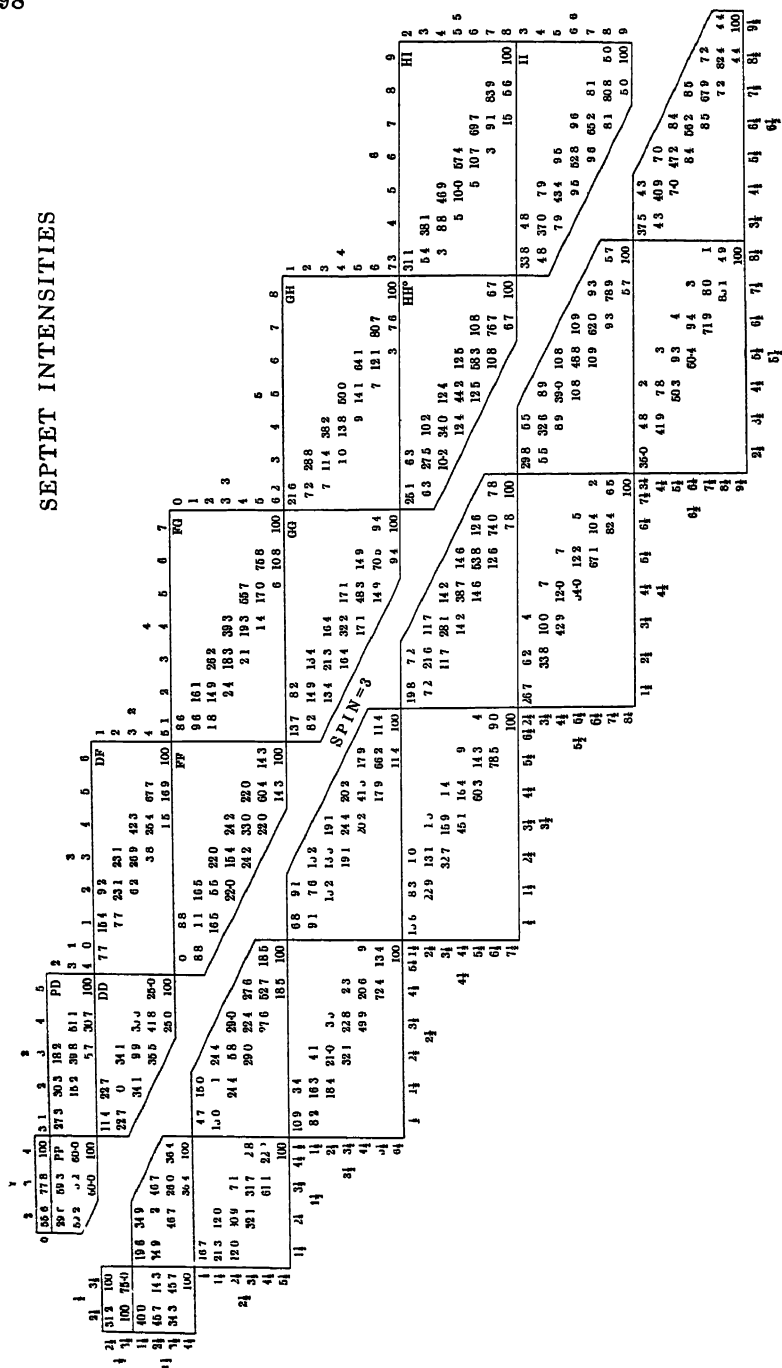
Fig 176 Multiplet intensities The intensity of each component is given as a percentage of the strongest line of the multiplet The numbers outside the frames are *L* and *J*, the former in heavy type The tables may be applied to (jj) coupling (p 150), related multiplets (p 104) and hyperfine structure (p 183) (After White and Elason, *PR*, 1933, 44 753)

QUINTET INTENSITIES



22

SEPTET INTENSITIES



OCTET INTENSITIES

The equations for the jump $L \rightarrow (L-1)$ may be illustrated by the three lines ${}^3D_{1,2,3} \rightarrow {}^3P_2$. In the 3D term L is 2 and S is 1, so that in the jump

$$\text{from } J=3 \text{ to } J=2 \quad I_- = \frac{1}{3}(5 \cdot 6 - 1 \cdot 2)(4 \cdot 5 - 2) = 168,$$

$$\text{from } J=2 \text{ to } J=2 \quad I_0 = 30,$$

$$\text{from } J=1 \text{ to } J=2 \quad I_+ = 2$$

This leads to the intensity scheme of Fig 17 5. Incidentally, too, this result satisfies the early qualitative rule of Sommerfeld, which stated simply that

$$I_- > I_0 > I_+$$

Fig 17 6 gives a list of the intensities calculated by these formulae, each line being expressed as a percentage of the

Terms	${}^6P_{1\frac{1}{2}}$	${}^6P_{2\frac{1}{2}}$	${}^6P_{3\frac{1}{2}}$
${}^6D_{\frac{1}{2}}$	3079 6 (20) 18 3	—	—
${}^6D_{1\frac{1}{2}}$	3073 1 (28) 25 8	3081 3 (12) 13 1	—
${}^6D_{3\frac{1}{2}}$	3062 1 (18 7) 18 7	3070 3 (36 6) 34 4	3082 1 (4 8) Lapped
${}^6D_{3\frac{3}{2}}$	—	3054 4 (51 4) 51 3	3066 0 (28 6) 26 1
${}^6D_{4\frac{1}{2}}$	—	—	3044 6 (100) 100

Fig 17 7 Intensities of the w ${}^6P^\circ \rightarrow 3d^5 ({}^6D) 4s {}^6D$ multiplet of Mn I. The upper figure is the wave-length, the middle the theoretical intensity, and the lower the observed intensity.

strongest line, this being a convenient practice because one per cent is about the accuracy which can be attained with the photographic technique developed in recent years. This technique leaves no doubt that these formulae are valid in very many spectra, as examples, Fig 17 7 shows a PD sextet from Mn I, and Fig 17 8 a DF quartet from Ti II.

The formulae are very seldom applied to the comparison of

different multiplets, but Wijk* has tried them out on the quartets and doublets of O II, a light atom in which the inter-system lines are very weak. This work roughly confirms the theoretical

$\begin{array}{c} 3d^3 \\ 3d^2 \ 4p \end{array}$	$^4F_{1+}$	$^4F_{2+}$	$^4F_{3+}$	$^4F_{4+}$
$^4D_+$	3161 19 (70) 72	—	—	—
$^4D_{1+}$	3154 18 (28) 28	3161 76 (112) 112	—	—
$^4D_{2+}$? (2 0) ?	3152 24 (37) 35	3162 56 (171) 168	—
$^4D_{3+}$	—	? (1 4) ?	3155 65 (29) 30	3168 52 (250) 249

Fig 17 8 Intensities of the $3d^2 4p \ ^4D \rightarrow 3d^3 \ ^4F$ multiplet of Tl II. The theoretical intensity is in brackets

quartet-doublet intensity ratio of 2 : 1, thus the intensities of the $^4P \rightarrow ^4S$ and $^2P \rightarrow ^2S$ lines were found to be as 1 : 6 : 1, and two other empirical ratios were 2 : 6 and 2 : 1.

3 The super-multiplet

In the mercury spectrum the division into singlets and triplets has little experimental justification, and many inter-system lines are strong, moreover, the sum rule in its simple form is but poorly obeyed. Accordingly, Ornstein and Burger† suggested that when the coupling is no longer Russell-Saunders, the sum rule ought to be extended. For this there is a precedent in the laws of the Zeeman effect, which define certain sums for a single multiplet when the coupling is Russell-Saunders, but only for all terms of a configuration when the coupling is abnormal.

In order to make the experimental work as significant as possible, Ornstein and Burger examined first a configuration of

* Wijk, *ZP*, 1928, 47 622

† Ornstein and Burger, *ZP*, 1926, 40 403

two electrons, which gives rise to a triplet and a singlet term. Moreover, they were careful that one of the two combining triplet terms should be unresolved. A super-multiplet satisfying these requirements is found in some DF combinations of calcium and strontium, if the intensities of the singlet and triplet lines are

Terms	1D_2	3D_1	3D_2	3D_3
1F	s	—	—	—
3F	—	t_1	t_2	t_3

Fig 17 9 Assumed intensities in a fundamental super-multiplet

written s and t_1 , t_2 and t_3 respectively, as in Fig 17 9, the sum rule applied to the vertical columns states that

$$\frac{s}{5} = \frac{t_1}{3} = \frac{t_2}{5} = \frac{t_3}{7}$$

Thus theory suggests that the intensity of the singlet should be equal to the mean intensity of the three triplet lines, whereas the intensities actually observed* and quoted in Fig 17-10

Transition	Ca		Ca		Sr	
	λ	Int	λ	Int	λ	Int
$^1D_2-^1F$	4878	4 3	4355	4 7	5156	4 4
$^3D_3-^3F$	4586	7 0	4099	6 9	4892	6 8
$^3D_2-^3F$	4581	5 0	4095	5 0	4869	5 0
$^3D_1-^3F$	4579	2 5	4093	2 8	4855	3 4
Mean triplet intensity		4 8		4 9		5 1

Fig 17 10 Observed intensities in three fundamental super-multiplets

show that it is rather weaker. But the singlet is of considerably longer wave-length than the triplet, and it has been shown* that when the intervals are large the sum rule should be applied not to the intensity itself, but to the intensity divided by ν^4 , and, in fact, if the intensity 4 3 of the singlet line is multiplied by $(4878/4582)^4$ the corrected intensity is 5 5, a figure which is as much larger than the mean intensity of the triplet as the first figure was too small.

* Ornstein, Coeltingh and Eymers, *ZP*, 1927, 44 653

With this satisfactory result Ornstein and Burger* were ready to tackle a group of lines in the mercury spectrum. The lines

Terms	1D_2	3D_3	3D_2	3D_1
1P_1	s	—	v_1	v_2
3P_2	i_4	t_1	t_3	t_6
3P_1	v_3	—	t_2	t_5
3P_0	—	—	—	t_4

Fig 17 11 Assumed intensities in a diffuse super-multiplet.

which should appear are shown in Fig 17 11, and the sum rule applied to this figure leads at once to the equations

$$\left. \begin{aligned} \frac{s + v_1 + v_2}{3} &= \frac{i_4 + t_1 + t_3 + t_6}{5} = \frac{v_3 + t_2 + t_5}{3} = \frac{t_4}{1} \\ \frac{s + i_4 + v_3}{5} &= \frac{t_1}{7} = \frac{v_1 + t_3 + t_2}{5} = \frac{v_2 + t_6 + t_5 + t_4}{3} \end{aligned} \right\}.$$

The eleven lines concerned in the relation differ so in wavelength that if all the intensities were measured close agreement with experiment could not be expected unless the ν^4 correction were applied, and at the time this work was done the ν^4 correction had not been tested out. But the intervals of the D terms are small, and as these equations require that

$$\frac{t_3 + t_6 + v_4}{t_1} = \frac{4}{21} = 19 \text{ per cent},$$

a check may be applied with measurements on only those four lines which arise from the combination of 3P_2 with the D terms

Wave length	Combination	Intensity
3650	t_1	100
3655	t_3	10 8
3663	t_6	} 18 7
3663	v_4	

Fig 17 12 Observed intensities in a diffuse super-multiplet of Hg I

The empirical results are shown in Fig 17 12. The line t_6 is so weak that if measured alone its intensity would certainly be less

* Ornstein and Burger, *ZP*, 1926, 40 403

than 1 per cent of t_1 , but it lies so close to t_4 that the two may be conveniently measured together after widening the slit of the spectroscope. The figures show that when the inter-system line is included the sum rule is satisfied, whereas if it is omitted ($t_3 + t_6$) is only 12 per cent of t_1 instead of the 19 per cent predicted.

These experiments clearly show that in certain spectra the sum rule is valid only if the singlet and triplet lines are treated as parts of a single whole, if this is a general phenomenon, then the intensity of successive lines in singlet and triplet series should decrease according to the same law, so that the relative intensity may be independent of the serial number, and in fact Ornstein and Burger* have confirmed this prediction.

4. The iron-frame elements

In the analysis of the iron frame elements at the Bureau of Standards, Russell and his co-workers have relied more on intensities than on multiplet intervals or magnetic splitting factors. Indeed, if the general intensity laws were not obeyed these spectra would probably still await analysis. The strongest lines arise from transitions in which only a single electron orbit changes, and this is in agreement with the correspondence principle, which indicates that those terms between which strong combinations appear must be built up from the same state of the ion. In contrast, transitions involving a change in the ion are much less probable, and the lines resulting are either absent or fainter even than the weak inter-system lines. Moreover, when only one electron jumps, the intensities of related multiplets can be obtained from the formulae designed to give the intensities of related lines, Russell† suggested a law of this kind, while Kronig used theory to show that if the coupling is Russell-Saunders the figures are identical and Fig 17 6 can be used. For this purpose S is replaced by ${}_iL$, L by l and J by L , where ${}_iL$ is the orbital moment of the ionic term, l the orbital moment of the jumping electron and L their resultant.

In the iron frame elements, though the relative intensities of

* Ornstein and Burger, *ZP*, 1926, 40 403

† Russell and Meggers, *Bur of Standards, Sci P* 1927, 22 332

the multiplets are admirably regular, the relative intensities within the multiplets are often abnormal. Russell's* visual estimates showed that in Ni I components of small J give fainter lines than they should, and later measurements amply confirm him. Not only are individual intensities irregular, but also the sums taken over all the terms of a multiplet with the same J

 ^5G ^5F

Statistical weight	5	7	9	11	13	Sum	Quotient
3	(30)	—	—	—	Theory	(29.7)	(9.9)
	9.5	—	—	—	Expt	9.5	3.2
5	(8)	(41)	—	—	—	(49.5)	(9.9)
	11	16	—	—	—	27	5.4
7	(0.5)	(12)	(57)	—	—	(69.2)	(9.9)
	~ 1	24.5	25	—	—	50.5	7.2
9	—	(0.6)	(12)	(76)	—	(89.0)	(9.9)
	—	~ 1	29	44	—	74	8.2
11	Theory	—	(0.3)	(8)	(100)	(108)	(9.9)
	Expt	—	~ 1	41	100	142	12.9
Sum	(38.5)	(53.8)	(69.2)	(84.6)	(100)	—	—
	21.5	41.5	55	85	100	—	—
Quotient	(7.7)	(7.7)	(7.7)	(7.7)	(7.7)	—	—
	4.3	5.9	6.1	7.7	7.7	—	—

Fig 17.13 Comparison of theoretical and experimental intensities in a $\text{F} \rightarrow \text{G}$ quintet of Ni I , this quintet arises as $d^8s ({}^4\text{F}) 5s {}^5\text{F} \rightarrow d^8s ({}^4\text{F}) 4p {}^5\text{G}^\circ$

Fig 17.13 shows this, for when the sum rule is valid, the intensity sum divided by the statistical weight yields a constant, but in Ni I and Co I the quotient varies from one J to another.

In a general way spectroscopists have long realised that the Russell-Saunders coupling, which predominates on the left-hand side of the periodic table, gives way to less regular coupling as one passes to the right, so that no one was surprised when titanium

* Russell, *PR*, 1929, 34 825

was shown to obey the intensity laws more closely than nickel. Nickel and cobalt indeed form the ultimate members of a series, which grows progressively less regular. In TiII 62 per cent of the lines obeyed the formulae to within 5 per cent, but in TiI, where Harrison* measured twenty-six strong multiplets, the proportion was down to 58 per cent, in chromium† and manganese‡ still more violations were observed, while in cobalt and nickel hardly a single multiplet is regular §

Various efforts have been made to trace the cause of the irregularity. In ZrI many intensities are abnormal because two terms of the same configuration and the same J have also nearly the same energy, these terms share their intensities, just as they share their magnetic splitting factors. In particular, the transition $4d^2\ 5s\ 5p^1F_3^{\circ} \rightarrow 4d^3\ 5s\ ^5F$ is observed because $d^2\ sp^1F_3^{\circ}$ lies near $d^2\ sp^5D_3^{\circ}$, their energies being 24,387 and 23,889 cm^{-1} respectively || And as the multiplet separations become greater as one passes from left to right across the table, the multiplets overlap and perturb one another more ¶ But adequate as this explanation may be in its place, it is necessarily unable to show why lines involving small values of J are weak compared with those in which J is larger—unless indeed many lines of lower multiplicity and therefore in general smaller J remain unidentified. Moreover, this explanation would suggest strong correlation with departures from the interval and Zeeman rules, but in fact Frenchs,** having examined selected multiplets from some elements of the iron frame, found that the correlation with the interval rule is poor, while Harrison in his detailed study of titanium found no correlation with either rule. Often those multiplets which split irregularly in the magnetic field and have irregular intervals obey the intensity laws well, while those which have regular values of g obey the intensity laws badly. And an attempt to

* Harrison, *JOSA*, 1928, 17 389

† Allen and Hesthal, *PR*, 1935, 47 926

‡ Seward, *PR*, 1931, 37 344

§ Ornstein and Buoma, *PR*, 1930 36 679

|| Kiess, C. C and Kiess, H. K., *BSJ*, 1931, 6 621

¶ Harrison and Johnson, *PR*, 1931, 38 773

** Frenchs, *AP*, 1926, 81 842

attribute the abnormal weakness of certain lines to the abstraction of energy by inter-system lines was no more successful

5 Alkali doublets

In general the intensity ratio in a series of multiplets is independent of the serial number, but this is not true of the principal doublets of the alkalis nor of the similar doublets of TI I

The controversy* about the alkali doublets has lingered on for many years because of the great experimental difficulties, the most serious being self-absorption, which can be avoided only by working at low temperatures and low current densities, but the general features are now clear (Fig 17 14) The diffuse and funda-

Element	Doublet		Intensity ratio	
	Transition	Wave length, Å	Calculated Feim	Experimental
Na	$2^1P \rightarrow 1^1S$	5890-96	2 0	Ro ¹ 1 98
K	$2^1P \rightarrow 1^1S$	7665-99	2 0	Ra 1 91
	$3^1P \rightarrow 1^1S$	4044-47	2 16	Ra 2 10
Rb	$2^1P \rightarrow 1^1S$	7800-947	?	Ra 1 85
	$3^1P \rightarrow 1^1S$	4202-16	2 60	H 2 55, K H 2 71, Ro ² 2 58
	$1^1P \rightarrow 1^1S$	3587-92	2 97	H 3 25, K H 3 32, Ro ² 2 90
Cs	$3^1P \rightarrow 1^1S$	4555-93	4 3	FW 3 3, Ra 3 85
	$1^1P \rightarrow 1^1S$	3877-89	7 15	K H 8 50, Ro ² 7 40, FW 4 6, H 8 0

Fig 17 14 Intensity ratios of some principal doublets of the alkalis

Observers

- F W Fuchtbauer and Wolff, *AP* 1929, 3 359 Extrapolated to allow for self absorption
H Hubner, *AP*, 1933, 17 781 Photographic comparison of lines emitted by burner at 2800°
K H Kohn and Hubner, *PZ*, 1933, 34 278 Emission spectrum
Ra Rasetti, *N Cim* 1924, 1 115 Anomalous dispersion
Ro ¹ Roschlostwenski, *IP*, 1912, 39 307 Anomalous dispersion
Ro ² Roschlostwenski, *T Opt I*, Petrograd, 1921, 13 1 Anomalous dispersion

mental doublets give the normal ratios of 9 5 1 and 3 2 in caesium,† and presumably in all other spectra, in the principal series, on the other hand, the normal ratio of 2 1 is found only in sodium and potassium‡, the deviation increases rapidly with atomic number

* Joos, *Hb d Expt Phys* 1929, 22 313

† Filippov, *ZP*, 1927, 42 495

‡ Fuchtbauer and Wolff, *AP*, 1929, 3 359

and is quite unmistakable in caesium. In the latter Rasetti* and Roschdostwenski,† both of whom used the accurate method of anomalous dispersion, found ratios of 3.85 and 7.40 in the second and third principal doublets respectively, while Sambursky‡ states that after rising to a maximum value of 25.1 in the first doublet, the intensity ratio decreases to 5.1 in the eighth. In the first doublet the ratio deviates very little from the normal value of 2 even in caesium.

This much was known when Fermi§ applied the quantum mechanics to the problem, and showed that if certain terms ordinarily neglected, are taken into account, deviations very similar to those observed should arise. Thus theory shows that the deviation will increase rapidly with atomic number, but should not affect the first doublet of the principal series, while the introduction of numerical values leads to intensity ratios of 4.3 and 7.15 for the second and third doublets of caesium, for the first doublets theory gives a ratio somewhat less than 2, but the difference is too small to measure. The agreement here obtained with experiment is probably as close as can be expected.

In ThI , which like the alkalis has a single electron outside closed shells, similar deviations occur. In the $m^2P_{1\frac{1}{2}} \rightarrow 2^2S_{\frac{1}{2}}$ series experiment shows that the intensity ratios when $m = 4, 5, 6$ are 4.4, 6.6, 6.0 and 5.2 ||

6 The Zeeman multiplet

As with the normal so with the Zeeman multiplet, certain simple rules have been established, but these suffice to determine the intensity ratios only in the simpler transition, in the more complex, reliance must be placed on formulae deduced with the aid of the quantum mechanics.

Three rules are usually cited,¶ but of these the first states only the well-known fact that the Zeeman multiplet is symmetrical about the undisplaced line. The second adds that the intensities

* Rasetti, *N. Cim* 1924, 1 115

† Roschdostwenski, *T. Opt. I, Petrograd*, 1921, 13.1

‡ Sambursky, *ZP*, 1928, 49 731

§ Fermi, *ZP*, 1930, 59 680.

|| Williams and Herlihy, *PR*, 1932, 39 802

¶ Ornstein and Burger, *ZP*, 1924, 28 135

sum of all lines originating in one Zeeman level is equal to the intensity sum of all lines originating in any other Zeeman level, and this holds true if the word 'ending' is substituted for 'originating'. This law appears at first analogous to the Burger and Dorgelo sum rule, the statistical weight of each Zeeman level being unity, but further examination shows that the relation is closer than analogy, for a term splits to $(2J+1)$ Zeeman components, so that the normal multiplet rule is a necessary consequence of the Zeeman rule

M		Total intensities	
3S_1	3P_2	σ	π
1	2	a_3	—
1	1	—	b_1
0	1	a_2	—
1	0	a_1	—
0	0	—	b_0
-1	0	a_1	—
0	-1	—	—
-1	-1	—	b_1
-1	-2	a_3	—

Fig 17 15 Assumed intensities of the Zeeman components of a $^3S_1 \rightarrow ^3P_2$ line

The third rule concerns the polarisation, and states that if the various components emitted in any direction are combined the resulting beam must be unpolarised. Thus in the normal Zeeman triplet observed transverse to the magnetic field, the sum of the intensities of the two σ components must be equal to the intensity of the π component. These rules appear simple enough, but the simplicity is in part only apparent, for the intensities mentioned in the sum and polarisation rules are not the same, the sum rule applies to the total radiated intensity, while the polarisation rule applies to the intensity observed in a particular direction. Thus when a pattern is observed transverse to the field, those oscillations which produce σ components vibrate in a circle whose plane is perpendicular to the magnetic axis, one component of this vibration is along the line of sight and so invisible, thus only half the radiated intensity of a σ component is observed. On the other hand the oscillators which are producing π components vibrate

along the magnetic axis, so that the whole of the radiated intensity reaches an eye looking transverse to the field

As an example of the way in which these three rules are applied consider the ${}^3S_1 \rightarrow {}^3P_2$ transition * Fig 17 15 gives on the left the possible values of the magnetic quantum number M , and on the right the total intensity radiated in each transition, the π and σ components being separated for convenience The consequences of the symmetry rule are embodied in the notation, the sum rule, applied to the components of the 3P_2 term, states that

$$b_0 + 2a_1 = b_1 + a_2 + a_3,$$

while applied to the 3S_1 term it shows that

$$b_0 + 2a_2 = b_1 + a_1 + a_3$$

In order to apply the polarisation rule, elect to observe the pattern transverse to the magnetic field, then the argument

Element	Line	Intensity	σ			π			σ			$\Sigma\pi$	$\Sigma\sigma$
		Theoretical	5	15	30	30	40	30	30	15	5	100	100
Mg	5183	Observed	5	16	30	32	39	32	30	14	3	103	98
Ca	6162	Observed	4	13	29	30	41	28	28	14	6	99	94
Zn	4810	Observed	5	15	29	32	41	31	29	15	3 5	104	97
Cd	5085	Observed	5	15	29	32	41	31	27	14	5	104	95

Fig 17 16 Observed intensities of the Zeeman components of a ${}^3S_1 \rightarrow {}^3P_2$ line in various spectra

given above shows that the total radiated intensity of the π components must be equal to half the total radiated intensity of the σ components, that is,

$$\begin{aligned} b_0 + 2b_1 &= \frac{1}{2} \{2a_1 + 2a_2 + 2a_3\} \\ &= a_1 + a_2 + a_3 \end{aligned}$$

The three simple rules thus determine the four unknown ratios, being satisfied by the values

$$\begin{aligned} a_1 &= 1, & a_2 &= 3, & a_3 &= 6, \\ b_0 &= 4, & b_1 &= 3 \end{aligned}$$

These predictions for the ${}^3S_1 \rightarrow {}^3P_2$ line have been amply confirmed by Van Geel,† as Fig 17 16 shows, moreover, similar

* Ornstein and Burger, *ZP*, 1924, 29 241

† Van Geel, *Diss Utrecht*, 1928, 60

predictions for the two other lines of the triplet are equally satisfactory (Fig 17 17)

Line	Transition	Intensity	σ		π		σ		$\Sigma\pi$	$\Sigma\sigma$
6122	$^3S_1 \rightarrow ^3P_1$	Theoretical	15	15	30	30	15	15	60	60
		Observed	17	16	30	28	15	15	58	63
6102	$^3S_1 \rightarrow ^3P_0$	Theoretical	10		20		10		20	20
		Observed	9		20		10		20	19

Fig 17 17 Intensities of the Zeeman components of two calcium lines

Though these three rules suffice when the J values of the two terms concerned are small, in more complex transitions resort must be had to the correspondence principle* or to the quantum mechanics † Calculations based on these principles show that for the transitions $J \rightarrow (J-1)$, the intensities are given by

$$\sigma \text{ Jump } M \rightarrow (M-1) \quad I_- = \frac{1}{2}p(M)p(M-1),$$

$$\pi \text{ Jump } M \rightarrow M \quad I_0 = p(M)q(M),$$

$$\sigma \text{ Jump } (M-1) \rightarrow M \quad I_+ = \frac{1}{2}q(M)q(M-1),$$

while for the transitions $J \rightarrow J$

$$\pi \text{ Jump } M \rightarrow M \quad I_0 = r^2(M),$$

$$\sigma \text{ Jump } M \rightarrow (M-1) \quad I_{\pm} = p(M)q(M-1)$$

$$\text{or } (M-1) \rightarrow M$$

In these equations the transitions considered make J and M the larger of the two quantum numbers concerned, while p , q and r are abbreviations, defined by

$$p(M) = J + M,$$

$$q(M) = J - M,$$

$$r(M) = M$$

The intensities given by these formulae are the total radiated intensities The formulae for the transition from $(J-1)$ to J are not quoted, because the intensities are independent of the direction in which the electrons jump, so that one may consider always $J \rightarrow (J-1)$

Intensities calculated from these formulae are given in Fig 17 18, as the formulae do not contain L or S the Zeeman inten-

* Honl, *ZP*, 1925, 31 340, *AP*, 1926, 79 288, Kromg, *ZP*, 1925, 31 885

† Heisenberg and Jordan, *ZP*, 1926, 37 263

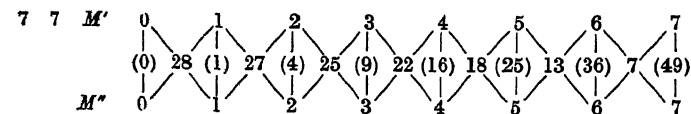
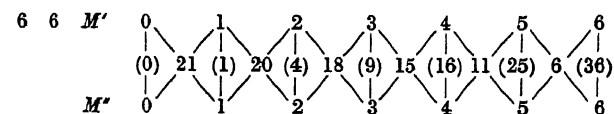
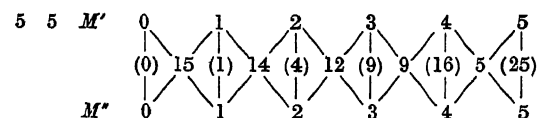
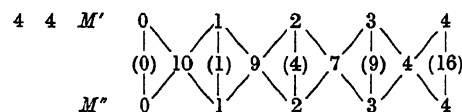
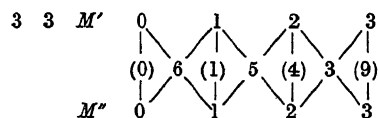
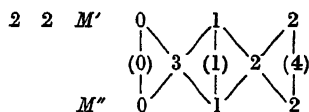
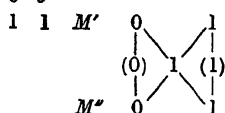
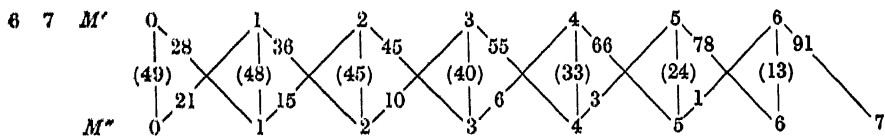
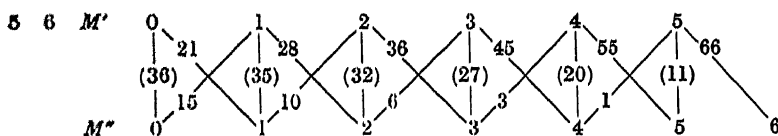
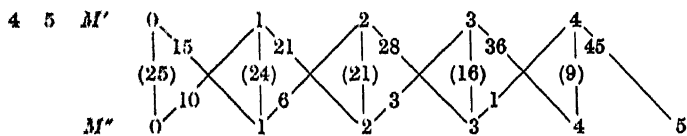
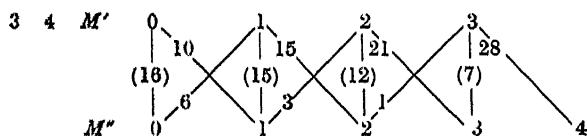
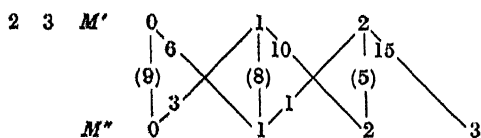
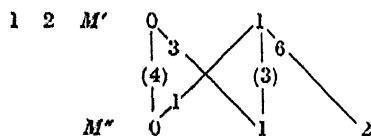
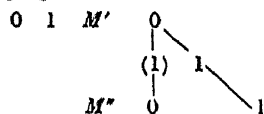
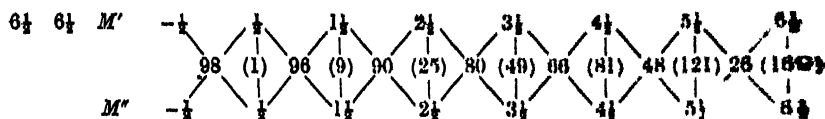
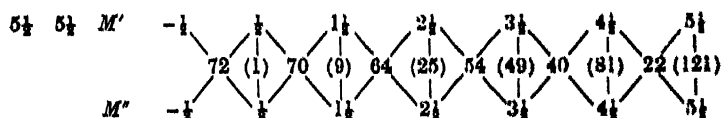
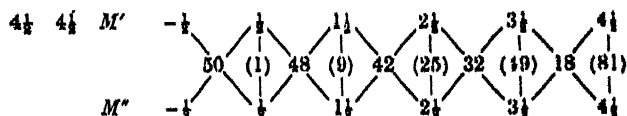
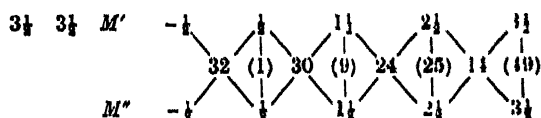
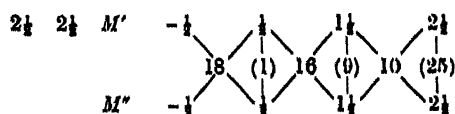
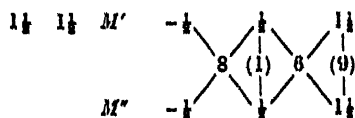
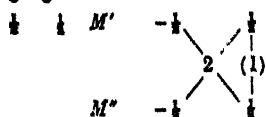
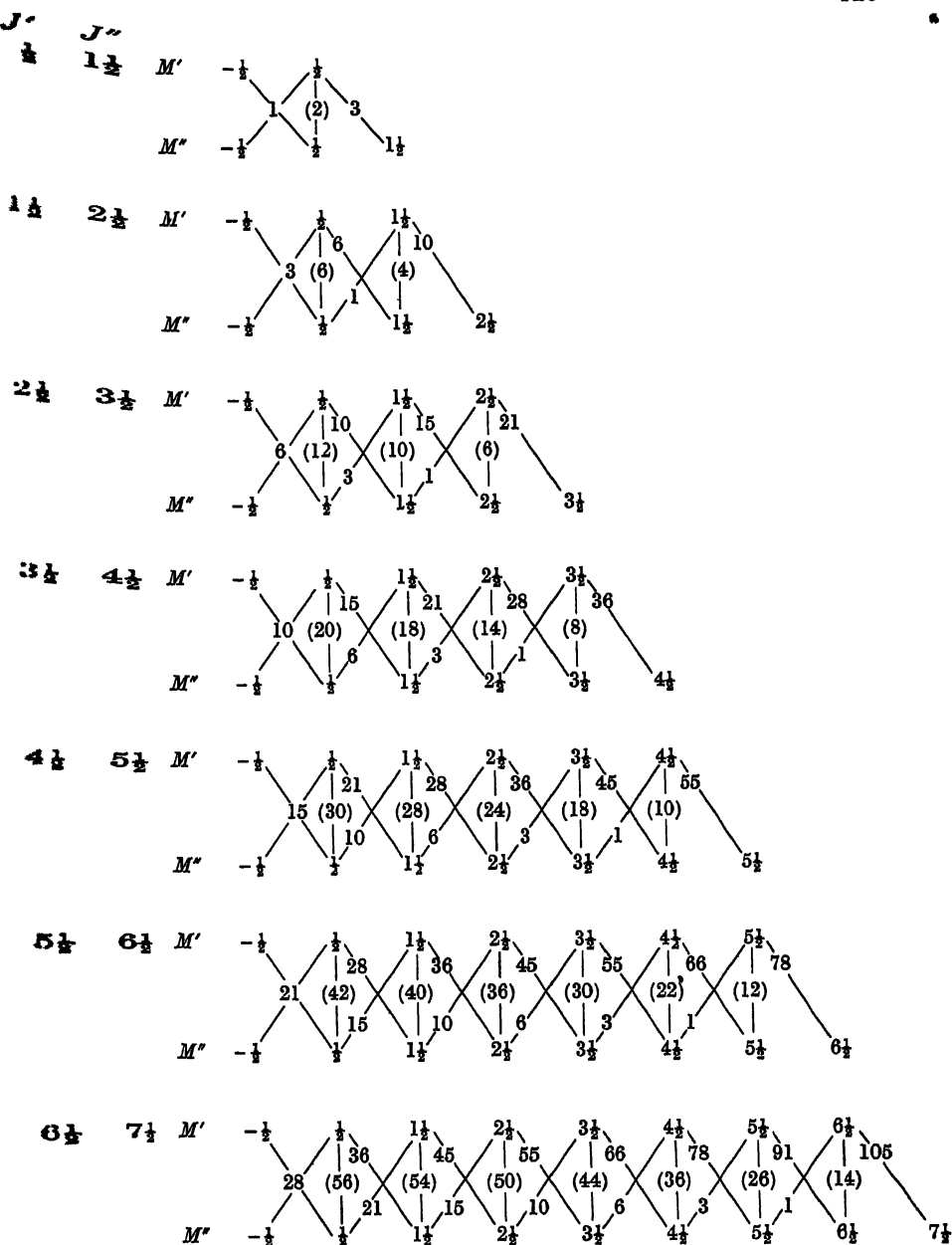
Fig 17 18 α Integral, $\Delta J = 0$

Table of Zeeman intensities The table is divided into four sections according as J is integral or half integral, and according as ΔJ is 0 or ± 1

$J' J''$ Fig 17 18 b J integral, $\Delta J = \pm 1$

$J' J''$ Fig 17-18 c. J half-integral; $\Delta J = 0$

Fig 17 18 d J half-integral, $\Delta J = \pm 1$

sities are a function of J only, so that the components of the lines $^3P_2 \rightarrow ^3S_1$ and $^7F_2 \rightarrow ^7D_1$ have the same intensity ratios, though they do not occupy the same positions. Like the simpler rules these formulae have been confirmed by the experiments of Van Geel, who has measured lines in the octet system of Mn I, and in the septet and quintet systems of Cr I (Figs 17 19–17 21), but

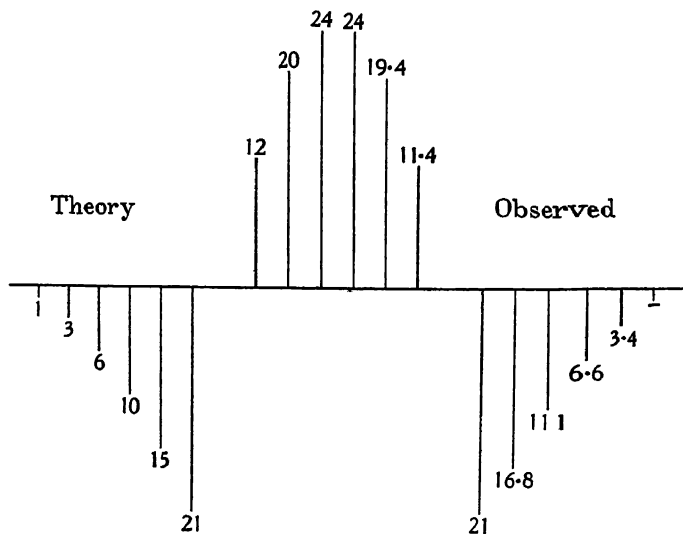


Fig 17 19 Intensities of the Zeeman components of the 4754 Å line of Mn I, this line arises as $a^8S_{3/2} \rightarrow z^8P^{\circ}_{2+}$. The pattern is (1) (3) (5) 9 11 13 15 17 19/7. On the left are the theoretical, and on the right the measured intensities, the brightest π and σ components being adjusted to fit. After Van Geel, Diss. Utrecht, 1928, 65

work on the iron row shows that the agreement is not always as good as that shown in the lines chosen here as illustrations

Having obtained satisfactory intensity formulae for normal multiplet lines, two extensions suggest themselves; first, one may enquire whether the formulae are applicable to inter-system lines, and second, what intensities are to be expected in the partial Paschen-Back effect. To these questions also Van Geel has offered some answer *

As an inter-system line he chose $^1D_2 \rightarrow ^3P_2$, 3663.3 Å, of Hg I,

* Van Geel, Diss. Utrecht, 1928 and ZP, 1928, 47 615

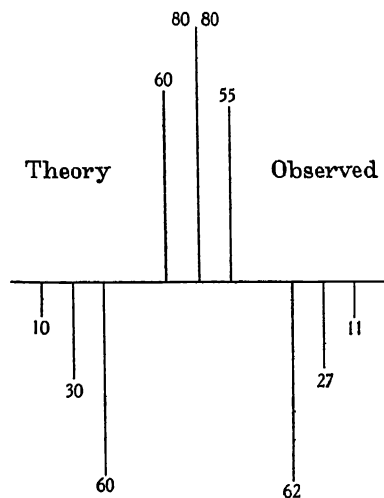


Fig 17 20 Intensities of the Zeeman components of the 5205 Å line of Cr I, this line arises as $z^5P_1 \rightarrow a^5S_2$, the pattern is (0) (1) 3 4 5/2 After Van Geel, Diss Utrecht, 1928, 68

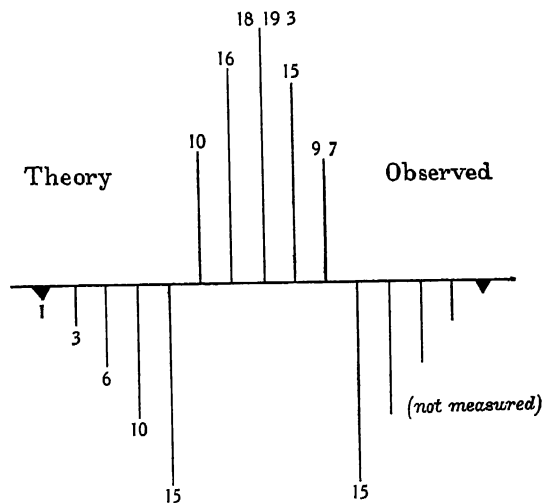


Fig 17 21 Intensities of the Zeeman components of the 5208 Å line of Cr I, this line arises as $z^5P_3 \rightarrow a^5S_2$, the pattern is (0) (1) (2) 3 4 5 6 7/3 After Van Geel, Diss Utrecht, 1928, 67

and obtained the intensity pattern shown in Fig. 17 22 This is certainly rather irregular, but then so is the pattern of the neighbouring triplet line, $^3D_1 \rightarrow ^3P_2$, 3662.9 Å (Fig 17-23), and

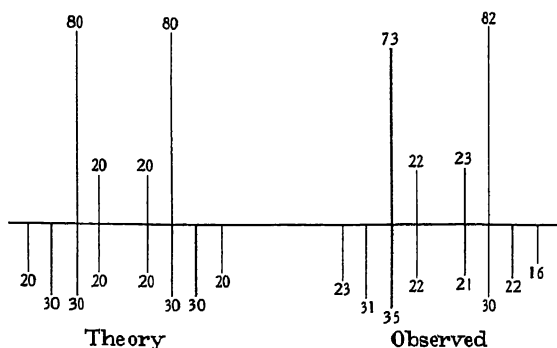


Fig. 17 22 Intensities of the Zeeman components of the 3663 Å line of Hg I; this line arises as $3^1D_2 \rightarrow 2^3P_2$, and the pattern is (1) (2) 1 2 3 4/2 After Van Geel, Diss Utrecht, 1928, 77

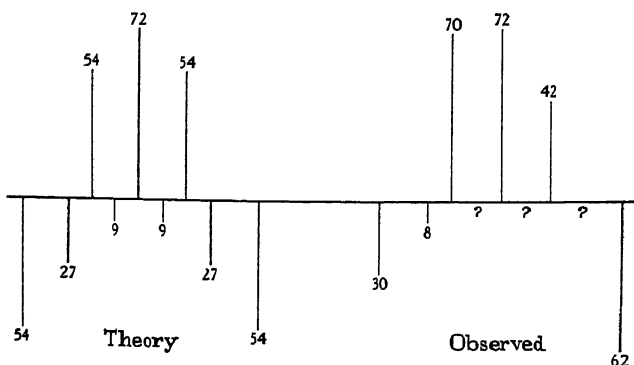


Fig 17 23. Intensities of the Zeeman components of the 3662.9 Å line of Hg I, this line arises as $3^3D_1 \rightarrow 2^3P_2$, the pattern is (0) (2) 1 3 5/2 After Van Geel, Diss Utrecht, 1928, 77

as this sd configuration produces two diads instead of a singlet and a triplet, the coupling must be abnormal

For the intensities of the partial Paschen-Back effect appeal must be had to the quantum mechanics, which has been applied by Darwin* and others to predict both the intensity and displace-

* Darwin, C G, *PRS*, 1927, 115 1

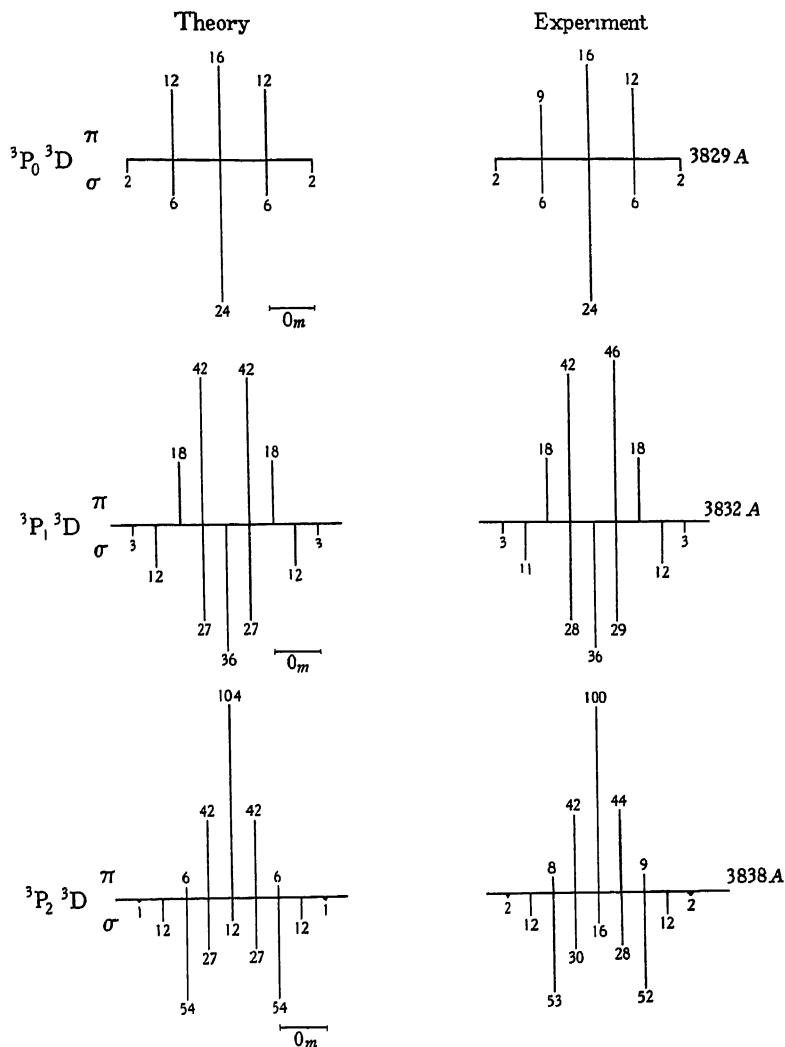


Fig 17.24 Intensities of the Zeeman components of a diffuse triplet of Mg I, the lines arise as $3^3D \rightarrow 2^3P_{0,1,2}$. This is an example of the partial Paschen Back effect. After Van Geel, Diss Utrecht, 1928, 84

ment of the magnetic components of any normal multiplet in any field. That this theory is valid few doubt, for it is of very general application and is known to give correct results in weak and strong fields. Further confirmation in intermediate fields is however still welcome, so that Van Geel's measurements on a Mg I triplet, which exhibits the partial Paschen-Back effect, are worth quoting (Fig. 17.24). They confirm the theory admirably.*

7. *Rares ultimes*

A common method of analysing an unknown salt is to add a little to a carbon arc,† if the powder happens to be a pure calcium salt, a great many calcium lines will appear, but as the proportion

Arc spectra			Spark spectra		
Element	Wave-length	Terms	Element	Wave-length	Terms
K	7665	$^2P_{1/2} \rightarrow ^2S_{1/2}$	Ca	3934	$^2P_{1/2} \rightarrow ^2S_{1/2}$
Ca	4227	$^1P_1 \rightarrow ^1S_0$	Sc	3614	$^3F_4 \rightarrow ^3D_3$
Sc	4779	$^2F_{3/2} \rightarrow ^2D_{3/2}$	Ti	3349	$^4G_{3/2} \rightarrow ^4F_{4/2}$
Ti	3635	$^3G_5 \rightarrow ^3F_4$	V	3093	$^5G_5 \rightarrow ^5F_5$
V	3185	$^4G_{5/2} \rightarrow ^4F_{4/2}$	Cr	2836	$^6F_{5/2} \rightarrow ^6D_{4/2}$
Cr	4254	$^7P_4 \rightarrow ^7S_3$	Mn	2576	$^7P_4 \rightarrow ^7S_3$
Mn	4031	$^6P_{3/2} \rightarrow ^6S_{3/2}$	Fe	2382	$^6F_{5/2} \rightarrow ^6D_{4/2}$
Fe	3720	$^5F_5 \rightarrow ^5D_4$	Co	2389	$^5G_5 \rightarrow ^5F_5$
Co	3452	$^4G_{5/2} \rightarrow ^4F_{4/2}$	Ni	2416	$^4G_{5/2} \rightarrow ^4F_{4/2}$
Ni	3415	—	Cu	—	—

Fig. 17.25 Some *rares ultimes*.

of calcium in the powder is diminished step by step, the weaker lines successively disappear until finally only one is left, this is known as the *rare ultime*, and is in fact the 4227 Å line, arising as $^1P_1 \rightarrow ^1S_0$. This method of analysis was developed by de Gramont.‡

The *rares ultimes* are not necessarily the strongest lines in the spectrum as ordinarily produced, nor are they in general the lines

* See also Back's measurements on a diffuse doublet of copper, Vol. I, 117

† Twyman and Smith, *Wavelength tables of spectrum analysis*, 1931. Ryde and Jenkins, *Sensitive arc lines of 50 elements*, 1930. Both book and pamphlet are published by Adam Hilger

‡ De Gramont, *Comptes Rendus*, 1920, 171 1106.

which require the least energy to excite them, the resonance lines. Instead they are determined by four conditions, of which two are energy conditions * The first states that the lower term is always the ground term, and the second that the higher term is of the same system As inter-system lines are in general weaker than lines arising within a system, this condition is surprising only as excluding lines, such as the 2536 Å $^3P_1 \rightarrow ^1S_0$ line of Hg I, which are exceptionally strong for other reasons Subject to these two conditions, and in part also to the fourth, the energy required to excite the line must be as small as possible The fourth condition states that when the energies are nearly equal, the *rare ultime* will usually arise from a transition in which ΔL is -1 in preference to one in which it is $+1$, but in aluminum and its homologues the 2D term lies so much higher than the 2S that the *rare ultime* is $^2S_{\frac{1}{2}} \rightarrow ^2P_{\frac{1}{2}}$, the fourth condition notwithstanding

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Frerichs in the *Handbuch der Physik*, 1929, 21, deals with theory and its empirical justification Van Geel, *Intensiteitsverhoudingen van Magnetsch Gesplitste Spectraallijnen*, 1928, contains a thorough study of Zeeman intensities, this is a dissertation presented to the University of Utrecht

For experimental methods

Dobson, Griffith and Harrison, *Photographic photometry*, 1926,
Ornstein, Moll and Burger, *Objektive Spektralphotometrie*, 1932

* Laporte and Meggers, *JOSA*, 1925, 11 459 Meggers and Scribner, however, have proposed a new rule 'A *rare ultime* originates in a simple interchange of a single electron between an s and a p state, usually preferring configurations in which only one electron occurs in these states' The *rare ultime* of Hf II is $5d^2 \ 6p \ ^4G_{5\frac{1}{2}} \ 5d^2 \ 6s \ ^4F_{4\frac{1}{2}}$, though 4F is not the ground term *BSJ*, 1934, 13 657, 1935, 14 629

CHAPTER XVIII

THE SUM RULES AND (JJ) COUPLING

1 Deviations from the Russell-Saunders coupling

The low terms of the light elements are easily divided into multiplets, and the multiplets into configurations. In general the components of a multiplet differ in energy by an amount which is small compared with the height which separates one multiplet from another, and similarly the interval separating two terms of a configuration is usually small compared with the height which separates one configuration from another. Each level is characterised by a certain value of J , and each multiplet by values of L and S . J and L are determined primarily by selection rules and S by the number of components in a multiplet, but in the allotment of quantum numbers the interval ratios, magnetic splitting factors and intensities all have to be considered. Moreover, J is the vector sum of L and S . All these regularities are regarded as arising in the Russell-Saunders coupling of the electrons, and conversely any irregularity is attributed to some distortion of the coupling.

Deviations from these rules occur frequently in elements of high atomic weight, and in the high terms of a spectrum, they are rather more common on the right-hand side of the periodic table than on the left. But the various rules are not equally sensitive, the interval ratios are often abnormal when the g factors and intensities are normal, while the g factors are rather more easily disturbed than the intensities. This must not be taken to mean that terms in which the interval ratio is normal and the intensities abnormal do not sometimes occur, but in the analysis of an irregular spectrum like Ni I intensities usually give the surest clue to the names of the terms. There are spectra, however, such as argon, in which none of these clues are worth much, for the terms are no longer divided into multiplets but only into configurations, values of L and S cannot then be assigned. Finally, there exist

in which different configurations lie at the same energy, perturb one another.

For an abnormal spectrum, the J values of each component are unchanged, and there is strong evidence that if the g factors and intensities are summed for terms of the same values of J , then the result is the same in all cases. The question remains over how many terms the sum is to be taken. It is not clear in this case the energy seems to be crucial, in simple cases in which each multiplet is well separated from other multiplets the rule is valid for each multiplet separately, when the multiplets are intermingled, but the configurations distinct, the rule must be taken over all terms of the configuration, and in cases in which the sum rules are usually met. But when the configurations overlap, then the sum must extend over both.

3. The g sum

For the Zeeman and Paschen-Back effects the sum of the g factors of all terms, which have the same M and J , from a single multiplet, is proportional to the field, the components of energy are measured in terms of the normal Zeeman effect, then their sum is independent of the field, and

$$\sum_J M g = \sum_{L, S} (M_L + 2M_S)$$

which makes possible the calculation of the weak field Zeeman effect without the aid of the quantum mechanics, but it does, however, only to a single multiplet, it is only when the coupling is Russell-Saunders.

This may be generalised, however, for an electron configuration by summing over all terms of a configuration. For if the splitting is independent of the field intensity, it should not change. It is strong evidence that each electron vector precesses independently about the magnetic axis, the coupling is then

$$\{(\mathbf{l}_1 \mathbf{H})(\mathbf{l}_2 \mathbf{H}) \quad (\mathbf{s}_1 \mathbf{H})(\mathbf{s}_2 \mathbf{H}) \quad \}$$

The rule reads

$$\sum M g = \sum (m_l + 2m_s)$$

For example, $\sum (m_l + m_s)$. As an example, consider a pd configuration which rises to the six terms 3P , 1P (Fig. 18.1)

Among these there are four with components in which M is 3, and in these $\Sigma(m_l + 2m_s)$ is 14. In a weak field these components appear as parts of the 3F_4 , 3F_3 , 3D_3 and 1F_3 terms, whose magnetic splitting factors are $\frac{5}{4}$, $\frac{13}{12}$, $\frac{4}{3}$ and 1 respectively, so that Σg is $4\frac{2}{3}$ and ΣMg is 14 as theory requires

$l_1 \ l_2$	$m_{l_1} \ m_{l_2}$	$m_{s_1} \ m_{s_2}$	$\Sigma(m_l + m_s)$	$\Sigma(m_l + 2m_s)$	$\Sigma m_l m_s$
1 2	1 2 1 0 -1 -2	$\pm\frac{1}{2} \ \pm\frac{1}{2}$	4 3 3 2	5 3 3 1	$1\frac{1}{2} \ \frac{1}{2} \ -\frac{1}{2} \ -1\frac{1}{2}$
			3 2 2 1	4 2 2 0	1 0 0 -1
			2 1 1 0	3 1 1 -1	$\frac{1}{2} \ \frac{1}{2} \ -\frac{1}{2} \ -\frac{1}{2}$
			1 0 0 -1	2 0 0 -2	0 1 -1 0
			0 -1 -1 -2	1 -1 -1 -3	$-\frac{1}{2} \ 1\frac{1}{2} \ -1\frac{1}{2} \ \frac{1}{2}$
	0 2 1 0 -1 -2	$\pm\frac{1}{2} \ \pm\frac{1}{2}$	3 2 2 1	4 2 2 0	1 -1 1 -1
			2 1 1 0	3 1 1 -1	$\frac{1}{2} \ -\frac{1}{2} \ \frac{1}{2} \ -\frac{1}{2}$
			1 0 0 -1	2 0 0 -2	0 0 0 0
			0 -1 -1 -2	1 -1 -1 -3	$-\frac{1}{2} \ \frac{1}{2} \ -\frac{1}{2} \ \frac{1}{2}$
			-1 -2 -2 -3	0 -2 -2 -4	-1 1 -1 1
	-1 2 1 0 -1 -2	$\pm\frac{1}{2} \ \pm\frac{1}{2}$	2 1 1 0	3 1 1 -1	$\frac{1}{2} \ -1\frac{1}{2} \ 1\frac{1}{2} \ -\frac{1}{2}$
			1 0 0 -1	2 0 0 -2	0 -1 1 0
			0 -1 -1 -2	1 -1 -1 -3	$-\frac{1}{2} \ -\frac{1}{2} \ 0 \ \frac{1}{2}$
			-1 -2 -2 -3	0 -2 -2 -4	-1 0 0 1
			-2 -3 -3 -4	-1 -3 -3 -5	$-1\frac{1}{2} \ -\frac{1}{2} \ \frac{1}{2} \ 1\frac{1}{2}$

Fig 18.1 Magnetic quantum numbers of a pd configuration in a field so strong that the electronic vectors precess independently

Thus far the sum has been assumed independent only of the field, but if it remains unchanged in a field so strong that all electronic vectors are uncoupled, it must surely be independent of the coupling. And a logical consequence of this hypothesis is that the sum of the splitting factors is independent of the coupling when summed over all those terms of a configuration which have the same value of J . For let the sum of the g values of terms having $J = a$ be Σg_a , and when $M = b$ let $\Sigma Mg = \sigma_b$, then if the highest value of J is J' , when $M = J'$,

$$J' \Sigma g_{J'} = \sigma_{J'}$$

or

$$\Sigma g_{J'} = \sigma_{J'} / J',$$

and as the right-hand side is independent of the coupling, so also is $\Sigma g_{J'}$. When $M = (J' - 1)$,

$$\Sigma g_J + \Sigma g_{(J'-1)} = \sigma_{(J'-1)} / (J' - 1)$$

Again, the right-hand side is independent of the coupling, and so

therefore must $\Sigma g_{J'-1}$ be. Clearly this argument can be extended to all values of J

The constancy of the g sum was first deduced by Pauli* and Landé† from theory, but in the last decade experiment has amply confirmed it. In the spark spectrum of chromium‡ the ${}^4P_{\frac{1}{2}}$ and

Term	g factor	
	Landé	Empirical
${}^6F_{\frac{1}{2}}$	-0 667	-0 671
${}^6D_{\frac{1}{2}}$	3 333	2 841
${}^4D_{\frac{1}{2}}$	0	0
${}^4P_{\frac{1}{2}}$	2 667	3 145
Sum Σg	5 333	5 315

Fig 18 2 g factors of those terms of the $3d^4({}^5D)4p$ configuration of Cr II, which have $J = \frac{1}{2}$. An example of the sum rule

Term	g values		g sums	
	Landé	Empirical	Landé	Empirical
${}^4P_{\frac{3}{2}}$	1 60	1 60	1 60	1 60
${}^4P_{\frac{1}{2}}$	1 73	1 63	} 3 06	3 06
${}^2P_{\frac{1}{2}}$	1 33	1 43		
${}^4P_{\frac{1}{2}}$	2 67	2 53	} 3 34	3 34
${}^2P_{\frac{1}{2}}$	0 67	0 81		

Fig 18 3 g factors of the $p^4({}^3P)5s$ terms of A II, showing that the sum rule is obeyed

${}^6D_{\frac{1}{2}}$ terms of the $3d^4({}^5D)4p$ configuration show very irregular g values, but the sum for the four terms which have a J value of $\frac{1}{2}$ agrees closely with that predicted by Landé (Fig 18 2). Other examples of the sum rule are to be found in the spark spectra of neon,§ argon|| and krypton,¶ in La I, La II** and Ag II †† Fig 18 3, which cites the $p^4({}^3P)5s$ configuration of A II,

* Pauli, *ZP*, 1923, 16 155

† Landé, *ZP*, 1923, 19 112

‡ Krömer, *ZP*, 1928, 52 542 Cr II

§ Bakker and de Bruin, *ZP*, 1931, 69 19 Ne II

|| Bakker, *K Akad Amsterdam, Proc* 1928, 31 1041 A II

¶ Bakker and de Bruin, *ZP*, 1931, 69 36 Kr II

** Russell and Meggers, *BSJ*, 1932, 9 665 La I, La II

†† Shenstone and Blair, *PM*, 1929, 8 765 Ag II

brings out also one important corollary of the sum rule, $^4P_{2\frac{1}{2}}$ is the only term arising from this configuration which has a J value of $2\frac{1}{2}$, and so its g value must be that predicted by Landé whatever the coupling

Configurations in which the g sum is abnormal are rare, but not unknown, usually the one which deviates is known to overlap another in the energy scale. In $K II^*$ the g sum of terms having a J value of 1 in the configuration $3p^5(^2P)4s$ is 2.57 ± 0.02 instead of 1.50, this configuration overlaps $3p^5 3d$, but not all the $3d$ levels are yet known, so that the combined g sum cannot be calculated. In $Rb II$ a similar deviation occurs, where two configurations overlap †

That the g sum rule is broken when two configurations overlap is of interest, because the quantum mechanics shows that the sum of the magnetic energies of all states having the same projection of angular momentum on the axis of the magnetic field is independent of the coupling between the vectors. As the sum has to be taken over all states of the atom, the theory in this form does not give much information, if, however, only states which have nearly the same energy influence one another, ‡ all empirical results are seen to hang together. The rule is valid for a single multiplet in spectra whose coupling is Russell-Saunders, since a single multiplet is there isolated, it is valid for a configuration when a configuration is isolated, and when two configurations overlap it is still probably true if the sum is extended over both, though this point has not yet been tested.

3 Invariance of the Γ sum

In the Paschen-Back effect evidence is found that the sum of the displacements of those components of a multiplet, which have the same M , remains constant when the strength of the magnetic field varies. That the similar law for the g sum remains valid for any coupling provided the sum is extended to all terms of a configuration, suggests a similar extrapolation for the Γ sum.

Compared with the g sum, the Γ sum is unsatisfactory, whereas

* Whitford, *PR*, 1932, 39 898

† Laporte, Miller and Sawyer, *PR*, 1931, 38 843.

‡ Goudsmit, *PR*, 1931, 37 664

theory can dictate absolute g values, it can dictate displacements only relative to an undetermined centroid, and only as a multiple of an undetermined constant A . Accordingly, the Γ sum rule is not susceptible of direct verification as the g sum rule is, and indeed had there not been some regularities which cried aloud for explanation, it seems hardly probable that Goudsmit could have pushed ahead on so flimsy a scaffolding

Parent spectrum		Derived spectrum	
Spectrum	Interval	Spectrum	Interval
C II Si II P III S IV Cl V Ge II Sn II	$p \Delta^2P$ 64 287 560 950 1495 1768 4253	C I Si I P II S III Cl IV Ge I Sn I	$(^2P) ns \Delta^3P$ 3s 60 4s 45 5s 54 4s 275 5s 283 4s 527 5s 547 4s 450 5s 924 4s 1446 5s 1661 6s 1740 7s 1765 6s 3988 7s 4201
			$(^2D) ns \Delta^3D$ 3s 20
			$(^2P) ns \Delta^3P$ 3s 17
			$(^2P) ns \Delta^3P$ 3s 777 4s 780 5s 778 6s 781 3s 1357 4s 1410 5s 1397 6s 1414 7s 1433 4s 2642 5s 5220 6s 9129
			$(^2D) ns \Delta^3D$ 4s 1508 5s 1506 6s 1506 5s 3530 6s 3532 7s 3539 6s 10132
O II	$p^3 \Delta^2D$ 30 $p^3 \Delta^2P$ 5	O I	
Ne II Na III Ar II K III Kr II Xe II	$p^5 \Delta^2P$ 782 1371 1431 2164 5371 10117	Ne I Na II Ar I K II Kr I Xe I	
Ni II Pd II Pt II	$d^9 \Delta^2D$ 1507 3539 —	Ni I Pd I Pt I	

Fig 18.4 Comparison of doublet intervals with those of the triplets derived from them, when an s electron is added

The regularities mentioned connect the arc and spark spectra of an element. Thus in a number of spectra the triplet resulting from the addition of an s electron to a doublet ground term has the same extreme interval as the doublet. The $^3P_{2,0}$ interval from the $s^2 p(^2P)ns$ configuration of Si I is equal to the 2P interval of Si II, and this is true also of isoelectronic spectra. Again, the $(^2P)ns^3P_{2,0}$ interval of the inert gas spectra is independent of the

chief quantum number n and equal to the 2P interval of the ionic ground term, and the same is true of the very similar $d^9(^2D)ns$ configuration in nickel and palladium (Fig. 18 4) Standing in contrast to these three groups of spectra in which the arc and spark intervals are identical stand the alkaline earth triplets whose extreme intervals obey the doublet laws, just as their spark doublets do

To account for these regularities, assume that the Γ sum rule, like the g sum rule, is valid in a field so strong that the electronic spin and orbital vectors precess independently round the field axis, provided the sum be then taken over all those terms of a configuration which have the same value of M Then construct a table of all the values which ${}_iM_L$ and ${}_iM_S$ can assume in the 2P term of an ion, and of m_l and m_s for the s electron Then if a sufficiently strong field be postulated,

$$\Gamma = A {}_iM_L {}_iM_S$$

and

$$\gamma = a m_l m_s,$$

where A and a are the interval quotients of the ground term of the ion and the electron respectively, A is thus determined by the ionic term, and a presumably by the Landé doublet formula, however, when Fig 18 5 is made out from these equations, it shows that a vanishes provided that m_l is always zero, as it is when the electron moves in an s orbit

${}_iM_L$	m_l	${}_iM_S$	m_s	${}_i\Gamma/A$	γ/a	M_L	M_S	M	Γ/A
1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1	1	2	$\frac{1}{2}$
0				0		0		1	0
-1				$-\frac{1}{2}$		-1		0	$-\frac{1}{2}$
1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	1	0	1	$\frac{1}{2}$
0				0		0		0	0
-1				$-\frac{1}{2}$		-1		-1	$-\frac{1}{2}$
1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	0	1	$-\frac{1}{2}$
0				0		0		0	0
-1				$\frac{1}{2}$		-1		-1	$\frac{1}{2}$
1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	1	-1	0	$-\frac{1}{2}$
0				0		0		-1	0
-1				$\frac{1}{2}$		-1		-2	$\frac{1}{2}$

Fig 18 5 Electronic and atomic displacements in a system consisting of an ion in a 2P state and an s electron

The Γ sums are to be taken over terms having the same value of M , so that this table must be re-arranged in the form of Fig. 18.6

M	2	1	0	1	2
1			1	0	$\frac{1}{2}$
0		1	0	$\frac{1}{2}$	
0		$\frac{1}{2}$	0	$\frac{1}{2}$	
1	$\frac{1}{2}$	0	$\frac{1}{2}$		
$\Sigma \Gamma^2$	$\frac{1}{2}$	0	1	0	$\frac{1}{2}$

Fig. 18.6 Atomic displacements of the system of Fig. 18.5 in a strong field

But the Γ sum is independent of the coupling, so that these sums must be those found empirically in a weak field. If Γ_2 , Γ_1 , Γ_1' and Γ_0 are the displacements of the 3P_2 , 3P_1 , 1P_1 and 3P_0 terms respectively, then arranging these by their M values we obtain Fig. 18.7

M	2	1	0	1	2
2	Γ_2	Γ_1	Γ_2	Γ_1	Γ_2
1		Γ_1'	Γ_1	Γ_1'	
1		Γ_1	Γ_1'	Γ_1	
0			Γ_0		

Fig. 18.7 Atomic displacements of the system of Fig. 18.5 in a weak field

Accordingly

$$\left. \begin{array}{l} \Gamma_2 \\ \Gamma_1 + \Gamma_1' \\ \Gamma_0 \end{array} \right\} \begin{array}{l} \frac{1}{2}A \\ \frac{1}{2}A \\ A \end{array}.$$

These equations show that in the atom

$${}^3P_2 - {}^3P_0 = \Gamma_2 - \Gamma_0 = \frac{3}{2}A,$$

whatever the coupling, and moreover that this is equal to the 2P interval of the ion.

When the addition of an s electron to a 2D term is considered, a similar argument shows that

$${}^3D_3 - {}^3D_1 = \frac{3}{2}A = {}^2D_{3/2} - {}^2D_{1/2},$$

A being now the interval quotient of the ionic 2D term. While

when the method is applied to the alkaline earth triplets, in which a p or d electron is added to a 2S ground term, it shows that the extreme triplet interval should depend only on the electronic quotient a and not at all on the ionic quotient A . Thus the invariance of the displacement sum satisfactorily explains the intervals observed in three different columns of the periodic table.

But if these were the points chiefly needing explanation, they do not limit the usefulness of the theory, with its aid Slater* has explained why the alkali doublets fit a formula developed for X-ray spin doublets, while Goudsmit† has related together the interval quotients of the numerous terms which may arise from a single configuration.

As an example of an X-ray spin doublet, consider the $L\Pi L\Pi$ doublet which arises from a p^5 group of electrons. The orbital and spin vectors of this group, restricted as they are by the exclusion principle, appear in Fig. 18 8

m_l	m_s					M_L M_S	γ/a					M	Γ/a
1 1 0 0 -1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1 $-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	$1\frac{1}{2}$	$-\frac{1}{2}$
1 1 0 -1 -1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0 $-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
1 0 0 -1 -1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1 $-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
	$-\frac{1}{2}$					$-\frac{1}{2}$	$-\frac{1}{2}$					$-1\frac{1}{2}$	$-\frac{1}{2}$

Fig 18 8 Electronic and atomic displacements of a p^5 configuration

These permitted combinations are rearranged according to their M values in Fig 18 9, and the sum of the displacements is obtained. But the only two permitted values of J are $\frac{1}{2}$ and $1\frac{1}{2}$, so

M	$-1\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$
M_S				
$\frac{1}{2}$	—	$\frac{1}{2}$	0	$-\frac{1}{2}$
$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	—
$\Sigma \Gamma_{sl/a}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

Fig. 18 9 Atomic displacements of a p^5 configuration in a strong field arranged by the corresponding values of M and M_S

* Slater, *PR*, 1926, 28 291

† Goudsmit, *PR*, 1929, 31 946

that if the displacements in zero field are $\Gamma_{\frac{1}{2}}$ and $\Gamma_{\frac{1}{2}}$, the displacements in a weak field must be those shown in Fig 18 10

M	$-1\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$
J				
$1\frac{1}{2}$	$\Gamma_{1\frac{1}{2}}$	$\Gamma_{1\frac{1}{2}}$	$\Gamma_{1\frac{1}{2}}$	$\Gamma_{1\frac{1}{2}}$
$\frac{1}{2}$	—	$\Gamma_{\frac{1}{2}}$	$\Gamma_{\frac{1}{2}}$	—
$\Sigma\Gamma_w$	$\Gamma_{1\frac{1}{2}}$	$(\Gamma_{\frac{1}{2}} + \Gamma_{1\frac{1}{2}})$	$(\Gamma_{\frac{1}{2}} + \Gamma_{1\frac{1}{2}})$	$\Gamma_{1\frac{1}{2}}$

Fig 18 10 Atomic displacements of a p^5 configuration in a weak field

Comparison of these two tables shows that, if the Γ sum is independent of the coupling, then

$$\left. \begin{aligned} \Gamma_{1\frac{1}{2}} &= -\frac{1}{2}a \\ \Gamma_{\frac{1}{2}} + \Gamma_{1\frac{1}{2}} &= \frac{1}{2}a \end{aligned} \right\},$$

so that

$$\Gamma_{1\frac{1}{2}} - \Gamma_{\frac{1}{2}} = -\frac{3}{2}a,$$

showing that the interval is equal to that of an alkali P doublet

To extend this method to other groups of equivalent electrons Goudsmit had to postulate Russell-Saunders coupling of the atomic vectors. Thus the d^2 configuration gives rise to the terms 3F , 3P , 1G , 1D and 1S , or six unknowns, since Γ is zero in all singlet terms. Calculations based on an assumed strong field however give only five equations, one for each value of M from 0 to 4, so that the Γ values are indeterminate. Should we assume however that the 3F and 3P terms obey the Landé interval rule, then the

M	-4	-3	-2	-1	0	1	2	3	4
M_S									
1			$-1\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$
					$-\frac{1}{2}$	0	$\frac{1}{2}$		
0				$1\frac{1}{2}$	1	$1\frac{1}{2}$	1	$-\frac{1}{2}$	0
			1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	
		$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-1\frac{1}{2}$	-1		
	0	$-\frac{1}{2}$	-1	$-1\frac{1}{2}$	-2				
-1			$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-1\frac{1}{2}$		
	$1\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$				
$\Sigma\Gamma/a$	$1\frac{1}{2}$	1	$-\frac{1}{2}$	-1	-2	-1	$-\frac{1}{2}$	1	$1\frac{1}{2}$

Fig 18 11 Atomic displacements of a d^2 configuration in a strong field, arranged by the values of M and M_S

displacements of their components may be stated in terms of only two unknowns A_F and A_P . This is shown in Figs 18.11, 18.12

$\begin{smallmatrix} M \\ L J \end{smallmatrix}$	-4	-3	-2	-1	0	1	2	3	4
3F_4	$3A_F$	$3A_F$	$3A_F$	$3A_F$	$3A_F$	$3A_F$	$3A_F$	$3A_F$	$3A_F$
3F_3		$-A_F$	$-A_F$	$-A_F$	$-A_F$	$-A_F$	$-A_F$	$-A_F$	
3F_2			$-4A_F$	$-4A_F$	$-4A_F$	$-4A_F$	$-4A_F$		
3P_2			A_P	A_P	A_P	A_P	A_P		
3P_1				$-A_P$	$-A_P$	$-A_P$			
3P_0					$-2A_P$				
1G_4	0	0	0	0	0	0	0	0	0
1D_2			0	0	0	0	0		
1S_0					0				

Fig 18.12 Atomic displacements of a d^2 configuration in a weak field, arranged by the values of M and L_J

The Γ sum rule shows that

$$\begin{aligned}
 \text{when } M &= 4 & 3A_F &= 1\frac{1}{2}a, \\
 &= 3 & 2A_F &= a, \\
 &= 2 & -2A_F + A_P &= -\frac{1}{2}a, \\
 &= 1 & -2A_F &= -a, \\
 &= 0 & -2A_F - 2A_P &= -2a
 \end{aligned}$$

And these equations are satisfied if

$$A_P = A_F = \frac{1}{2}a$$

Proceeding in this way Goudsmit* was able to calculate the interval quotient A of different components of many supermultiplets in terms of the interval quotients a of p or d electrons, the results are summarised in Fig 18.13

This theory has been strikingly successful in predicting experimental facts. In the d^7 configuration of Ru II all the terms should be inverted except the 2F term, and so in fact experiment shows that they are †. While in the d^3 configuration of Ti II ‡ the relative separations are in very fair agreement with theory, Fig 18.14 shows this, the separation of the 3H term having been fitted to the experimental value §.

* Goudsmit, *PR*, 1928, 31 948

† Meggers and Shenstone, *PR*, 1930, 35 868a

‡ Russell, *Ast. P. J.* 1927, 66 283

§ Pauling and Goudsmit, *Structure of line spectra*, 1930, 163

Con-figuration	Multiplet	Interval quotient	Extreme interval	Con-figuration
p	2P	a	$1\frac{1}{2}a$	$-p^5$
p^2	3P	$\frac{1}{2}a$	$1\frac{1}{2}a$	$-p^4$
p^3	2D	0	0	$-p^3$
	2P	0	0	
d	2D	a	$2\frac{1}{2}a$	$-d^9$
d^2	3F	$\frac{1}{2}a$	$3\frac{1}{2}a$	$-d^8$
	3P	$\frac{1}{2}a$	$1\frac{1}{2}a$	
d^3	4F	$\frac{1}{3}a$	$3\frac{1}{3}a$	$-d^7$
	4P	$\frac{1}{3}a$	$\frac{1}{3}a$	
	2H	$\frac{1}{3}a$	$1\frac{1}{3}a$	
	2G	$\frac{1}{3}a$	$\frac{2}{3}a$	
	2F	$-\frac{1}{6}a$	$-\frac{1}{3}a$	
	2D	$\frac{1}{3}a$	$\frac{1}{3}a$	
	2P	$\frac{1}{3}a$	a	
d^4	5D	$\frac{1}{4}a$	$\frac{5}{4}a$	$-d^6$
	3H	$\frac{1}{10}a$	$\frac{1}{10}a$	
	3G	$\frac{1}{10}a$	$\frac{2}{5}a$	
	3F	$\frac{1}{12}a$	$\frac{1}{12}a$	
	3D	$-\frac{1}{12}a$	$-\frac{1}{12}a$	
	3P	$\frac{1}{2}a$	$1\frac{1}{2}a$	
d^5	All	0	0	$-d^5$

Fig 18 13 Atomic interval quotients of terms arising from shells of p and d electrons in terms of the electronic interval quotients

J	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{2}$
Term						
b^4F	Observed	75 8	103 4	128 4	—	—
	Calculated	74 1	103 6	133 2	—	—
a^4P	32 0	122 3	—	—	—	—
	44 4	74 1	—	—	—	—
a^3H	—	—	—	—	—	97 8 (97 8)
a^3G	—	—	—	120 5 120 0	—	—
b^3F	—	—	-59 9 -51 8	—	—	—
b^3D	—	129 4	—	—	—	—
$-^3D$	—	— ^a	—	—	—	—
		$\Sigma = 74 1$	—	—	—	—
a^3P	125 0 88 9	—	—	—	—	—

^a This state has not been identified

Fig 18 14 Term intervals in the d^3 configuration of Ti II, to obtain the calculated values the 3H term has been fitted to the experimental value

Nevertheless, in spite of its successes the theory has its difficulties, of these one example will suffice. In the $p^5 ns$ terms of neon, theory shows that Γ_0 , $(\Gamma_1 + \Gamma_1')$ and Γ_2 are independent of the coupling and therefore of the chief quantum number. Now Γ has always been interpreted as the displacement of a term from the centroid of the multiplet, so that for the 1P term Γ_1' should be zero, indeed Γ has been assumed zero for all singlet terms in some of the above calculations. But if this assumption is made ($\Gamma_1 - \Gamma_0$)

Term	1P_1 ms_2	3P_0 ms_3	3P_1 ms_4	3P_2 ms_5		
Inter- val m	$\Gamma_1' - \Gamma_0$ $\Gamma_0 - \Gamma_1$ $\Gamma_1 - \Gamma_2$				$\Gamma_0 - \Gamma_2$	$\Gamma_1' + \Gamma_1 - 2\Gamma_0$
1	38040 7 1070 1	39110 8	39470 2 417 4	39887 6	776 8	7107
2	(14506 5) (145 4)	(14651 9) (489 6)	(15141 5) (290 7)	(15432 2)	(780 3)	(-344 2)
3	7272 9 50 2	7323 1	8016 7 693 6	8101 3 84 6	778 2	-6
4	4201 8 21 7	4223 5	4962 1 738 6	5004 8 42 7	781 3	-7169
5	2605 4 11 2	2616 6	3372 4 755 8	3396 7 24 3	780 1	-7446
6	1667 7 7-4	1675 1	2440 0 764 8	2456 1 16 2	781 0	-7574
7	1072 4 4-9	1077 3	1848 5 771 2	1858 1 9 6	780 8	-7663

Figures in brackets have been intrapolated from the series formula

Fig 18 15 Energies and intervals of the s terms of neon; the term values are measured down from the series limit

should be independent of n just as $\Gamma_2 - \Gamma_0$ is; a prediction which experiment does not support. Moreover, the most obvious way of dodging the difficulty is blocked, for if one suggests plausibly enough that when the coupling is no longer Russell-Saunders one may not rightly speak of a singlet term, still though Γ_1' need not be zero, yet $(\Gamma_1 + \Gamma_1')$ must be constant. And if this be admitted, theory predicts that

$$(\Gamma_1 + \Gamma_1') - 2\Gamma_0 = (\Gamma_1 - \Gamma_0) + (\Gamma_1' - \Gamma_0)$$

should be constant, a prediction quite at variance with experiment (Fig 18 15)

4 The intensity sum

That the intensity sum is proportional to the statistical weight $(2J + 1)$ is a thesis, which has been developed by successive stages from the SP combination, in which it is true of a single line, through the general multiplet, PD, to the super-multiplet or configuration in which the sum must be taken over all lines which originate in terms having the same value of J *

This development is so closely analogous to the development of the g and Γ sums, that one expects similar deviations to occur when configurations overlap. And in fact abnormal intensities in Ba I have been ascribed to this cause †

5 General coupling of two electrons

The coupling of electron vectors postulated by Russell and Saunders explains that division of levels into multiplets which is characteristic of light atoms. It explains, for example, why in the $s^2 p$ configuration of Cl the four levels are divided into a triplet below and a singlet above, it even explains the 2:1 interval ratio, but when in Pb I these four levels divide into two diads, the Russell-Saunders coupling fails. What then is to be put in its place?

In the model the interaction of ion and electron is represented as the coupling of four vectors. This means that there are six interactions, but these belong to only four different types, since $(l_1 s_1)$ and $(l_2 s_2)$ are identical and so are $(l_1 s_2)$ and $(l_2 s_1)$, of these four types, $(s_1 s_2)$, $(l_1 l_2)$, $(l_1 s_1)$ and $(l_1 s_2)$, the quantum mechanics states that the fourth may be neglected, unless the atom is extremely light, but the other three are all important. In spectra which conform to the Russell-Saunders type, the spin coupling $(s_1 s_2)$ determines the separation of terms of different multiplicity, the orbital coupling $(l_1 l_2)$ that of terms of different name, and the orbit-spin coupling $(l_1 s_1)$ the multiplet intervals. As the separation of the terms is a measure of the coupling, and in Russell-Saunders spectra terms of different multiplicity are

* Harrison and Johnson, *PR*, 1931, 38 758, give the full theory and compare it with experiment

† Langstroth, *PRS*, 1933, 142 286

widely separated, the $(s_1 s_2)$ coupling must be strong. The $(l_1 l_2)$ coupling is weaker than the $(s_1 s_2)$ but stronger than the $(l_1 s_1)$, for terms with different names but the same multiplicity are less widely separated than terms of different multiplicity but more widely than different components of the same multiplet.*

If the evidence of the quantum mechanics is to be accepted, and the $(l_1 s_2)$ coupling ignored, there are still three alternatives to the coupling postulated by Russell and Saunders. If the latter is written

$$(A) \quad \{(l_1 l_2)(s_1 s_2)\} = \{LS\} = J,$$

the other three are

$$(B) \quad \{(l_1 s_1)(l_2 s_2)\} = \{J_1 J_2\} = J,$$

$$(C) \quad \{[(l_1 s_1)s_2]l_2\} = \{[j_1 s_2]l_2\} = \{j' l_2\} = J,$$

$$(D) \quad \{[(l_1 s_1)l_2]s_2\} = \{[j_1 l_2]s_2\} = \{j'' s_2\} = J$$

These three all show an ion, whose orbital and spin vectors are not at once unlinked when a second electron is added, but differ in the influence the ion has on the coupling of the series electron. Thus in (B) the coupling $(l_2 s_2)$ is preserved and the atomic resultant J appears as the sum of two electronic vectors $j_1 j_2$. In (C) the ion shows a special attraction for the spin vector s_2 of the second electron, and so breaks the $(l_2 s_2)$ coupling, while in (D) the ion attracts particularly the l_2 vector.

All three coupling types have been considered by those who have tried to interpret abnormal spectra, in particular, the magnetic g factors have been calculated and compared with experiment in more than one spectrum,[†] but the result of this work has so far been to show that types (C) and (D) have no advantage over the simpler (JJ) coupling of type (B). Incidentally, too, the three are identical when the electron added occupies an s orbit.

The further discussion of abnormal spectra will therefore be simplified by treating the (JJ) coupling as though it were the only alternative to that postulated by Russell and Saunders.

* Hund, *Linienspektren*, 1927, 91 f

† For g factors of all four coupling types, see p. 145

6 (jj) coupling

In the change from weak to strong magnetic field, while the energies change, the number of states and their magnetic quantum numbers, M , do not, similarly, in the change from (LS) to (jj) coupling, while the relative energies change, the J values do not. Accordingly, in the complex spectrum of a heavy metal, such as platinum, values of J can be assigned to the empirical terms, but the Russell-Saunders notation, depending as it does on L and S , is of little use

l_1	l_2	j_1	j_2	J	Names if coupling is R S
0	0	$\frac{1}{2}$	$\frac{1}{2}$	0 1	1S_0 3S_1
0	1	$\frac{1}{2}$	$\frac{1}{2}$ $\frac{3}{2}$	0 1 1 2	3P_0 3P_1 1P_1 3P_2
1	1	$\frac{1}{2}$	$\frac{1}{2}$ $\frac{3}{2}$ $\frac{3}{2}$ $\frac{1}{2}$ $\frac{3}{2}$ $\frac{1}{2}$ $\frac{3}{2}$	0 1 1 2 1 2 0 1 2 3	1S_0 3P_0 1P_1 3S_1 3P_1 1D_1 1D_2 3P_2 3D_1 3D_3

Fig 18 16 Terms arising from the jj coupling of two unlike electrons in s s, sp and p p states, the J values are the same as those of the corresponding Russell-Saunders terms

To illustrate the invariance of J , consider the terms arising from the (jj) coupling of two electrons. The values of j permitted a single electron are $(l \pm \frac{1}{2})$, and the terms arising from the combination of two electrons can be deduced by combining the two j vectors. When the electrons are not equivalent, \mathbf{J} is simply the vectorial sum of \mathbf{j}_1 and \mathbf{j}_2 , and there are no restrictions, thus two electrons having j values of $1\frac{1}{2}$ and $2\frac{1}{2}$ produce four states having J values of 1, 2, 3, 4. Other examples are given in Fig 18 16. On the other hand, when the two electrons are equivalent, the Pauli exclusion principle does not allow two electrons with the same

resulting terms. The two ionic levels are defined by j_1 , which here assumes the values $1\frac{1}{2}$ and $\frac{1}{2}$. The addition of the second electron splits each of the levels in two, for j_2 can orient itself parallel or anti-parallel to j_1 . j_2 is here identical with s_2 , since l_2 is zero. Thus the atomic level scheme consists of two 'diads', and the

Con- figura- tion	Number of elec- trons in which j is			Number of levels in which J is						
	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	0	1	2	3	4	5	6
p^2	2			1						
	1	1		1	1	1				
d^2		2		1						
		1	1	1	1	1	1	1		
d^4		4		1						
		3			1		1	1		
		2	1	2	1	4	2	3	1	1
		1	3	1	2	2	3	2	1	1
Con- figura- tion										
	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{2}$	$6\frac{1}{2}$
	2	1			1					
	1	2		1	1	1				
d^3		3			1					
		2		1	1		1	1		
		1	1	1	2	2	2	1	1	
			3		1	1		1		
d^5		4	1			1				
		3	2		1	2		1	1	
		2	3		2	3	3	3	1	
		1	4		1	2	2	1	1	1
			5			1				

Fig 18 19 Terms arising from various numbers of equivalent p and d electrons

distance between the diads, being due to the coupling ($l_1 s_1$), must be greater than the interval of either diad, for the latter is due to ($j_1 j_2$). Further, the relative positions of the two terms can be foretold, for the energy is due primarily to the interaction of l_1 and s_1 and secondarily to the interaction of s_1 and s_2 . Hund's rule states that the energy of interaction of s_1 and s_2 is small

when the angle between them is small, so that in the lower level of each diad s_1 and s_2 will be parallel. The levels constructed on this principle are shown in Figs 18 20 and 18 22, the former

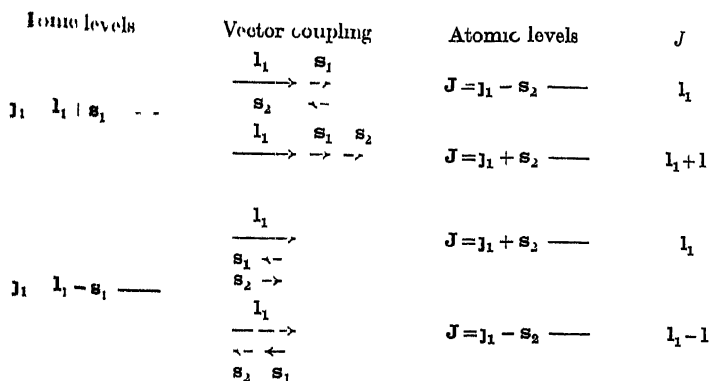


Fig 18 20 Energy levels resulting when an s electron is added to an ion in an erect doublet state, jj coupling is assumed

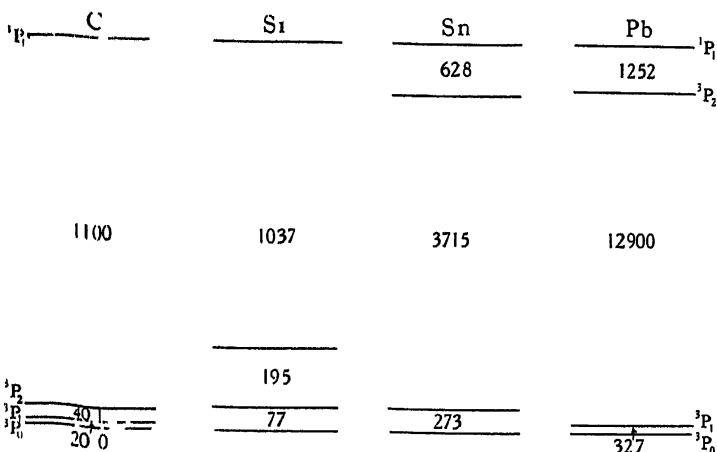


Fig 18 21 Intervals of low s^2p ns terms in four elements of column IV, the scale is adjusted so as to make the $^1P_1 - ^3P_0$ interval the same in all elements. Note the change from LS coupling in Cr to jj in Pb.

representing an erect and the latter an inverted parent term, the first agrees well with the empirical s^2p ns terms of tin and lead (Fig 18 21), and the second with the $p^5(^2P)$ ns terms of the inert gases (Fig 18 23)

But one may ask, why does the addition of an s electron to a 2P ground term produce two diads in columns IV and VIII, when the addition of a p electron to a 2S ground term produces triplets and singlets in column II. The couplings which have to be contrasted are

$$\{\mathbf{s}_1 (\mathbf{l}_2 \mathbf{s}_2)\} = \{\mathbf{s}_1 \mathbf{j}_2\} \quad \text{in the alkaline earths,}$$

$$\text{and} \quad \{(\mathbf{l}_1 \mathbf{s}_1) \mathbf{s}_2\} = \{\mathbf{j}_1 \mathbf{s}_2\} \quad \text{in column IV}$$

In both the coupling of the electronic vector \mathbf{j} with the isolated spin vector will decrease rapidly in strength with increase of the chief quantum number, the coupling energy being roughly proportional to n^{*-3} . But the interaction of the orbital vector with its own spin vector differs in the two columns, in the alkaline earths the orbital vector concerned is the orbital vector of the series electron, so that the coupling $(\mathbf{l}_2 \mathbf{s}_2)$ will decrease in strength with increase of the chief quantum number just as the $\{\mathbf{j}\mathbf{s}\}$ coupling does, in column IV, on the other hand, the orbital vector concerned is the orbital vector of the ion, and the coupling of this with its own spin vector remains unchanged throughout the series, being determined by the doublet interval of the ion.*

In the addition of an s electron to a 2P term, the energy sequence of J values happens to be identical with that produced by (LS) coupling, but the two will not always agree. In the $5s^2 5p^4(^3P) 6s$ configuration of Ir^\dagger the five levels can be named $^4P_{2\frac{1}{2}, 1\frac{1}{2}, \frac{1}{2}}$, $^2P_{1\frac{1}{2}, \frac{1}{2}}$, for the J values occur in that order from below upwards, but the intervals are 1459, 4803, 924 and 4530 cm^{-1} , and so are actually more consistent with a (JJ) coupling, for they divide the terms into a lower diad, which we interpret as $(^3P_2 6s)_{2\frac{1}{2}, 1\frac{1}{2}}$, a middle diad $(^3P_1 6s)_{\frac{1}{2}, 1\frac{1}{2}}$ and an upper monad $(^3P_0 6s)_{\frac{1}{2}}$, the critical will not fail to observe, however, that if the (JJ) coupling was rigid, the J values of the middle diad would be interchanged.

But in fact even in those spectra in which the terms arising from different ionic states are most widely separated, spectra such as Rn I , Cs II , Xe I and Xe II , the individual terms of a configuration are arranged in no consistent order. In Rn I the four

* Pauling and Goudsmit, *Structure of line spectra*, 1930, 104

† Evans, S. F., *PRS*, 1931, 133 417

terms (${}^2P_{1\frac{1}{2}} P_{1\frac{1}{2}}$) might lie consistently below the two terms (${}^2P_{1\frac{1}{2}} P_{\frac{1}{2}}$), but in fact the six empirical terms cannot be separated into two groups

7. Calculation of g for any coupling

Less important than the sum rule at the moment because data is scanty, but destined perhaps to be equally useful, is the calculation of g for any coupling. Following Landé, Goudsmit and Uhlenbeck* have shown that if any vector \mathbf{Z} is the resultant of two other vectors \mathbf{X} and \mathbf{Y} , so that the permissible values of Z are determined by

$$\begin{aligned} \text{then} \quad & |X + Y| \geq Z \geq |X - Y|, \\ g(Z) = & \frac{Z(Z+1) + X(X+1) - Y(Y+1)}{2Z(Z+1)} g(X) \\ & + \frac{Z(Z+1) + Y(Y+1) - X(X+1)}{2Z(Z+1)} g(Y) \end{aligned}$$

With this assumption consider the four coupling types of an earlier section

$$(A) \quad \{(l_1 l_2)(s_1 s_2)\} = \{\mathbf{L}\mathbf{S}\} = J$$

J is the sum of the \mathbf{L} and \mathbf{S} , so

$$\begin{aligned} g(J) = & \frac{J(J+1) + L(L+1) - S(S+1)}{2J(J+1)} g(L) \\ & + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} g(S). \end{aligned}$$

Now \mathbf{L} is the sum of the vectors \mathbf{l}_1 and \mathbf{l}_2 , so

$$\begin{aligned} g(L) = & \frac{L(L+1) + l_1(l_1+1) - l_2(l_2+1)}{2L(L+1)} g(l_1) \\ & + \frac{L(L+1) + l_2(l_2+1) - l_1(l_1+1)}{2L(L+1)} g(l_2) \end{aligned}$$

$$= 1, \text{ if } g(l) = 1 \text{ for all values of } l$$

And similarly $g(S) = 2$, if $g(s) = 2$ for all values of s . Substituting these values in the expression for $g(J)$ gives at once the usual Landé formula

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

* Goudsmit and Uhlenbeck, *ZP*, 1926, **35** 618

This analysis brings out, perhaps more clearly than any other, the cause of the whole anomalous Zeeman effect, which is nothing else than the double magnetism of the electron

If there are more than two electrons active one may assume that the binding of s_1 and s_2 is tighter than that of their resultant $s_{1,2}$ with s_3 , then one may first work out $g(s_{1,2})$ and then combine $s_{1,2}$ with s_3 . But for the Russell-Saunders coupling $g(L)$ is always 1 and $g(S)$ always 2

$$(B) \quad \{(l_1 s_1)(l_2 s_2)\} = (j_1 j_2) - J$$

J is compounded of j_1 and j_2 , so

$$g(J) = \frac{J(J+1) + j_1(j_1+1) - j_2(j_2+1)}{2J(J+1)} g(j_1) \\ + \frac{J(J+1) + j_2(j_2+1) - j_1(j_1+1)}{2J(J+1)} g(j_2)$$

The $g(j_1)$ of this expression appears in Landé's table simply as the g factor of some doublet term, for j_1 is the sum of l_1 and s_1 , only one electron being active

Very similar to this (JJ) coupling for two electrons is the binding of the series electron near the limit of a term sequence, indeed the equation is unaltered, save that the ground term of the ion is not necessarily a doublet

(C) In the coupling

$$\{[(l_1 s_1) s_2] l_2\} = \{[j_1 s_2] l_2\} = \{j' l_2\} = J,$$

J is the resultant of j' and l_2 , so

$$g(J) = \frac{J(J+1) + j'(j'+1) - l_2(l_2+1)}{2J(J+1)} g(j') \\ + \frac{J(J+1) + l_2(l_2+1) - j'(j'+1)}{2J(J+1)} g(l_2)$$

In this equation $g(l_2) = 1$, and j' is the resultant of j_1 and s_2 , so that

$$g(j') = \frac{j'(j'+1) + j_1(j_1+1) - s_2(s_2+1)}{2j'(j'+1)} g(j_1) \\ + \frac{j'(j'+1) + s_2(s_2+1) - j_1(j_1+1)}{2j'(j'+1)} g(s_2)$$

Here $g(j_1)$ may be taken from Landé's doublet table, while $g(s_2)$ is 2, as always for a single electron

(D) The coupling $\{[(1s_1)l_2]s_2\}$ is so similar to (C) that it need not be worked out in detail

The values to be predicted by each of these four coupling types have been worked out for certain spectra whose Zeeman splitting is irregular, but no close agreement with experiment has ever been found. Coupling types (B), (C) and (D) often give better

J	Coupling scheme							g values					J
	A	B		C	D			A	B	C	D	Em- pirical	
		j_1	j_2	j'	l_2	j''	s_2						
$3\frac{1}{2}$	$^4D_{3\frac{1}{2}}$	2	$1\frac{1}{2}$	$2\frac{1}{2}$	1	3	$\frac{1}{2}$	1.43	1.43	1.43	1.43	1.43	$3\frac{1}{2}$
	$^4P_{2\frac{1}{2}}$	2	$1\frac{1}{2}$	$2\frac{1}{2}$	1	2	$\frac{1}{2}$	1.60	1.44	1.53	1.53	1.60	$2\frac{1}{2}$
	$^4D_{2\frac{1}{2}}$	1	$1\frac{1}{2}$	$1\frac{1}{2}$	1	2	$\frac{1}{2}$	1.37	1.40	1.40	1.40	1.33	
	$^2D_{2\frac{1}{2}}$	2	$\frac{1}{2}$	$1\frac{1}{2}$	1	3	$\frac{1}{2}$	1.20	1.33	1.24	1.24	1.24	
								Σg	4.17	4.17	4.17	4.17	
$1\frac{1}{2}$	$^4P_{1\frac{1}{2}}$	2	$1\frac{1}{2}$	$1\frac{1}{2}$	1	1	$\frac{1}{2}$	1.73	1.47	1.49	1.50	1.73	$1\frac{1}{2}$
	$^4D_{1\frac{1}{2}}$	0	$1\frac{1}{2}$	$1\frac{1}{2}$	1	2	$\frac{1}{2}$	1.20	1.33	1.29	1.30	1.20	
	$^4S_{1\frac{1}{2}}$	1	$\frac{1}{2}$	$2\frac{1}{2}$	1	1	$\frac{1}{2}$	2.00	1.67	1.84	1.83	2.00	
	$^2D_{1\frac{1}{2}}$	2	$\frac{1}{2}$	$\frac{1}{2}$	1	2	$\frac{1}{2}$	0.80	1.22	1.11	1.10	0.90	
	$^2P_{1\frac{1}{2}}$	1	$1\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	1.33	1.38	1.33	1.33	1.23	
								Σg	7.06	7.06	7.06	7.06	
$\frac{1}{2}$	$^4P_{\frac{1}{2}}$	1	$\frac{1}{2}$	$1\frac{1}{2}$	1	0	$\frac{1}{2}$	2.67	1.78	2.11	2.00	2.67	$\frac{1}{2}$
	$^4D_{\frac{1}{2}}$	0	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	0.00	0.67	0.67	0.67	0.00	
	$^2P_{\frac{1}{2}}$	1	$1\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	0.67	1.22	0.89	1.00	0.99	
	$^2S_{\frac{1}{2}}$	2	$1\frac{1}{2}$	$1\frac{1}{2}$	1	1	$\frac{1}{2}$	2.00	1.67	1.67	1.67	1.68	
								Σg	5.33	5.33	5.33	5.34	

Fig. 18.24. g factors of the p^4 (1P) $4p$ terms of A II compared with values calculated for four different couplings. The sum rule is valid although the g factors are abnormal. (Bakker, *K. Akad. Amsterdam*, 1928, 31.1041.)

agreement for certain terms, but none of them has been found to give close agreement even for one configuration in one element. A good example of results obtained is afforded by the (3P) $4p$ configuration of A II (Fig. 18.24).

As there is evidence that departures from the interval rule and Landé's g formula are both signs of a break from the Russell-Saunders coupling, a study of the variation of g in a series or in a sequence of homologous spectra has been a long felt want. Recently Pogány* has supplied the need by studying the change in g in the p^5 (2P) ns terms of the inert gases. This configuration produces two terms having a J value of 1, and in neon both in-

* Pogány, *ZP*, 1935, 93.376

tensity laws and magnetic splitting factors show that the lower is 3P_1 and the higher 1P_1 . * Of these 3P_1 approaches the lower limit of the ion, and must therefore flow to the (jj) coupling term of ($^2P_{1\frac{1}{2}}$ s), with a g value of $7/6$, while the higher term approaches the higher limit and must flow to ($^2P_{\frac{3}{2}}$ s) $_1$ with a g value of $4/3$. Thus as the coupling changes the magnetic splitting factor of

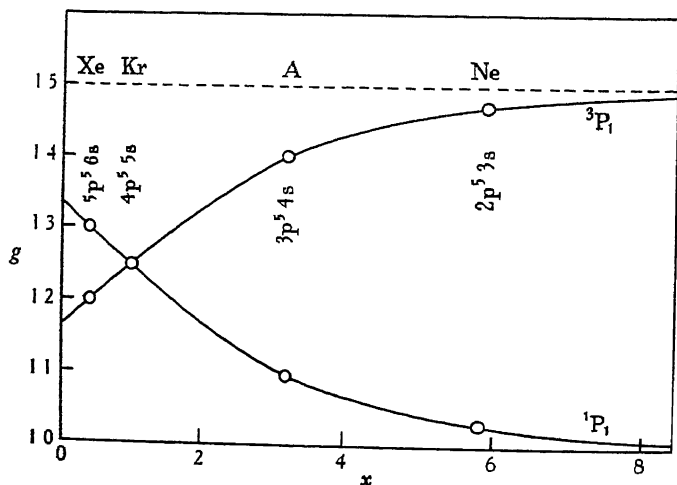


Fig 18.25 g factors of the s terms of the inert gases, showing the transition from LS to jj coupling (After Pogány, ZP, 1935, 93-376)

the 3P_1 term flows from 1.5 to 1.17, while the g factor of the 1P_1 term flows from 1.0 to 1.33, this shows that the g values must be equal that for some particular coupling the two g values must be equal. Experiment has not yet revealed the steps in this transition in the series of neon, but the low terms of all inert gases are readily accessible and Pogány has shown five steps in the analogous transition from the (LS) coupling characteristic of light to the (jj) characteristic of heavy atoms, in neon the g values approach the ideal values of 1.5 and 1.0, being 1.47 and 1.03, whereas in krypton they are equal, and in xenon become 1.20 and 1.30 (Fig 18.25). A similar transition can be detected in the p terms (Fig. 18.26).

* See p. 42

(LS) coupling	Neon (² P) 3p	Argon (² P) 4p	Krypton (² P) 5p	Xenon (² P) 6p	(J) coupling
Term g	Term g	Term g	Term g g	Term g	J_1 J_2 g
³ P ₀ 0 ¹ S ₀ 0	P ₁ 0 P ₃ 0	P ₁ 0 P ₅ 0	P ₁ 0 0 P ₅ 0 0	P ₁ 0 P ₅ 0	$\frac{1}{2}$ $\frac{1}{2}$ 0 $1\frac{1}{2}$ $1\frac{1}{2}$ 0
³ P ₁ 1.5 ¹ P ₁ 1.0 ³ D ₁ 0.5 ³ S ₁ 2.0	P ₂ 1.340 P ₅ 0.999 P ₇ 0.699 P ₁₀ 1.984	P ₂ 1.379 P ₄ 0.819 P ₇ 0.840 P ₁₀ 1.962	P ₃ 1.425 1.384 P ₄ 0.619 0.635 P ₇ 1.028 1.046 P ₁₀ 1.891 1.820	P ₂ P ₄ P ₇ 1.02 P ₁₀	$\frac{1}{2}$ $1\frac{1}{2}$ 1.5 $\frac{1}{2}$ $\frac{1}{2}$ 0.67 $1\frac{1}{2}$ $1\frac{1}{2}$ 1.33 $1\frac{1}{2}$ $\frac{1}{2}$ 1.5
³ P ₂ 1.5 ³ D ₂ 1.17 ¹ D ₂ 1.0	P ₄ 1.301 P ₆ 1.229 P ₈ 1.137	P ₃ 1.248 P ₆ 1.302 P ₈ 1.121	P ₂ 1.163 P ₆ 1.400 1.406 P ₈ 1.116 1.110	P ₃ 1.183 P ₆ 1.402 P ₈ 1.113	$\frac{1}{2}$ $1\frac{1}{2}$ 1.17 $1\frac{1}{2}$ $1\frac{1}{2}$ 1.33 $1\frac{1}{2}$ $\frac{1}{2}$ 1.17
³ D ₃ 1.33	P ₉ 1.329	P ₉ 1.333	P ₉ 1.333 1.333	P ₉	$1\frac{1}{2}$ $1\frac{1}{2}$ 1.33

Fig 18.26 g factors of the p terms of the inert gases, showing the transition from LS to J coupling. These terms arise as p⁵ (²P) η p

Sources: Ne Back, *AP*, 1925, 76 330, which is in good agreement with Murakawa and Iwama, *Inst Phys and Chem Tokyo*, 1930, 13 289. A. Pogány, *ZP*, 1935, 93 364, which agrees well with Terrien and Dijkstra, *J de Phys* 1934, 5 443. Kr and Xe Pogány, *ZP*, 1935, 93 376.

8 Electronic displacements

In an earlier chapter, while trying to explain why terms are inverted if they arise from a shell more than half full, resort was had to the theory that the atomic displacement Γ may be regarded as the sum of electronic displacements γ_1 and γ_2 , and it was shown that provided the coupling is Russell-Saunders, the relation between the two is

$$\begin{aligned}\Gamma &= \gamma_1 + \gamma_2 = \cos(\mathbf{LS}) \sum a_1 l_1 \cos(l_1 \mathbf{L}) s_1(s_1 \mathbf{S}) \\ &= LS \cos(\mathbf{LS}) \sum a_1 \frac{l_1}{L} \cos(l_1 \mathbf{L}) \frac{s_1}{S} \cos(s_1 \mathbf{S}).\end{aligned}\quad (13.2)$$

The quantity summed in this expression is commonly abbreviated as the interval quotient A , so that

$$A = \sum a_1 \frac{l_1}{L} \cos(l_1 \mathbf{L}) \frac{s_1}{S} \cos(s_1 \mathbf{S}) \quad (18.1)$$

If this equation is applied to the addition of an electron to an ion, the expression for the interval quotient A will in general contain two interval quotients a_1 and a_2 , and no method is known

separating these. But under certain conditions the expression simplifies, thus if either the ion or the electron is in an s state, or if the ion is in a singlet state, one of the interval quotients vanishes, while if the two electrons are equivalent a_1 and a_2 are equal. The method cannot be extended to the combination of three electrons, unless two are assumed so tightly linked that they are quite undisturbed by the third electron.

Thus consider the addition of an electron ($l_2 s_2$) to an ionic S term defined by ($l_1 s_1$), then since l_1 is zero, a_1 vanishes from the expression for A , and there is left simply

$$A = a_2 \frac{l_2}{L} \cos(l_2 \mathbf{L}) \frac{s_2}{S} \cos(s_2 \mathbf{S}) \quad (18\ 2)$$

In this equation

$$\begin{aligned} \frac{l_2}{L} \cos(l_2 \mathbf{L}) &= \frac{l_2(l_2+1) + L(L+1) - l_1(l_1+1)}{2L(L+1)} \\ &= 1, \text{ when } l_1=0 \text{ and } L=l_2, \end{aligned}$$

so that the expression for A simplifies to

$$A = a_2 \frac{s_2(s_2+1) + S(S+1) - s_1(s_1+1)}{2S(S+1)} \quad (18\ 3)$$

Now when an electron is added to a term with spin vector s_1 , two terms will be produced with spin moments $(s_1 + \frac{1}{2})$ and $(s_1 - \frac{1}{2})$, if these have interval quotients A_+ and A_- ,

$$\begin{aligned} A_+ &= a_2 \frac{\frac{3}{4} + (s_1 + \frac{1}{2})(s_1 + \frac{3}{2}) - s_1(s_1+1)}{2(s_1 + \frac{1}{2})(s_1 + \frac{3}{2})} \\ &= a_2/(2s_1+1) \end{aligned}$$

And similarly it may be shown that

$$A_- = \frac{-a_2}{(2s_1+1)}$$

Applied to the alkaline earth spectra the expression for A_+ leads directly to the results already obtained by the sum rule, but applied to the $d^5(^6S)np$ configuration of Cr I and Mo I , the formulae lead to results which are new, thus the 7P term should be erect and the 5P term inverted, while both should have the same interval quotient. In Cr I the two terms obey the interval rule well, and the 5P term is inverted, but the interval quotients

instead of being equal bear a ratio of 9 : 1 (Fig 18 27) In Mo I the terms do not obey the interval rule and both are erect, while if the theory is applied to the $p^3 (^4S) np$ configuration of column VI the results are even more unsatisfactory

Agreement, however, is only to be expected if the p^3 and d^5 groups are so firmly bound in an S state that the addition of the

Terms		Cr I		Mo I	
		$\Delta\nu$	A	$\Delta\nu$	A
3d ⁵ (⁶ S) 4p for Cr I and 4d ⁵ (⁶ S) 5p for Mo I	⁷ P ₂	81.4	27.1	257	86
	⁷ P ₃	112.5	28.1	449	112
	⁷ P ₄				
	⁵ P ₁	-5.7	-2.9	121	61
	⁵ P ₂	-8.8	-2.9	87	29
	⁵ P ₃				

Fig 18 27 Intervals of the $d^5 (^6S) p$ configuration in Cr I and Mo I

p electron does not disturb them, and if the coupling of the ion and electron is Russell-Saunders. That experiment does not here support theory shows only that these assumptions are invalid, for a very valuable application of the same theory has been made to the displaced terms of beryllium, magnesium and isoelectronic spectra. The $3s 3p^3 P^o$ and $3p^2 3P$ of the magnesium-like spectra have many of them identical intervals as Fig 18 28 shows. Now according to the above theory the intervals should be determined by

$$\Gamma = ALS \cos(\mathbf{LS}),$$

$$\text{where } A = a_1 \frac{l_1}{L} \cos(\mathbf{l}_1 \mathbf{L}) \frac{s_1}{S} \cos(\mathbf{s}_1 \mathbf{S}) \\ + a_2 \frac{l_2}{L} \cos(\mathbf{l}_2 \mathbf{L}) \frac{s_2}{S} \cos(\mathbf{s}_2 \mathbf{S}) \quad (18.4)$$

For the $3s 3p$ configuration the first term of this equation is zero, and there remains simply $A = \frac{1}{2}a_2$. For the $3p^2$ configuration, on the other hand, the two terms of this equation are equal, and since $L = S = l_1 = l_2 = 1$ and $s_1 = s_2 = \frac{1}{2}$, there results again $A = \frac{1}{2}a_2$. Thus the interval quotients are equal, and since both are 3P terms their displacements and intervals are also equal.

Bechert* has advanced even further than this and has been able to apply a theory of Goudsmit's† to account for the discrepancies from the 2 : 1 interval ratio, but the theory involves the use of the quantum mechanics and is beyond the scope of this book

	3s 3p $^3P^{\circ}$			3p 2 3P		
	$\Delta^3P^{\circ}_{1,2}$	$\Delta^3P^{\circ}_{0,1}$	$\Delta^3P^{\circ}_{1,2}/\Delta^3P^{\circ}_{0,1}$	$\Delta^3P_{1,2}$	$\Delta^3P_{0,1}$	$\Delta^3P_{1,2}/\Delta^3P_{0,1}$
Mg I	40.6	20.1	2.02	40.6	20.5	1.98
Al II	122.6	64.1	1.91	122.6	59.7	2.05
Si III	261.3	130.7	2.00	261.3	130.1	2.01
P IV	469.1	227.1	2.07	469.1	234.7	2.00
S V	767.1	360.4	2.13	767.1	412.4	1.86
Cl VI	1183.7	521.3	2.27	1183.7	668.0	1.77

Fig. 18.28. $sp\ ^3P^{\circ}$ and $p^2\ ^3P$ intervals in spectra isoelectronic with Mg I

9. Abnormal Intensities

Kronig has shown that the intensity tables given in the previous chapter may be applied to configurations in which the coupling is (jj), if S is replaced by j_1 and L by j_2 , where j_1 is taken to be the quantum number which does not change in the transition ‡

Abnormal intensities due to (jj) coupling must be distinguished from those due to the overlapping of multiplets and configurations, to obtain clear evidence of the validity of this formula, one would wish to measure the intensities of lines arising in a configuration well separated from all others and exhibiting the intervals characteristic of (jj) coupling

10. Perturbed terms

There is one factor producing abnormal series, interval ratios, g factors and intensities, which has only recently received the attention it deserves. It is known as perturbation, and is best introduced by a study of a perturbed series

Of perturbed series, the diffuse series of calcium affords a clear

* Bechert, *ZP*, 1931, 69 735

† Goudsmit, *PR*, 1930, 35 1325

‡ White and Eliason, *PR*, 1933, 44 753. Bartlett, *PR*, 1929, 34 1247, derived formulae for (jj) coupling

example The first four members of the 3D term sequence contract in the usual way, but the next three expand anomalously, the expansion reaches a maximum at the seventh term, the succeeding triplets contracting again as they approach the series limit. This may be shown in two ways, by plotting the displacement of

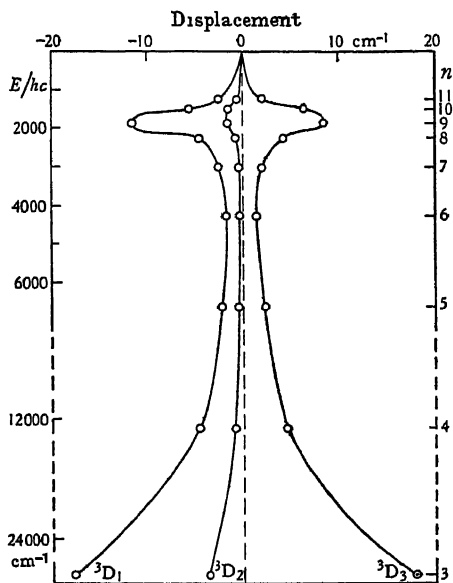


Fig 18.29 The $4s\ nd\ ^3D$ term series of calcium showing perturbation. The abscissae are the displacements from the centroid, which is shown dotted.

each term from the centroid against the energy (Fig 18.29) and by plotting the effective quantum number against the energy (Fig 18.30).

Similar abnormalities have been noticed in the 3F series of $Al\ \Pi^*$ and the 2P series of $Cu\ I$,† and their cause has been elucidated by Russell and Shenstone‡. They show that these anomalous series only occur when a term of another series intrudes, thus between the sixth and seventh terms of the $4s\ nd\ ^3D$ sequence of

* Sawyer and Paschen, *AP*, 1927, 84.1

† Shenstone, *PR*, 1929, 34.1623

‡ Russell and Shenstone, *PR*, 1932, 39.415. White, *Introduction to atomic spectra*, 1934, gives a clear and full account of this work.

calcium there lies the $3d\ 5s\ ^3D$ term, a term be it noted with the same values of L , S and J , but with an interval much greater than that to be expected at this height in the $4snd$ series (Fig 18 32) When this intruding term is excluded from the sequence, all succeeding values of the quantum defect are reduced by one, and the graph of the defect against the energy comes to resemble an anomalous dispersion curve (Fig 18 31) Moreover, if a formula

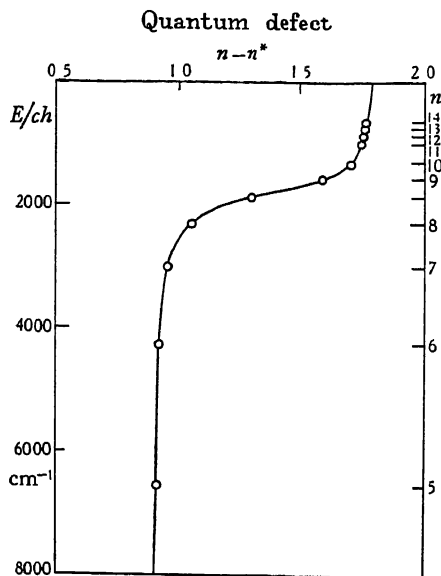


Fig 18 30 The quantum defect ($n - n^*$) for the 3D term sequence of Ca I with the intruding term included

is developed on anomalous dispersion lines,* very good agreement with experiment can be obtained

Clearly all spectroscopic terms do not perturb one another, analysis alone does not reveal what conditions are essential, but the quantum mechanics provides an answer to this important question. If two levels have the same characteristic quantum numbers and lie close together, the eigenfunction of each level contains some components of the eigenfunction of the other, so that both levels belong in part to both configurations. Stated in other words this means that when two states perturb one another,

* Langer, *PR*, 1930, 35 649a

there is a certain probability that the atom will jump back and forth between the two without energy being radiated. The assignment of a given level to a definite electron coupling is therefore indefinite, it becomes more definite the further the levels are apart. More precisely two levels perturb one another when both have the same J and both are odd or both even, the two terms

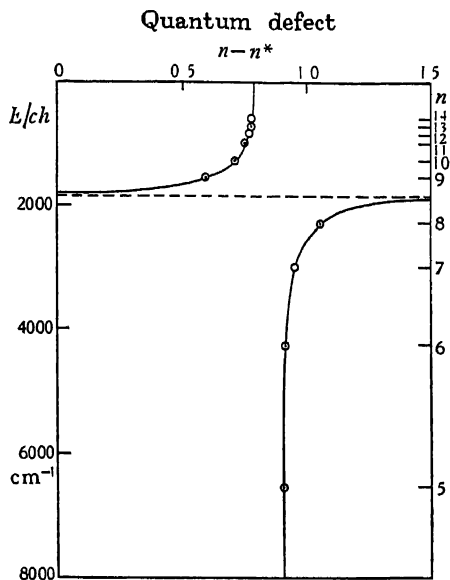


Fig. 18.31 The quantum defect ($n - n^*$) for the $4D$ term sequence of $Ca\ I$ with the intruding term removed. Note how similar this graph is to that characteristic of anomalous dispersion.

need not belong to the same configuration, or be derived from the same state of the ion, but experiment shows that the perturbation is greater when the two terms have the same values of L and S if the coupling is (LS) or the same values of j_1 and j_2 if the coupling is (JJ).

Since the assignment of electron coupling is indefinite, each perturbed level is likely to have some of the properties of the other, and in fact much evidence has accumulated to show that the perturbing levels share their interval factors, their magnetic splitting factors and their intensities.

The quantum mechanics shows that in terms of energy perturbation manifests itself essentially as a repulsion, but it is not difficult to show that repulsion leads logically to a sharing of intervals. Consider for example a narrow triplet lying close below a wide one, if one of these is odd and the other even, they will not perturb one another, and if the coupling is (LS) they will appear as shown on the left of Fig 18 33. If however both terms are

Observed term value cm ⁻¹	Configura- tion	<i>n</i>	<i>n</i> *	<i>n</i> - <i>n</i> *	Calculated term value cm ⁻¹	<i>T</i> (obs) - <i>T</i> (calc)
28969 1	4s 3d	3	1 946	1 054	26465	2504
11556 4	4s 4d	4	3 081	0 919	11556	0
6561 4	4s 5d	5	4 090	0 910	6562	- 1
4255 5	4s 6d	6	5 078	0 922	4252	+ 3
3002 4	4s 7d	7	6 045	0 955	3002	0
2268 3	4s 8d	8	6 955	1 045	2287	- 19
1848 9	3s 5d		7 702		Foreign term	
1551 3	4s 9d	9	8 410	0 590		0
1273 1	4s 10d	10	9 284	0 716	1276	- 3
1045 6	4s 11d	11	10 244	0 756	1048	- 2
889 8	4s 12d	12	11 232	0 768	873	- 3
734 0	4s 13d	13	12 227	0 773	737	- 3
628 0	4s 14d	14	13 219	0 781	630	- 2

Fig 18 32 Series calculations of the anomalous 4s nd ³D₁ series of calcium. The calculated term values are obtained from a formula designed on the lines of those used to explain anomalous dispersion. (After Russell and Shenstone, *PR*, 1932, 39 415.)

even, the levels with the same *J* will repel one another, and as the repulsion is greater the nearer the levels are to one another, the ³P₀ terms which are nearest undergo the greatest displacement, thus the narrow triplet is widened and the wide triplet is narrowed, this is shown on the right side of the figure. Moreover, if the narrow term lies above the wide, the same sharing of the intervals results.

Of the sharing of magnetic splitting factors much evidence has accumulated. Thus in Zr I the ³P₂ and ¹D₂ terms of the 4d² 5s² configuration should have *g* values of 1.5 and 1.0, whereas in fact both have the same *g* value, 1.25, these terms are separated by 915 cm⁻¹*. Further examples from Zr I are shown in Fig 18 34,

* Kiess, C. C. and Kiess, H. K., *BSJ*, 1931, 6 660

while the phenomenon has been observed in Ti I , Cu I , Ni II^* and La II . In all these spectra the two perturbing terms belong to the same configuration, but the deviating g sums of K II , Rb II and Pn III make it probable that this is not an essential condition. In Pb III^\dagger the g value of the $6s\ 6p\ ^3P_2$ term, which

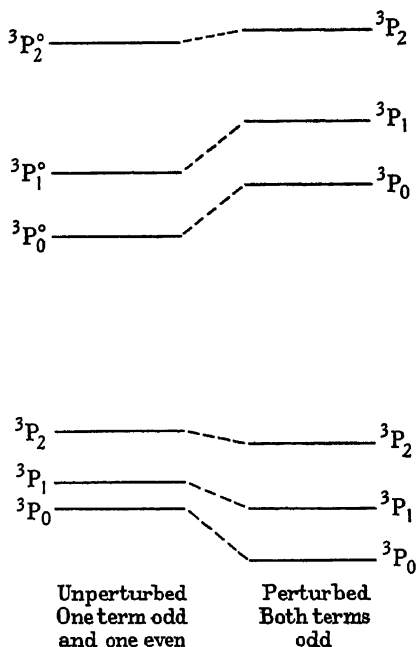


Fig 18 33 Changes produced by perturbation in wide and narrow triplets when these lie close to one another

should be 1.50 whatever the coupling, is in fact 1.35. This can be most reasonably explained if the term is perturbed, but what unidentified configuration is likely to lie near enough is not clear.

Short of complete sharing, the g factors may be more or less distorted. A method of calculating the precise change in g caused by neighbouring states was first developed by Houston ‡ for the

* Shenstone, *PR*, 1927, 30 264

† Green and Loring, *PR*, 1932, 41 389a

‡ Houston, *PR*, 1929, 33 297

ps configuration, Goudsmit* later extended the method to the ds, and Laporte and Inglis† to the p^5 s and d^9 s configurations. The only empirical constant which these formulae contain is one easily determined from the energies of the terms.

Though perturbed terms share their magnetic splitting factors, one must not too readily assume the converse, that when two terms share their splitting factors they are perturbing one another. The 3P_1 and 1P_1 terms of the $4p^5(^2P)5s$ configuration

Configuration	Term	E/hc	g theory	g observed
$4d^2 5s^2$	3P_2	4,186	1 500	1 25
	1D_2	5,101	1 000	
$4d^3 5s$	3F_4	12,342	1 250	1 15
	3G_4	12,761	1 050	
$4d^2 5s$ $5p$	1G_4	26,931	1 000	1 13
	3F_4	26,938	1 250	
$4d^3 5p$	5P_2	34,761	1 833	1 42
	1D_2	34,850	1 000	
$4d^3 5p$	1F_3	36,760	1 000	0 87
	3G_3	36,942	0 750	

Fig 18 34 Sharing of magnetic splitting factors in Zr I

of Kr I both have g factors of 1 25, but they are separated by 4930 cm^{-1} , and in fact their g factors have been explained as due to a particular coupling intermediate between (LS) and (jJ) (Fig 18 25).

The sharing of intensities is also well established. In Zr I singlet terms have been found combining with quintets, but only when the singlet has a quintet neighbour or the quintet a singlet and the two perturbing terms have the same J ‡. Moreover, in barium Langstroth§ has identified the terms which perturb the first three multiplets of the diffuse and fundamental series, and has shown that the experimental intensity sums are those required for (LS) coupling. In this comparison an uncalculable parameter appears connecting the intensities of two multiplets.

* Goudsmit, *PR*, 1930, 35 1325

† Laporte and Inglis, *PR*, 1930, 35 1337. Pogány, *ZP*, 1933, 86 729, confirms the predicted values in Kr I.

‡ Details on p 106

§ Langstroth, *PRS*, 1933, 142 286

arising in different configurations, but this may be determined experimentally provided some lines occur in each multiplet which are only slightly perturbed. A study of perturbed terms leads naturally to the identification of lines previously regarded as anomalous, for a weak multiplet may have one or more of its lines intensified by perturbation until they become strong enough to stand out in the spectrum. Several fragments of multiplets, whose other components are so weak that they escape observation, have in fact been found.

In barium no attempt was made to calculate the precise intensities of individual lines, for this is only possible when the parent configurations of all perturbing terms are known, and as yet the analysis of Ba II is insufficiently advanced. Indeed individual perturbed intensities seem to have been calculated and compared with experiment only once, by Kast* working on Sr I. He found satisfactory agreement, but the perturbation in the two terms considered was not as large as one could wish considering the magnitude of the experimental error.

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* Kast, *ZP*, 1932, 79 731

CHAPTER XIX

SERIES LIMIT

1 *J* values

Evidence has already been adduced to show that in the spectra of N, A II and Ne where the limit is a multiplet, some series converge to one component and some to another. To recall the point at issue consider the displaced terms of calcium, five terms of the

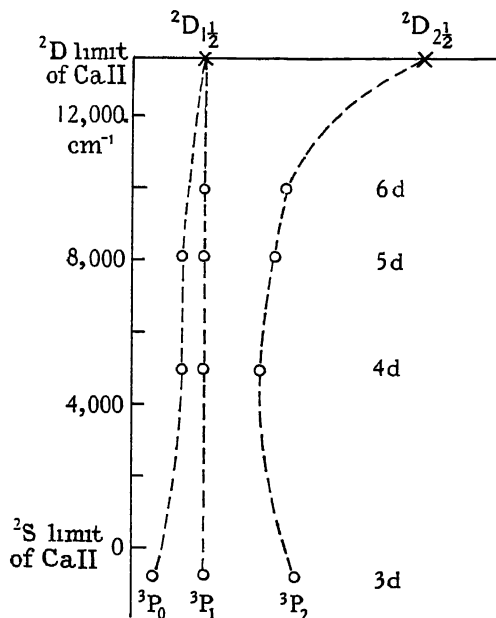


Fig 19 1 Splitting of the 3d *nd* 3P terms of Ca I showing that all three do not tend to the same limit

d np 3P series are known and the separations are plotted in Fig 19 1, showing quite clearly that 3P_2 converges to an upper limit, while 3P_1 and 3P_0 converge to a lower, these limits are naturally interpreted as the $^2D_{2\frac{1}{2}}$ and $^2D_{1\frac{1}{2}}$ terms of Ca II

All these terms have been named because they approximate to the Russell-Saunders ideal, but there are other terms such as the

(²P)p and (²P)d terms of neon which cannot be named though their series have been worked out, and the limits they approach. Naturally then the question arises cannot theory predict which terms will approach each limit? This was the problem which Hund set himself, and his solution will be considered in due course, but in that J is more easily determined than the orbital and spin vectors, it must take precedence.

Empirically the J values of unknown terms are always more easily determined than the multiplet structure. And theoretically the J values retain their significance even when the coupling is most irregular and L and S have no meaning. In particular, if both the series limit and the orbital vector of the series electron

J	0	1	2	3
Limit				
2P_1	P_1	P_2	P_5	P_4
$^3P_{1\frac{1}{2}}$	P_3	P_7	P_{10}	P_6
				P_8 P_9

Fig. 19.2 Paschen's p series in neon and the limits to which they tend

are known, then the J values of the terms approaching that limit are readily determined.

Ask, for example, which of the p terms of neon approach the lower ²P_{1½} limit of Ne II. When the chief quantum number of the series electron is sufficiently large the coupling is roughly (jj) and the problem is simply to find what J values can result from a term having $j_1 = 1\frac{1}{2}$ and a p electron having $j_2 = \frac{1}{2}$ or $1\frac{1}{2}$. When $j_2 = \frac{1}{2}$ the permitted values of J are 1 and 2, while when $j_2 = 1\frac{1}{2}$, J will assume the values 0, 1, 2 or 3. Thus the J values of the terms approaching the lower limit should be 0, 1, 1, 2, 2, 3, while those approaching the upper ²P₁ limit will have J values of 0, 1, 1, 2, and this is in fact the division actually observed.

And these facts may be simply summarised in a cell diagram, constructed to show the limits from top to bottom and the J values from left to right.

This diagram shows that the structure of certain terms is determined uniquely, thus the empirical p₁ term is the only term

arising by the addition of a p electron to a $^2P_{\frac{1}{2}}$ limit and having $J=0$, and it is thus the only term whose structure can be expressed symbolically as $(^2P_{\frac{1}{2}} p)_0$. But unfortunately no method is known of distinguishing two terms which have the same limit and the same value of J , though theory shows clearly that the one arises when the p electron has $j_2 = \frac{1}{2}$ and the other when $j_2 = 1\frac{1}{2}$. p_6 and p_8 must both be written $(^2P_{1\frac{1}{2}} p)_2$, for they cannot be distinguished as $(^2P_{1\frac{1}{2}} p_{\frac{1}{2}})_2$ and $(^2P_{1\frac{1}{2}} p_{1\frac{1}{2}})_2$.

The same method when applied to the $(^2P)d$ terms of neon or

J	0	1	2	3	4
Limit					
$^2P_{\frac{1}{2}}$		s_1'	s_1''	s_1'''	s_1''''
$^2P_{1\frac{1}{2}}$	d_6	d_2	d_5	d_1''	d_3
				d_1'	d_4
					d_4'

Fig 19 3 Limits of the $(^2P)nd$ terms of neon and argon

J	0	1	2
Limit			
$^2P_{1\frac{1}{2}}$			1P_1
$^2P_{\frac{1}{2}}$	3P_0	3P_1	3P_2

Fig 19 4 Limits of the $(^2P)ns$ terms of silicon

argon gives Fig 19 3, while applied to the $3p \ ns$ configuration of silicon it gives Fig 19 4

The J values of all known series fit in with these requirements, and the agreement is equally satisfactory whether the low terms are roughly Russell-Saunders as they are in nitrogen and the s terms of neon, or whether a large gap separates the terms tending to the upper and lower limits as in the p and d terms of argon

2. Hund's theories

The real problem, however, is not to account for the empirical terms by assuming a (11) coupling, but to name them with the Russell-Saunders symbols

Towards the solution of this problem Hund has made two

pronouncements In the first* he fixed the limit towards which a term series must converge, and fixed it given only the ion, the series electron and the symbol of the term, but though this theory is successful enough when both terms and limit are erect it fails badly when the limit is inverted as it is in neon † Thus the (2P) ns configuration yields two terms having $J = 1$, 3P_1 and 1P_1 , Hund's early theory states that of these 3P_1 will always approach the $^2P_{\frac{3}{2}}$ limit, and 1P_1 the $^2P_{\frac{1}{2}}$ limit, but though this prediction is correct in silicon it is incorrect in neon

Hund's revised prediction‡ is less precise and therefore less useful, it has usually been taken to mean that terms arising from

Term	$n=5$	$\Delta\nu$	$n=6$	$\Delta\nu$	Limit
3D_3	19533 2		108313 0		195,195
3D_2	18802 5	730 7	107817 0	496 0	194,824
3D_1	18398 6	403 9	107309 3	507 7	194,268
1D_2	16122 5	2276 1	107631 5	- 322 2	195,978

Fig 19 5 Values of the limits to which the four terms of the $4d\ ns$ configuration of Zr III tend

the same configuration and having the same J do not cross as they approach the limit, and in this form the theory is certainly in closer agreement with experiment Thus in silicon the 3P_1 term lies below the 1P_1 term and accordingly 3P_1 will approach the lower limit, namely $^2P_{\frac{3}{2}}$ of Si II, in neon 3P_1 , being s_4 in Paschen's notation, again lies below 1P_1 or s_2 , so that again 3P_1 approaches the lower limit, but this time the limit is inverted and the lower component is $^2P_{\frac{1}{2}}$

There are many similar successes to the credit of the revised theory, but it is not without its own problems Consider the $4d\ ns$ configuration of Zr III for example,§ the levels of this configuration are shown in Fig 19 5, measured upwards from the 3F_2 ground term Each of these pairs of terms is treated as part of a Rydberg sequence, and the limits calculated

These seem to show that 1D_2 , though below 3D_2 in the low

* Hund, *Linienspektren*, 1927, 184 f

† Shenstone, *N*, 1928, 121 619, 122 727

‡ Hund, *ZP*, 1929, 52 601

§ Kiess, C C and Lang, *BSJ*, 1930, 5 321

terms, yet tends to a higher limit, for the separation of the limits of 1D_2 and 3D_2 works out as 1154 cm^{-1} , which compares well with the interval of the 3D term of Zr IV , which is 1250 cm^{-1} . That the 1D_2 sequence should converge to the upper limit is in accord with Hund's early theory, but contrary to the usual interpretation of his later pronouncement.

A letter, however, written by Hund to Mack* shows that his statement has been interpreted too rigidly, for he specifically limited his prediction to terms which showed no symmetry property, and the letter states that any experimental evidence of crossing is to be taken as evidence of a symmetry property of the system. Accordingly the crossing of levels having the same J can never be in disagreement with Hund's conjecture, and the latter is in fact only a convention for naming levels where experiment does not distinguish between them. Mack pointed this out.

But even in this very limited form one may question whether the convention is useful, for if the Russell-Saunders notation is not to have any of its usual implications, it is surely better abandoned, and one which has no implications at all adopted in its place. And in fact Russell has advised that terms whose configuration and coupling are still undetermined shall be specified simply by numbers.

Present theory thus appears unable to make any general pronouncements, but it is not therefore valueless, experiment shows that in a large number of spectra, terms having the same J do not cross, and accordingly the consequences of this hypothesis are worthy of study, if only as an ideal with which the empirical may be compared. This ideal is the more useful in that if all terms obeyed Hund's energy rules only two types of convergence would be found, one when the limit is erect, the other when the limit is inverted. The erect type of convergence is identical with that dictated by Hund's first theory.

Take as an example the convergence of the terms of a $(^3P)nd$ configuration when the limit is first erect and then inverted (Figs. 19.6, 19.7). When the limit is inverted all components of a multiplet tend to approach the same limit, but when the limit is

* Mack, *PR*, 1929, 34.34

erect they tend to approach different limits, and in this the (3P)d terms are only one example of a general trend

A word should be added on the filling up of these figures, the number of cells in each row and their J values have been con-

J	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$				
Limit									
3P_2	$^2P_{\frac{1}{2}}$	$^4P_{\frac{1}{2}}$	$^2P_{1\frac{1}{2}}$	$^2D_{1\frac{1}{2}}$	$^2D_{3\frac{1}{2}}$	$^2F_{3\frac{1}{2}}$	$^2F_{5\frac{1}{2}}$	$^4D_{3\frac{1}{2}}$	$^4F_{4\frac{1}{2}}$
3P_1		$^4D_{\frac{1}{2}}$	$^4P_{1\frac{1}{2}}$	$^4D_{1\frac{1}{2}}$	$^4P_{2\frac{1}{2}}$	$^4D_{2\frac{1}{2}}$	$^4F_{3\frac{1}{2}}$		
3P_0				$^4F_{1\frac{1}{2}}$	$^4F_{2\frac{1}{2}}$				

Fig 19 6 Convergence of (3P)nd terms when the limit is erect, the low terms are assumed to obey Hund's energy rules and terms with the same J do not cross as they tend to the limit

J	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$				
Limit									
3P_0			$^2D_{1\frac{1}{2}}$	$^2D_{2\frac{1}{2}}$					
3P_1		$^2P_{\frac{1}{2}}$	$^2P_{1\frac{1}{2}}$	$^4P_{1\frac{1}{2}}$	$^2F_{2\frac{1}{2}}$	$^4P_{2\frac{1}{2}}$	$^2F_{3\frac{1}{2}}$		
3P_2	$^4P_{\frac{1}{2}}$	$^4D_{\frac{1}{2}}$	$^4D_{1\frac{1}{2}}$	$^4F_{1\frac{1}{2}}$	$^4D_{2\frac{1}{2}}$	$^4F_{2\frac{1}{2}}$	$^4D_{3\frac{1}{2}}$	$^4F_{3\frac{1}{2}}$	$^4F_{4\frac{1}{2}}$

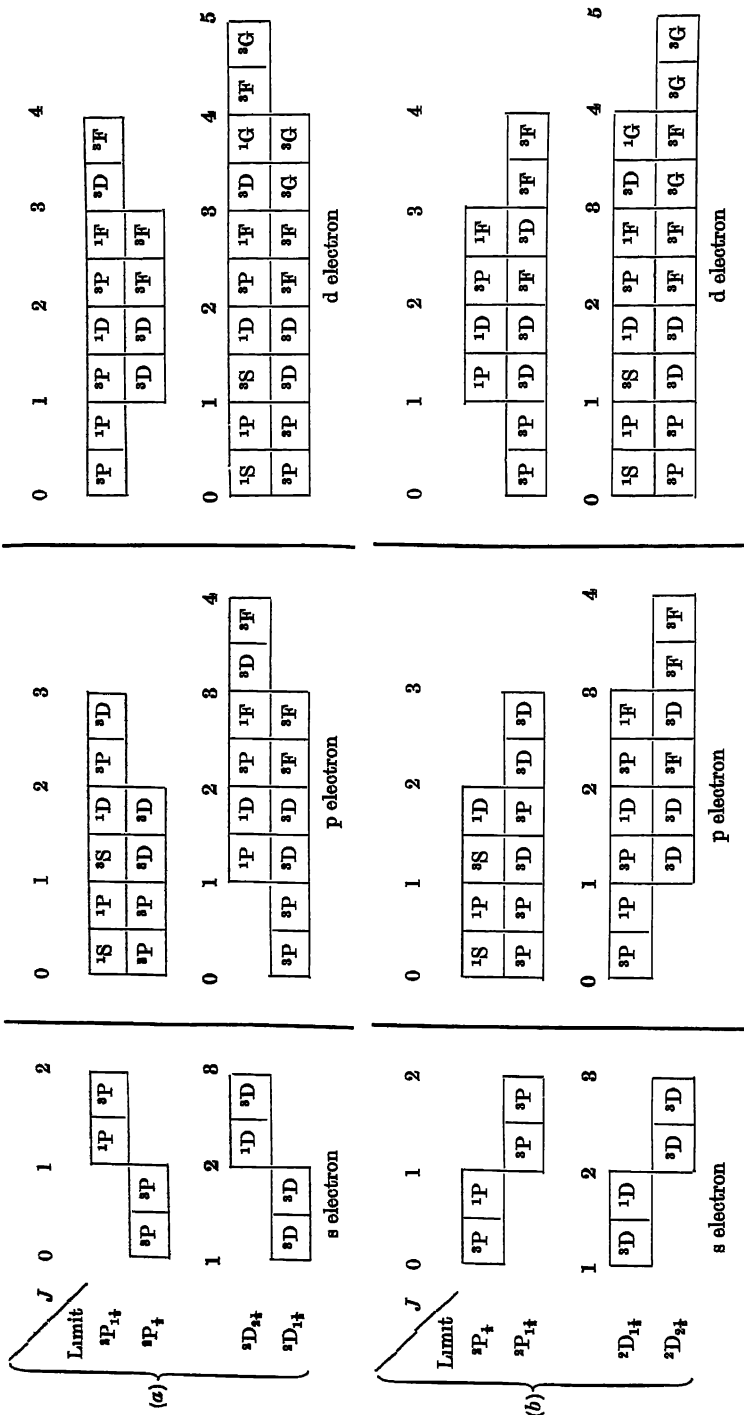
Fig 19 7 Convergence of (3P)nd terms when the limit is inverted, the assumptions being the same as in the previous figure

sidered earlier, the terms arising from (3P)d are 4F , 4D , 4P , 2F , 2D , 2P in that order from below upwards, accordingly, starting with 4F , insert each of the four components in the lowest cell which its J value permits, having finished 4F continue to 4D and so through 4P and $^2(FDP)$ until all seventeen cells are full

A number of these convergence types are given in Fig 19 8

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Chapter XIX of White's *Introduction to atomic spectra* may be usefully read, though its subject "Series Perturbations" is parallel to rather than a development of series limits



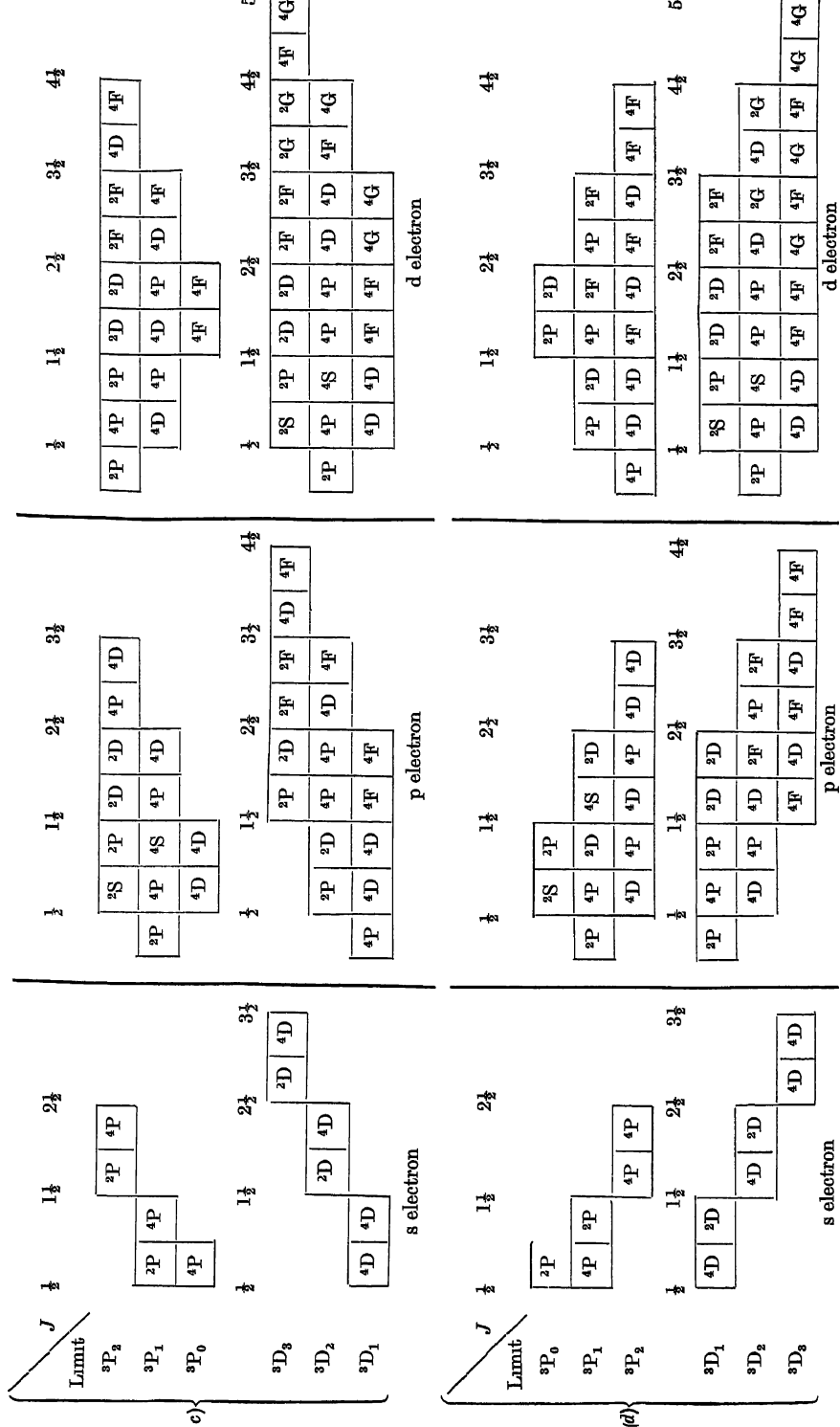


Fig 19.8 (a) Convergence of terms to erect doublet limits (b) Convergence of terms to inverted doublet limits (c) Convergence of terms to erect triplet limits (d) Convergence of terms to inverted triplet limits

CHAPTER XX

HYPERFINE STRUCTURE

1 Empirical

When the interferometers of Michelson, Fabry-Perot and Lummer-Gehrcke are applied to spectral lines, which appear single in a prism spectroscope, many are found to exhibit a complex structure with component intervals of 0.1 to 10 cm^{-1} . This much has been known since these high resolving instruments were first invented, the structure being commonly referred to as hyperfine.

The small intervals alone might suggest that the hyperfine levels can hardly be attributed to the same electron spin which produces the normal multiplet, and this suspicion gains support from the difficulty in fitting them into Hund's term scheme. Caesium, for example, must surely have a doublet structure, yet if a spectroscope of high resolving power is applied to certain lines, which should be simple, they appear as doublets with an interval of about 0.3 cm^{-1} . Time and again Hund's scheme has predicted how many terms an element should possess, and when the analysis has been completed, these and only these have been found. Yet when an element exhibits hyperfine structure, the extra levels cannot be fitted in.

Clearly then some new explanation must be sought. In 1924 Pauli* drew attention to the nucleus as a possible influence, but it was not until the following year that Schuler† brought forward the first experimental evidence in his work on the hyperfine structure of the 5485 Å line of Li II . This line arises from the $2p^3P_{0,1,2} \rightarrow 2s^3S_1$ transition, and should therefore be similar to the 7065 Å line of He I , for the latter arises from $3s^3S \rightarrow 2p^3P$, but whereas the helium line is a triplet, Schuler showed that the 5485 Å line of lithium has at least 14 components.

As both the helium atom and lithium ion have two orbital electrons, they differ only in their nuclei, and so to the nuclei

* Pauli, *Nu*, 1924, 12 741

† Schuler, *AP*, 1925, 76 292, *ZP*, 1927, 42 487

must be attributed the hyperfine structure. Further, lithium has only two isotopes, Li^6 and Li^7 , so that even if these did produce separate triplets they would account for only 6 out of the 14 components, the nuclear property responsible must therefore be something other than a simple mass effect.

2 Influence of nuclear mass

In both hydrogen and ionised helium, the series are governed by the relation

$$\nu = Z^2 R \left\{ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right\},$$

but the Rydberg constant, R , has a slightly different value in the two spectra, 109,678 in hydrogen and 109,722 in helium. This small difference Bohr explained as due to the different mass of the two nuclei, for if the mass is infinite

$$R = \frac{2\pi^2 e^4 m_e}{ch^3} \quad (2.9)$$

But when the mass of the nucleus is finite, the electron and nucleus both revolve about the common centre of gravity, to correct for this one substitutes for the mass of the electron, m_e , the quantity

$$\frac{M}{M + m_e} m_e$$

This theory clearly shows that the spectral lines of the heavy isotope of hydrogen will be displaced, each line of the Balmer series having a weak component on its short wave-length side, moreover, as the mass of the isotope is 2, the intervals of the first four lines, H_α to H_δ , should be 4.16, 5.61, 6.29 and 6.65 cm^{-1} *. Photographs show that in fact a weak satellite does occur in this position, and that the satellite is stronger when the concentration of H^2 is increased.

When there is more than one electron, the theory is much more complicated, but Hughes and Eckart† have provided a solution for systems of two and three electrons. The separations of the

* Urey, Brickwedde and Murphy, *PR*, 1932, 40.1

† Hughes and Eckart, *PR*, 1930, 36.694

lithium isotopes, Li^6 and Li^7 , is in reasonable agreement with theory * The measured intervals are

Li II	$2p\ ^3P \rightarrow 2s\ ^3S$	5485 Å	$1\ 06\ \text{cm}^{-1}$
Li I	$2p\ ^2P \rightarrow 2s\ ^2S$	6708 Å	$0\ 345\ \text{cm}^{-1}$
	$3p\ ^2P \rightarrow 2s\ ^2S$	3233 Å	$0\ 56\ \text{cm}^{-1}$

For atoms with more than three electrons, no theory has been evolved, and in fact a displacement due to mass alone seems to have been demonstrated only in neon. Each arc line of this element is accompanied by a faint companion of shorter wavelength. If the single electron theory was not invalid, it would suggest that the intervals should be given by $\frac{\delta\nu}{\nu} = 247 \cdot 10^{-8}$, while experiment shows that this ratio assumes the values $437 \cdot 10^{-8}$ in the lines $2p_m \rightarrow 1s_2$, and $368 \cdot 10^{-8}$ in the lines $2p_m \rightarrow 1s_{3,4,5}$. Thus the qualitative agreement is satisfactory †

3. The extended vector model

The first theory put forward to explain hyperfine structure‡ assigned to the nucleus an angular momentum \mathbf{I} , and made this combine with the electronic moment \mathbf{J} to form a resultant atomic moment commonly written \mathbf{F} . Then \mathbf{F} must be quantised as well as \mathbf{I} , and

$$\mathbf{J} + \mathbf{I} = \mathbf{F}$$

Previous experience with the similar linking of \mathbf{L} and \mathbf{S} to form \mathbf{J} suggests that possibly F will change only by ± 1 or 0 , and that the transition from $F=0$ to $F=0$ will be forbidden, further, one may hope that the multiplet intervals will satisfy the Landé interval rule. This hypothesis was at first only a guess, but evidence drawn both from experiment and from the calculations of the wave mechanics shows that the guess is fortunate.

Evidence will be adduced first in support of the interval and selection rules, thus certain lines of the bismuth spectrum were

* Hughes, *PR*, 1931, **38** 857. Cf. Granath, *PR*, 1932, **42** 44.

† Hansen, *N*, 1927, **119** 237, Nagaoka and Mishima, *Imp Acad Tokyo, Proc* 1929, **5** 200, 1930, **6** 143, Thomas and Evans, *E J, PM*, 1930, **10** 128.

‡ Pauli, *Nw*, 1924, **12** 721.

PLATE VII HYPERFINE STRUCTURE

1 *Potassium resonance lines*, 7699 and 7665 Å, $^2P \rightarrow ^2S$ Light from a potassium lamp was examined with a Fabry-Perot étalon, after passing through a beam of potassium travelling at right angles to the line of sight, the absorption pattern possesses a fine doublet structure, the Doppler width in absorption being much less than in emission

2 *Rubidium resonance lines*, 7800 and 7948 Å, $^2P \rightarrow ^2S$ Photographed with a reflection échelon, each line consists of four components, the weak outer components are due to Rb_{87} , and the strong inner components to Rb_{85} . As the hfs is the same for both lines, it arises in the common level 2S

3 *Caesium line*, 4555 Å, $^2P_{1/2} \rightarrow ^2S$ The hfs revealed by a Lummer-Gehrcke plate consists of doublets, arising in the 2S level, the intensity ratio of 1.27 : 1 shows that $I = 3/2$. The fringes of 4593 Å, $^2P_{3/2} \rightarrow ^2S$, appear very faint

4 *Gallium resonance line*, 4033 Å, $^2S \rightarrow ^2P_{1/2}$ A reflecting échelon grating reveals three lines with intensity ratio of 4.9 : 6.1 : 5. The 2S and 2P terms have nearly the same interval so that two of the four components overlap, the theoretical patterns are the same for $I = 1/2$ and $I = 1 1/2$, but the intensity ratios are 1 : 2 : 1, and 5 : 6 : 5 respectively, thus $I = 1 1/2$

5 *Indium line*, 4101 Å, $^2S \rightarrow ^2P_{1/2}$ The photograph on the left is taken with a reflecting échelon grating, while on the right this has been crossed with a Fabry-Perot étalon. Since the four streaks *A*, *B*, *C*, *D* lie on a diagonal, the pattern consists of these and not of *D'*, *A*, *B*, *C* as the left-hand figure might suggest. The intensity ratio of 2.72 : 1.82, 1.00 : 2.74 shows that $I = 4 1/2$

6 *Thallium line*, 5351 Å, $^2S \rightarrow ^2P_{1/2}$ The left-hand photograph was taken with a reflecting échelon, in the right hand this has been crossed with a Fabry-Perot étalon. The line consists of two close doublets, for the components *b'* and *B'* can only be *b* and *B* appearing in the next order, since on the right the étalon fringes appear at the same height. The small interval is due to isotope displacement, *a* and *b* being due to Tl_{203} and *A* and *B* to Tl_{205} , the larger interval is due to the hfs of the 2S term

7 *Thallium line*, 3776 Å, $^2S \rightarrow ^2P_{1/2}$ The six components are arranged in three close doublets, but the pair *B* and *b* in the *n*th order overlap the pair *C* and *c* in the (*n* + 1)th order, for when the échelon is crossed with a Fabry-Perot étalon, the fringes in these lines are double

All photographs were lent by Dr D. A. Jackson

1
7699 Å \ / 7665 Å



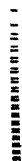
Potassium

2



Rubidium

3
4593 Å \ / 4555 Å



Caesium

4



Gallium

5(a)

$D' \setminus \begin{matrix} A & B & C & D & A' \end{matrix}$



5(b)

$D' \setminus \begin{matrix} A & B & C & D & A' \end{matrix}$



Indium

6(a)
 $b' B' a A b B$



6(b)



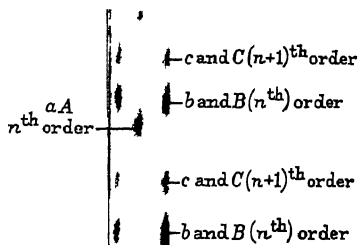
Thallium
5351 Å

7(a)

$a A b B$



7(b)



Thallium
3776 Å

Hyperfine structure

early analysed* and the 4722 Å line, arising from the $p^2 s 1_{\frac{1}{2}} \rightarrow p^3 {}^2D_{1\frac{1}{2}}$ transition may be quoted as an example. The observed pattern is shown in Fig 20 1 and analysis of this shows that the lower term, ${}^2D_{1\frac{1}{2}}$, has intervals of 0 152, 0 198, and 0 255, these best satisfy the interval rule if F is assigned the values 3 to 6, and the vector model allows just these values if I is $4\frac{1}{2}$, moreover, the model predicts always $(2J+1)$ or $(2I+1)$ components, according as I or J is the larger, and in fact the $1_{\frac{1}{2}}$, $8_{1\frac{1}{2}}$ and ${}^2D_{2\frac{1}{2}}$ terms do

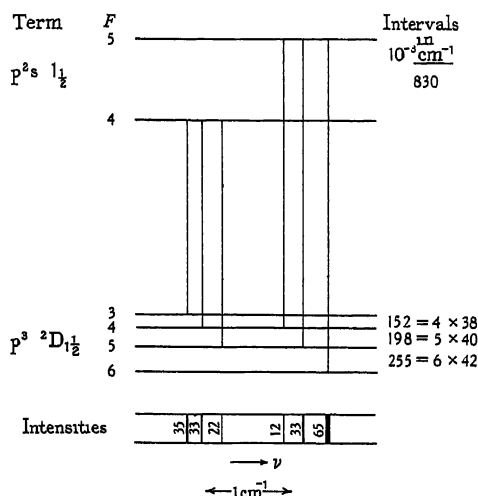


Fig 20 1 Level diagram showing the structure of the 4722 Å line of Bi I, the intervals given are empirical, the intensities theoretical (After Goudsmit and Back, *ZP*, 1927, 43 321)

split into 2, 4 and 6 levels respectively. Of these terms the two last obey the interval rule even better than the ${}^2D_{1\frac{1}{2}}$ term cited above (Fig 20 2)

The analysis of bismuth by the interval rule has been followed up by work on Mn, Pr, La and Cs. That bismuth was the first element successfully analysed was not due only to chance, but to the existence of only one isotope. For where several isotopes occur, each may have a different nuclear spin, and each spin then produces its own hyperfine pattern.

* Goudsmit and Back, *ZP*, 1927, 43 321. Zeeman, Back and Goudsmit, *ZP*, 1930 66 1.

Thus cadmium possesses six isotopes and these manifest themselves in the structure of the triplet 4678, 4800, 5086 Å, which arises from the transition $2^3S_1 \rightarrow 2^3P_{0,1,2}$. * The three hyperfine patterns have only one interval in common, namely 0.396 cm^{-1} , and the 2^3S_1 term must split therefore to two and only two components, but as the number of components is always $(2J+1)$ or $(2I+1)$, I must be $\frac{1}{2}$, and since this value must hold for all three P levels the term scheme and line pattern should be those shown in Fig 20.3. All the lines required by this scheme are in fact observed and have their theoretical intensities, but in

Term	F	2	3	4	5	6	7
$6p^3 \ ^2D_{1\frac{1}{2}}$	Interval	—	-0.152	-0.198	-0.255	—	—
	A	—	-0.038	-0.040	-0.042	—	—
$6p^3 \ ^2D_{3\frac{1}{2}}$	Interval	0.256	0.312	0.385	0.491	0.563	—
	A	0.085	0.078	0.077	0.082	0.080	—
$6p^3 \ 7s \ 8_{1\frac{1}{2}}$	Interval	—	0.379	0.473	0.563	—	—
	A	—	0.095	0.095	0.094	—	—

Fig 20.2 Examples of the interval rule in bismuth

addition each gross line has a strong component shown by the dotted line A at the foot. This strong component has been attributed to isotopes of cadmium having no nuclear moment and hence no hyperfine structure. Of the six isotopes of cadmium which Aston has identified therefore some have a nuclear moment of $\frac{1}{2}$ and some of zero, and the intensity of A relative to the other components makes it necessary to allocate the value $\frac{1}{2}$ to the odd isotopes 111 and 113, while all the even isotopes 110, 112, 114 and 116 have $I=0$.

Cadmium is peculiar in that the hyperfine levels are inverted, the levels with the largest values of F lying lowest, this arrangement is rather rare among hyperfine structures thus far analysed, though the $^2D_{1\frac{1}{2}}$ level of bismuth examined above happens also to be inverted.

The results thus far cited may be taken to prove that the hyperfine intervals satisfy the interval rule and that the selection rule is $\Delta F = \pm 1$ or 0, but no evidence has been adduced to show

* Schuler and Bruck, *ZP*, 1929, 56 291, 58 735

ther the transition from $F=0$ to $F=0$ is specifically forbidden. Consider therefore the 3776 Å line of TlI, a line which arises from jump $2^2S_{1/2} \rightarrow 2^2P_{1/2}$, examined for hyperfine structure this line

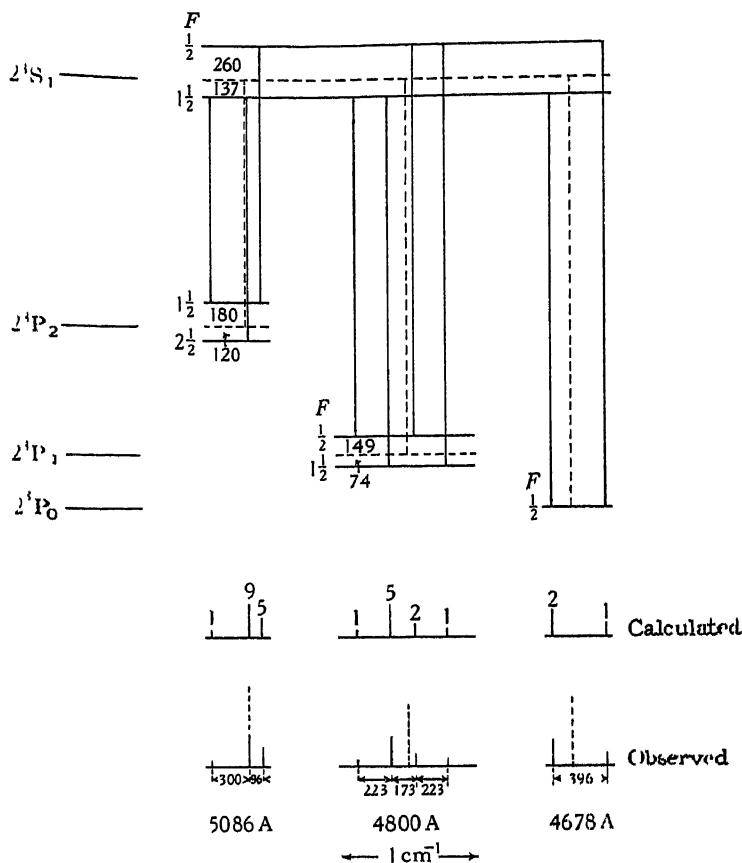


Fig. 20.3 Level diagram showing the structure of three cadmium lines

forms an unsymmetrical triplet (Fig. 20.4). * As the initial and final terms have the same value of J , this structure can be explained only if I is $\frac{1}{2}$, and if the jump from $F=0$ to $F=0$ is specifically forbidden, any other value of I would give rise to

Schuler and Bruck, *ZP*, 1929, 55 575 Schuler and Keyston, *ZP*, 1931,

four components, and should two of these coincide then the triplet would be symmetrical

That the same selection and interval rules are valid for both

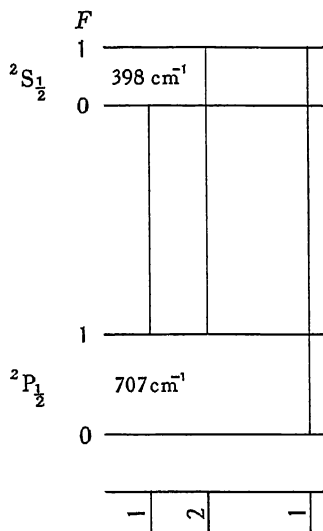


Fig 20 4 Level diagram of the 3776 Å, $^2S_{1/2} \rightarrow ^2P_{1/2}$, line of thallium

Gross structure		Hyperfine structure	
Resultant of spin moments	S	I	Nuclear spin
Resultant of orbital moments	L	J	Electronic moment
Vector sum of S and L	J	F	Vector sum of I and J
Magnetic moment of spin	$g(\mathbf{S}) \mathbf{S}$	$g(\mathbf{I}) \mathbf{I}$	Magnetic moment of nucleus
Magnetic moment of orbital vector	$g(\mathbf{L}) \mathbf{L}$	$g(\mathbf{J}) \mathbf{J}$	Magnetic moment of electronic vector
Projection on magnetic axis			Projection on magnetic axis
of S	M_S	M_I	of I
of L	M_L	M_J	of J
of J	M_J	M_F	of F

Fig 20 5 Comparison of vectors used to explain gross and hyperfine structures

gross and hyperfine structures suggests a far-reaching analogy, which is summarised and extended in Fig 20 5, so as to suggest rules for the Zeeman and Paschen-Back splitting. The angular momenta are given in units $\hbar/2\pi$ and the magnetic moments in Bohr magnetons

$$\frac{\hbar}{2\pi} \quad \frac{e}{2m_e c}$$

That the interval rule is valid shows that if the interaction energy of **I** and **J** is E , then

$$\begin{aligned} E/ch &= A I J \cos(\mathbf{IJ}) \\ &= \frac{1}{2} A \{F(F+1) - J(J+1) - I(I+1)\} \end{aligned} \quad (20\ 1)$$

The separation of the two levels F and $(F-1)$ is then

$$\frac{1}{2} A \{F(F+1) - (F-1)F\} = AF,$$

so that successive intervals in a multiplet term are proportional to the greater of the adjacent quantum numbers

Hyperfine multiplets usually obey the interval rule so well that exceptions attract an attention, which similarly deviating gross multiplets never obtain. These irregularities are to-day recognised as arising from at least two distinct causes, perturbation and absence of spherical symmetry in the electric field of the nucleus. As perturbation, however, produces also abnormal isotope displacements, and isotope displacements have not yet been considered, these irregularities are better postponed to a later section.

4 Zeeman and Paschen-Back effects

In work on the splitting of lines in the magnetic field, theory has long travelled ahead of experiment, so consider what splitting the vector model predicts.*

In a weak field **J** and **I** combine to form **F**, and the projection of **F** on the magnetic axis **H** is quantised. According to the theory of the Zeeman effect, the increment of energy will be ΔE , where

$$\Delta E/ch = M_F g(F) o_m \quad (20\ 2)$$

o_m is here an abbreviation for the Lorentz unit

$$\frac{eH}{4\pi m_e c^2} = 4\ 698\ 10^{-5} H\ \text{cm}^{-1},$$

H being measured in gauss, while M_F is restricted by the condition

$$F \geq M_F \geq -F$$

This energy equation gives the displacement of a level from the position it occupies when the magnetic field is zero, to obtain the

* Goudsmit and Bacher, *ZP*, 1930, 66 13

displacement from the centroid a second term must be added so that the energy becomes

$$E/ch = \frac{1}{2}A \{F(F+1) - I(I+1) - J(J+1)\} + M_F g(F) o_m \quad (20\ 3)$$

A is here the hyperfine interval quotient

In an earlier chapter $g(J)$ was derived on the assumption that if two vectors \mathbf{X} and \mathbf{Y} combine to form a third vector \mathbf{Z} , then

$$g(\mathbf{Z}) = g(\mathbf{X}) \frac{X}{Z} \cos(\mathbf{XZ}) + g(\mathbf{Y}) \frac{Y}{Z} \cos(\mathbf{YZ}),$$

this expression being itself derived from the wave mechanics. Now $\mathbf{F} = \mathbf{J} + \mathbf{I}$, so that

$$g(F) = g(J) \frac{J}{F} \cos(\mathbf{JF}) + g(I) \frac{I}{F} \cos(\mathbf{IF}). \quad (20\ 4)$$

The magnitude of the hyperfine splitting itself and the theory of its cause both suggest that $g(I)$ will be very small, about 1/2000 say, so that in all fields which can be applied to the atom $g(I)$ will be negligible, and $g(F)$ may be written as

$$g(F) = g(J) \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} \quad (20\ 5)$$

This completes the description of the weak field. In a strong field \mathbf{J} and \mathbf{I} are no longer linked together, but precess independently round the magnetic axis, so that their projections on the magnetic axis have to be quantised separately. As in the theory of the Paschen-Back effect, so here

$$E/ch = AM_I M_J + M_J g(J) o_m, \quad (20\ 6)$$

where $g(I)$ has been treated as negligibly small

This simple theory should give good agreement in fields so weak that $o_m \ll A$, and in fields so strong that $o_m \gg A$, but in intermediate fields resort must be had to the quantum mechanics. Moreover, in all fields the quantum mechanics should give more accurate numerical values.

A complex theory such as this is most satisfactorily tested, if applied, first to a transition producing a simple pattern, and afterwards to patterns of growing complexity. Accordingly,

Green and Wulff first examined the 3092 Å line of Tl II, which arises from $6s\ 7s\ ^1S_0 \rightarrow 6s\ 6p\ ^1P_1$,* and then turned to more complex lines in thallium and bismuth † They worked with the first and second spark spectra, because hyperfine intervals are there larger than in the arc, and they examined each line in three

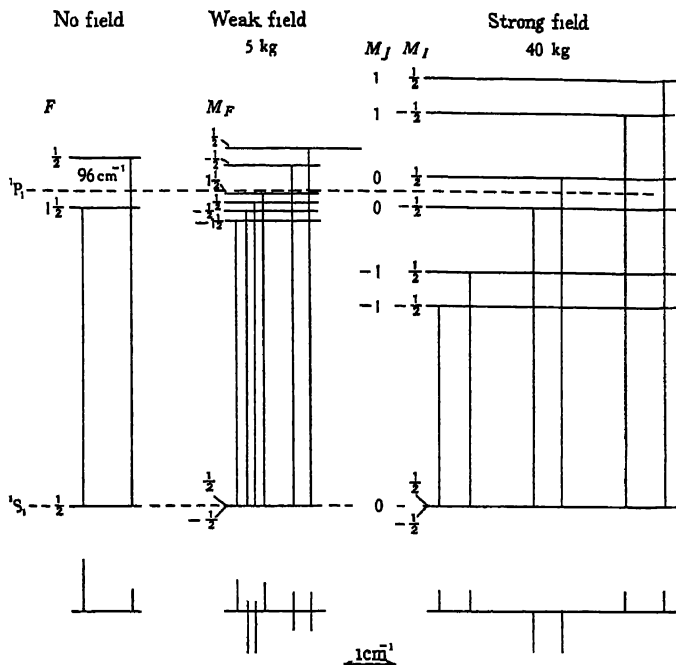


Fig 20 6 Level diagram showing the splitting of the 3092 Å, $^1S_0 \rightarrow ^1P_1$, line of Tl II in weak and strong magnetic fields. The displacements shown are calculated from the vector model

fields. In these heavy elements, the Doppler effect does not produce so wide a line as when the atom is light, moreover, thallium and bismuth provide a convenient contrast, for their nuclear moments are $\frac{1}{2}$ and $4\frac{1}{2}$ respectively, so that in a strong field the thallium lines split to two components while the bismuth split to ten.

Consider then the 3092 Å line of Tl II, of the two terms, 1S_0 and

* Green and Wulff, *ZP*, 1931, 71 593

† Green and Wulff, *PR*, 1931, 38 2176, 2186

1P_1 (Fig 20 6), from which it arises, the first does not split but the second has a hyperfine interval of -0.96 cm^{-1} , so that the interval quotient A is -0.64 cm^{-1} , while in the weakest field used, namely 14,700 gauss, the Lorentz unit α_m is 0.69 cm^{-1} . In this field then α_m is not 'much smaller' than $\Delta\nu$ and the splitting is likely to be characteristic of an intermediate rather than of a weak field, but in fact the simple Zeeman theory does give good qualitative agreement. For the $F=1\frac{1}{2}$ and $\frac{1}{2}$ components of the 1P_1 term $g(F)$ works out at $\frac{2}{3}$ and $\frac{4}{3}$ respectively, so that the patterns predicted are $\pm(1) 1\frac{3}{3}$ and $\pm(2) 2/3$, while the dis-

Field strength	14,700 g		43,350 g	
M_F	Vector model	Quantum mechanics	Vector model	Quantum mechanics
$1\frac{1}{2}$	0.37	0.37	1.51	1.51
$\frac{1}{2}$	1.10	1.06	2.15	2.17
	-0.09	-0.05	0.32	-0.025
$-\frac{1}{2}$	0.18	0.11	-0.32	0.005
	-0.55	-0.48	-1.51	-1.50
$-1\frac{1}{2}$	-1.01	-1.01	-2.15	-2.15

Fig 20 7 Displacements of the 1P_1 term of Tl II in weak and strong magnetic fields

placements calculated by equation (20 3) work out as 0.37, -0.09 , -0.55 and -1.01 for $F=1\frac{1}{2}$, and 1.10 and 0.18 for $F=\frac{1}{2}$. Fig 20 7 shows how these calculations compare with the more accurate predictions of the quantum mechanics.

In contrast a field of 43,350 gauss produces a Lorentz unit of 2.02 cm^{-1} , so that splitting characteristic of a strong field is to be expected. In fact theory shows that the 3092 Å line splits to six components, whose displacements are $\pm(0.32)$, 0.32, 1.51, 2.15. The pattern thus consists of three pairs of lines, one at the centroid of the hyperfine doublet and the other two arranged symmetrically on either side, and in general terms this is what experiment reveals, though the numerical agreement is not close.

Better numerical agreement is, however, obtained if the more precise theory of the wave mechanics is substituted for the

1.025 instead of 1 since the coupling is not pure (LS), but shows some signs of the (jj) type so that the g factor of the 1P_1 term is influenced by other terms which lie near; the numerical value of the correction is obtained by the method of Houston *

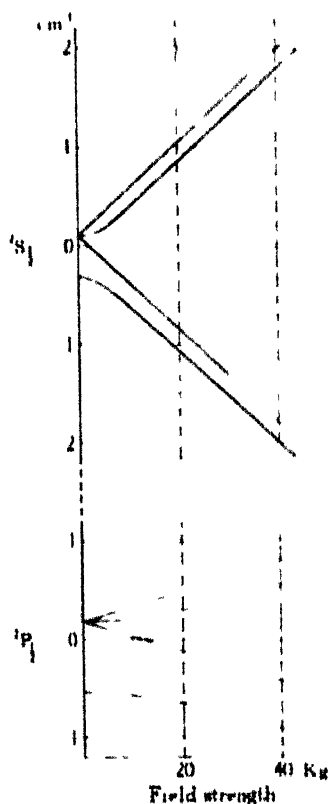


Fig. 20-10. Displacements of the hyperfine components of the $^2S_{1/2}$ and $^2P_{1/2}$ levels of ThI in various fields. The displacements are calculated by equations (20.8) of the quantum mechanics.

Thus theory interprets this simple pattern with great success; nor is it less successful with the more complex patterns of ThI. Back and Wulff† photographed the 3776 Å., $^2S_{1/2} - ^2P_{1/2}$, line in fields of 17,050, 29,700 and 43,350 gauss. In these fields the Lorentz unit assumes the values 0.80, 1.40 and 2.08 cm^{-1} , while

* Houston, *PR*, 1920, 33 297.

† Back and Wulff, *ZP*, 1930, 66 31

the hyperfine intervals of the $^2S_{\frac{1}{2}}$ and $^2P_{\frac{1}{2}}$ terms are 0.40 and 0.71 respectively, so that all three fields are technically 'strong'. The magnetic field and nuclear moment would thus separately split each term in two, the Zeeman components being determined by $M_J = \pm \frac{1}{2}$, and the hyperfine components by $M_I = \pm \frac{1}{2}$. According to the wave mechanics the displacements of these four levels from the centroid are given by the equations

$$\begin{array}{rclcl}
 M_J = \frac{1}{2} & M_I = \frac{1}{2} & M_F = 1 & E/ch = A/4 + \frac{1}{2}go_m & \\
 \frac{1}{2} & -\frac{1}{2} & 0 & = -A/4 + \frac{1}{2}\sqrt{A^2 + g^2o_m^2} & \\
 -\frac{1}{2} & \frac{1}{2} & 0 & = -A/4 - \frac{1}{2}\sqrt{A^2 + g^2o_m^2} & \\
 -\frac{1}{2} & -\frac{1}{2} & -1 & = A/4 - \frac{1}{2}go_m & (20.8)
 \end{array}$$

and shown graphically in Fig. 20.10. As previously g is here an abbreviation for $g(J)$.

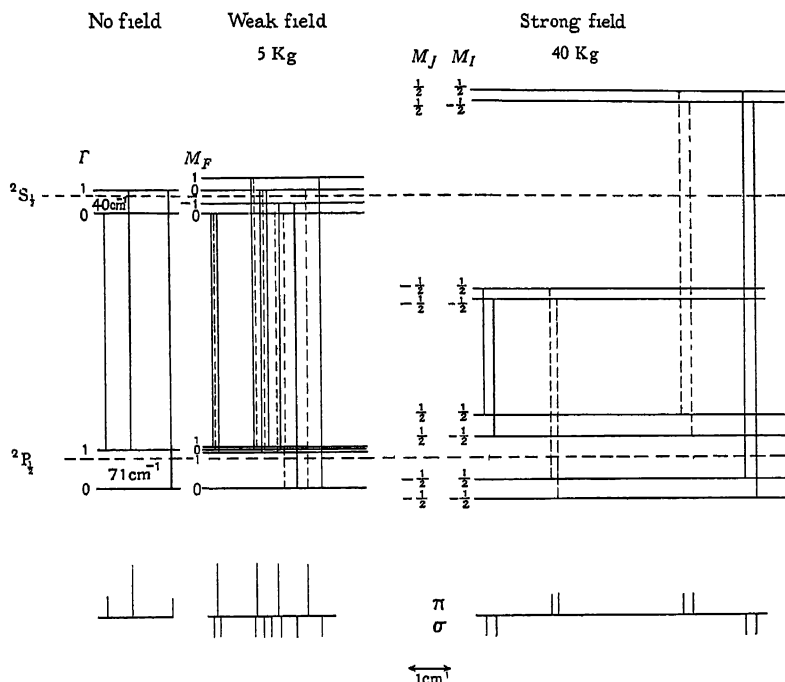


Fig. 20.11. Level diagram to show the structure of the 3776 \AA , $^2S_{\frac{1}{2}} \rightarrow ^2P_{\frac{1}{2}}$ line of Th I in weak and strong magnetic fields. The diagram assumes that the laws of the vector model are valid.

When the four levels of each gross term combine, they would give rise to six π and eight σ components in a weak field, but in a strong field two π and four σ components fade, for in a strong field transitions in which M_I changes its value are no longer allowed (Fig 20 11) The empirical displacements are compared with this theory in Fig 20 12, the centroid of the empirical pattern being adjusted so that the line of lowest frequency has the value dictated by theory The empirical intensities are estimated, not

Field in gauss	Parallel components				Perpendicular components			
	Position (cm ⁻¹)		Intensity		Position (cm ⁻¹)		Intensity	
	Calc	Obs	Calc	Obs	Calc	Obs	Calc	Obs
17,050	1 344	1 39	3 1	3	1 521	1 62	8 0	9
	0 456	0 50	6 9	10	0 810	0 93	9 85	9
	0 455		10 0		0 633	0 76	2 0	1
	-0 302	-0 24	6 9	8	0 278	0 34	0 15	0 5
	-0 609	-0 57	10	10	-0 075	-0 05	2 0	3
	-1 190	-1 19	3 1	3	-0 863	—	0 15	—
					-0 963	-0 94	8 0	6
					-1 368	-1 368	9 85	10
43,350	2 89	2 91	0 8	6	3 08	3 11	9 4	10
	1 36	1 50	9 2	7	2 45	2 46	10 0	10
	1 28	1 17	10	9	1 55	1 59	0 6	1
		-1 06		2				
	-1 20	(1 19)	9 2		1 09	—	0 02	—
		-1 32		7	-1 00	-0 97	0 6	1
	-1 44	(1 43)	10		-1 65	—	0 02	—
		-1 55		10				
	-2 74	-2 74	0 8	6	-2 52	-2 52	9 4	10
					-3 00	-3 00	10 0	10

Fig 20 12 Splitting of the 3776 Å line of Tl I compared with the predictions of the quantum mechanics in two magnetic fields

measured To prove that the vector model gives a reasonable approximation to these results is left to the reader

In bismuth the theory is of course unchanged, but one line may be cited to show how the nuclear moment was first determined by the use of a magnetic field * The 4722 Å line arises from the $1_{\frac{1}{2}} \rightarrow 2D_{\frac{1}{2}}$ transition, so that in the absence of hyperfine structure it should split to two π and four σ components When the line was examined in a field of 43,340 gauss, these components were found,

* Back and Goudsmit, *ZP*, 1928, 47 174 Zeeman, Back and Goudsmit, *ZP*, 1930, 66 1.

xx]

but all were exceptionally wide, and four were clearly resolved into ten components (Fig. 20 13) Now the energy of a magnetic level is

$$E/ch = AM_I M_J + M_J g(J) o_m,$$

where the second term determines the gross Zeeman level and the first term its hyperfine structure, as M_I can assume $(2I + 1)$

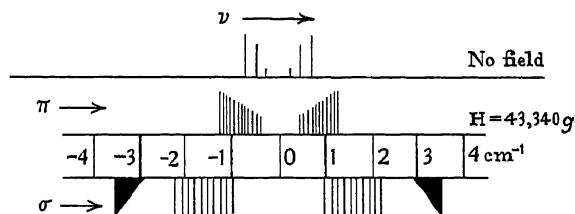


Fig. 20 13 Empirical structure of the 4722 Å, $1_{1/2} \rightarrow 2D_{1/2}$, line of Bi I in a strong magnetic field (After Back and Goudsmit, ZP, 1928, 47 179)

values, each gross term will have $(2I + 1)$ components (Fig 20 14) Moreover, as M_I is not allowed to change in a transition the gross Zeeman lines will also split to $(2I + 1)$ components, the interval being $A'M_J' - A''M_J''$ (Fig 20 15) Thus, if in fact the lines split to ten components the nuclear moment must be $4\frac{1}{2}$. And it is of some interest to note that the value of the interval was also confirmed, the hyperfine interval quotients of the $2D_{1/2}$ and $1_{1/2}$ terms are known to be -0.0403 and 0.166 cm^{-1} respectively, so that the interval of the π components should be

$$\begin{aligned} A'M_J' - A''M_J'' &= -0.0403(-\frac{1}{2}) - 0.166(-\frac{1}{2}) \\ &= 0.1031 \text{ cm}^{-1}, \end{aligned}$$

and of the outer σ components

$$\begin{aligned} A'M_J' - A''M_J'' &= -0.0403 \times \frac{1}{2} - 0.166(-\frac{1}{2}) \\ &= 0.0628 \text{ cm}^{-1} \end{aligned}$$

Now the width of the π components was measured as 0.880 cm^{-1} , so that the interval between successive components must be $0.880/9$ or 0.0977 cm^{-1} . The width of the outer σ components is 0.520 cm^{-1} , and the interval accordingly 0.0577 cm^{-1} . Both are in satisfactory agreement with theory

In Figs 20 16-17 evidence is presented for two still more complex lines, 2298 Å , $2^3S_1 \rightarrow 2^3P_2$ of Tl II and 5719 Å , $3^3P_0 \rightarrow 3^3P_1^o$ of

$B_1 \Pi, *$ in each figure the photometer curve appears above and the theoretical pattern below, and in both the agreement is all that can be desired

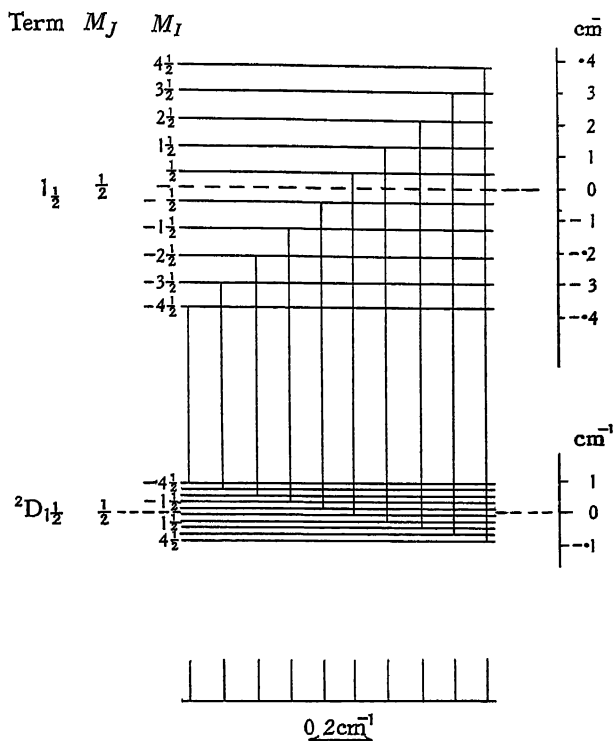


Fig 20 15 Level diagram to show the hyperfine structure of one gross σ component of the 4722 Å line of Bi I

5. Intensities

No absolute measurements of intensity seem to have been made, but the photometric curves obtained by Zeeman, Back and Goudsmit in 1930 showed that the intensity formulae deduced from the wave mechanics are qualitatively correct, while more recently these rules have been so widely applied to the analysis of the complex structures described in the next section and the results have been so satisfactory, that few serious deviations can exist

* Green and Wulff, *PR*, 1931, **38** 2182, 2186

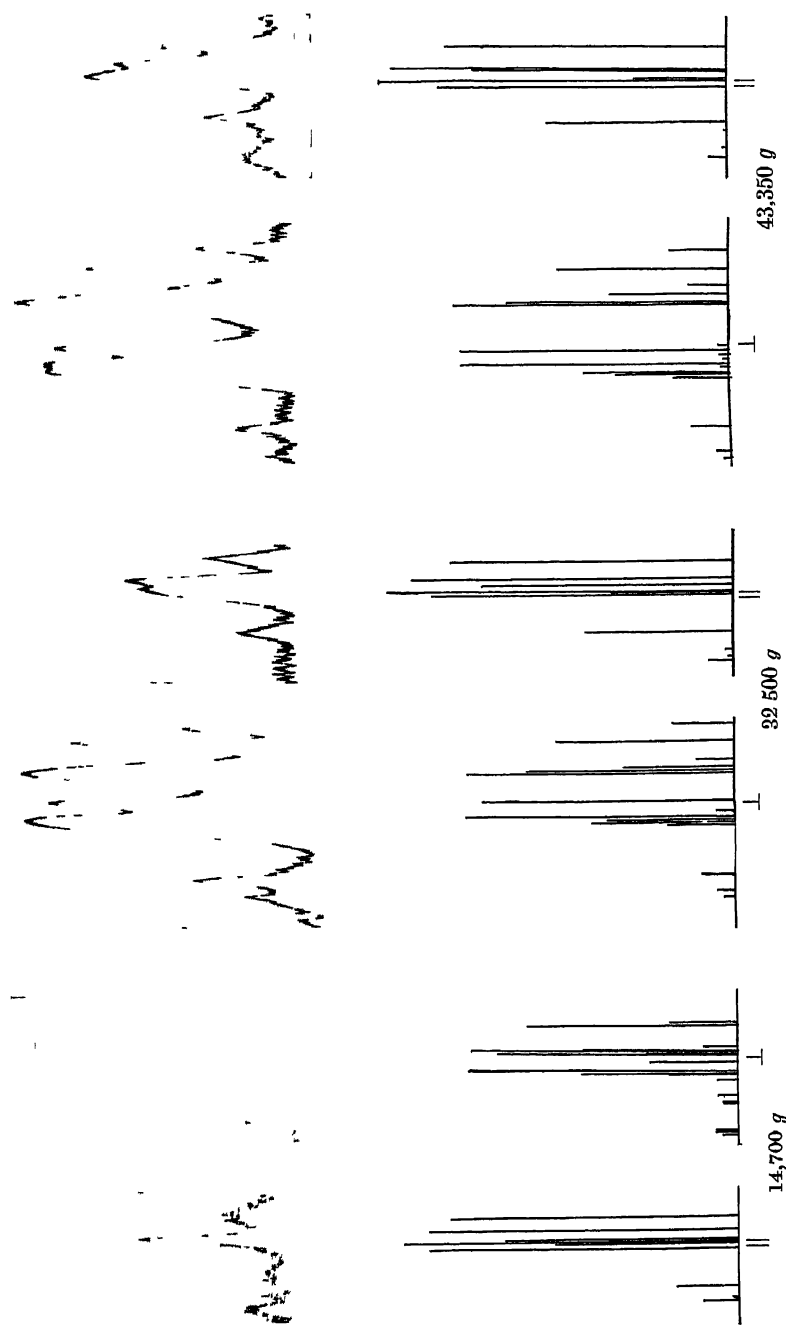


Fig 20 16 Splitting of the 2998 Å line of Tl II in magnetic fields of 14,700, 32,500 and 43,350 gauss. The photometer curve appears above and the theoretical pattern below The line arises as $6s\ 7s\ ^3S_1 \rightarrow 6s\ 6p\ ^3P_2$ (After Green and Wulff, *PR*, 1931, **38** 2182)

In order that the hyperfine components of a line may be observed with their theoretical intensities, however, the exciting source must fulfil certain conditions. Thus the temperature of the source must be as low as possible so as to reduce the Doppler broadening of the lines, the source is therefore worked in liquid air or liquid hydrogen. And secondly, the vapour pressure of the

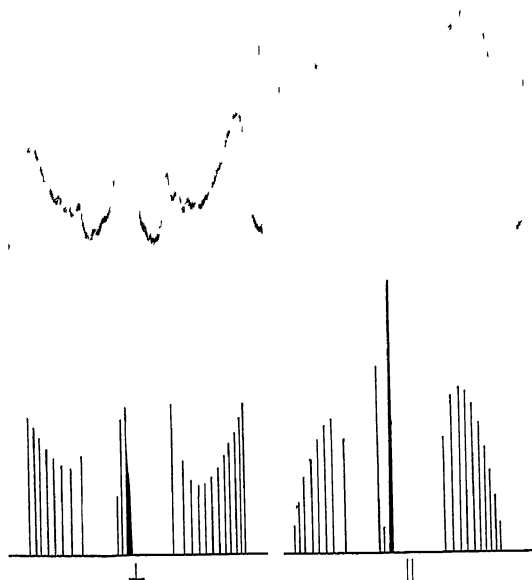


Fig 20 17a Splitting of the 5719 Å line of Bi II in a magnetic field of 14,700 gauss, again the photometer curve is above and theory below. The line arises as $6p\ 7p\ ^3P_0 \rightarrow 6p\ 7s\ ^3P_1$ (After Green and Wulff, *P.R.*, 1931, 38 2186)

element in the source must be kept low, for high pressure leads to self-absorption, besides hyperfine intensities seem in some way disturbed by inter-atomic fields, though why they should be is not yet clear *

The intensity formulae, deduced from the wave mechanics by Hill,[†] correspond to the multiplet intensity formulae with the transformation (Fig 20 5) already used in the interval rule

* Schuler and Keyston, *ZP.*, 1931, 71 413

† Hill, *Nat Acad Sci, Proc* 1929, 15 779

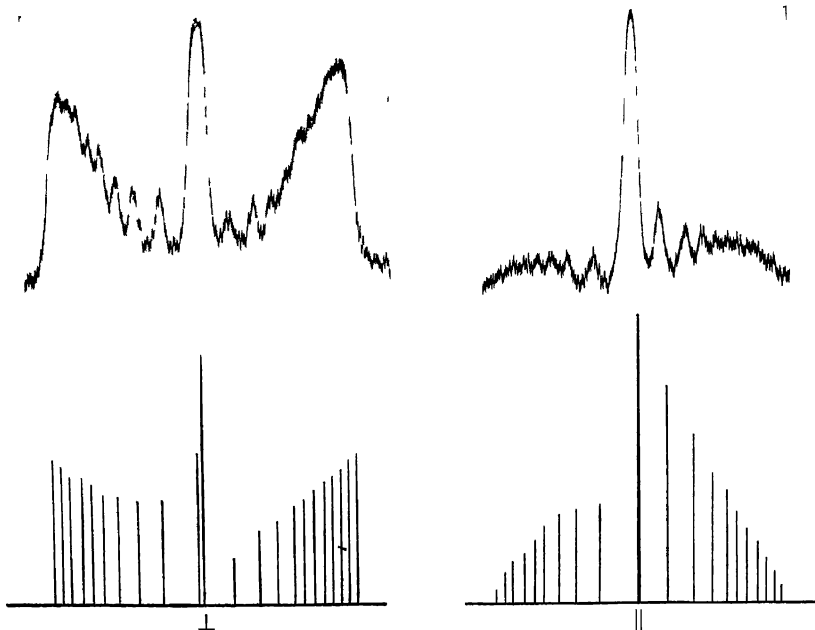


Fig 20 17b Splitting of the 5719 Å line of Bi II in a field of 32,500 gauss

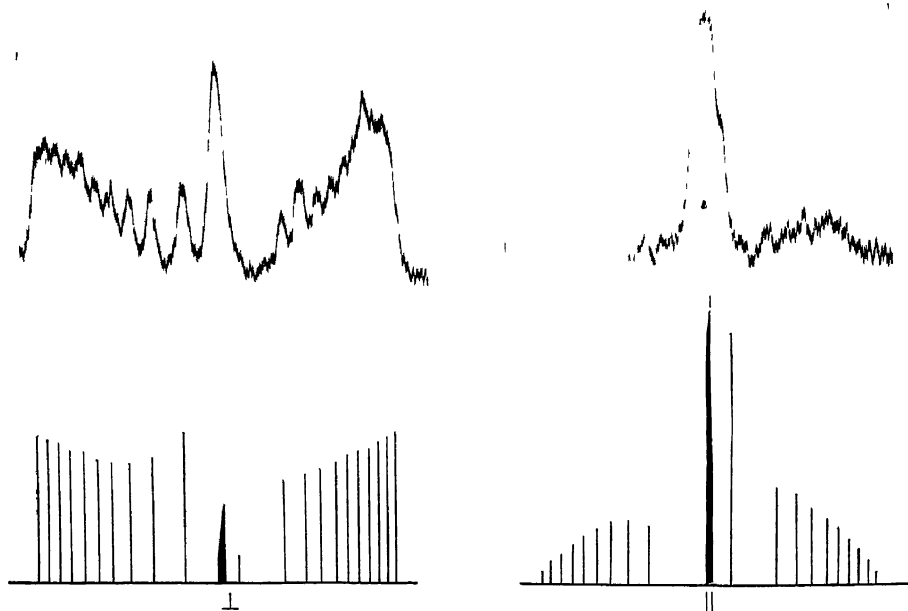


Fig 20 17c Splitting of the 5719 Å line of Bi II in a field of 43,350 gauss

In the jump $J \rightarrow (J-1)$ and

$$F \rightarrow (F-1) \quad I_- = \frac{1}{F} P(F) P(F-1),$$

$$F \rightarrow F \quad I_0 = \frac{(2F+1)}{F(F+1)} P(F) Q(F),$$

$$(F-1) \rightarrow F \quad I_+ = \frac{1}{F} Q(F) Q(F-1),$$

while in the jump $J \rightarrow J$, and

$$F \rightarrow F \quad I_0 = \frac{(2F+1)}{F(F+1)} R^2(F),$$

$$\begin{aligned} F \rightarrow (F-1) \\ \text{or } (F-1) \rightarrow F \end{aligned} \quad I_{\pm} = \frac{1}{F} P(F) Q(F-1),$$

where

$$P(F) = (F+J)(F+J+1) - I(I+1),$$

$$Q(F) = I(I+1) - (F-J)(F-J+1),$$

$$R(F) = F(F+1) + J(J+1) - I(I+1)$$

In all these formulae the transitions are so chosen that J and F are the larger of the two quantum numbers involved. Moreover, a term containing only J and I has been dropped since in fact the formulae are applied to the components of a single gross line.

The Zeeman intensity formulae may be transformed in the same way and compared with the visual estimates of Back and Wulff*.

6. Isotope displacement

The vector model, based on the hypothesis of nuclear spin, solves many of the problems of hyperfine structure, but there is clear evidence that when several isotopes exist, the vector model does not suffice.

Consider, for example, the Tl I line, 5351 Å,† which arises as $7s\ ^2S_{\frac{1}{2}} \rightarrow 6p\ ^2P_{\frac{1}{2}}$, work on the 3776 Å line of Tl has already been adduced to show that the nuclear moment of Tl is $\frac{1}{2}$, so that the 5351 Å line might be expected to split to three components with an intensity ratio of 5 : 2 : 1 as shown in Fig. 20.18, but in fact the

* Back and Wulff, *ZP*, 1930, 66.31

† Schuler and Keyston, *ZP*, 1931, 70.1

line splits to four components, and these have intensities roughly in the ratio of 6 9 3 2 3 1, that is two pairs with an intensity ratio of 3 1, now the three lines of theory can be reduced to two with an intensity ratio of 3 1 if the two components of the $^2P_{1\frac{1}{2}}$ term are not resolved. Accordingly, the empirical facts can be explained if there are two isotopes of Tl with an abundance ratio of 2 3 1, and if the term schemes of these two isotopes are displaced slightly relative to one another, in short that is if the term

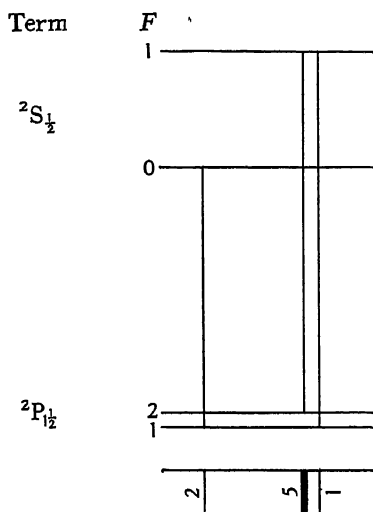


Fig. 20.18 Structure of a normal $^2S_{1/2} \rightarrow ^2P_{1/2}$ line

scheme of Fig. 20.19 is postulated. In this diagram the energy of the $^2S_{1/2}$ term is supposed the same in both isotopes, though in fact only the displacements of the one term relative to the other can be measured, experiment then shows that the $^2P_{1/2}$ term is displaced through 55 X in one of the isotopes 203 and 205. The existence of the isotopes 203 and 205 in the abundance ratio of 1 2 3 has been confirmed by the analysis of many Tl π lines, while more recently Aston* has found a ratio of 1 2 40.

Isotopes have been observed displaced in the lines of many elements, but of all mercury is the most complex, for it consists of

* Aston, *P.R.S.*, 1932, 134 571.

six isotopes,* of these the four of even atomic weight have zero moment, but the two odd isotopes have different nuclear moments, namely $\frac{1}{2}$ and $1\frac{1}{2}$, and to make confusion worse confounded the terms of one are inverted

To illustrate the methods by which such a complicated line

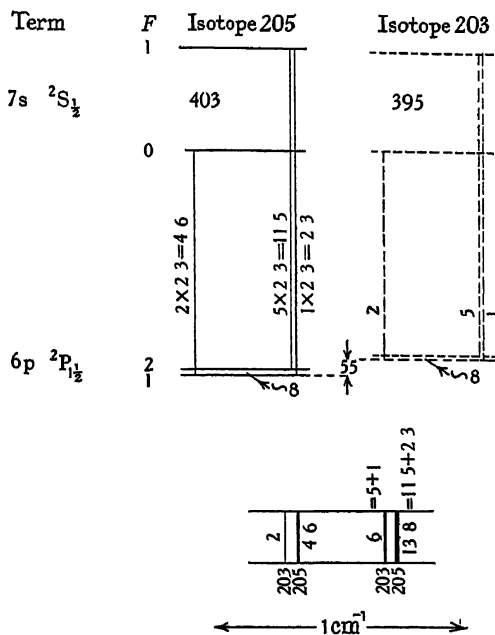


Fig 20 19 Structure of the 5351 Å, $^4S_{1/2} \rightarrow ^2P_{1/2}$, line of Th I (After Schuler and Keyston, ZP, 1931, 70 3)

structure can be analysed, consider two mercury arc lines, 2536 and 4078 Å,† which arise as $2^3P_1 \rightarrow 1^1S_0$ and $2^1S_0 \rightarrow 2^3P_1$, these lines are in fact particularly simple, for in one term J is zero

In the analyses described heretofore, three criteria have been used first, the picking of a common hyperfine interval from lines with a common gross structure level, second, the interval rule, and third the intensity rules Of these the first two suffice for the

* Recent work has revealed more than six isotopes, but in work on hyperfine structure only those present in a proportion of more than 1 per cent need usually be considered

† Schuler and Keyston, ZP, 1931, 72 423

analysis, when the structure is simple, but the last is the most valuable when the structure is complex. That the method has been so successful is perhaps the best evidence of the wide validity of the intensity formulae.

To obtain the theoretical intensities of the components of a line, the intensity ratios of the components due to a single isotope must first be calculated, and these must then be weighted with the

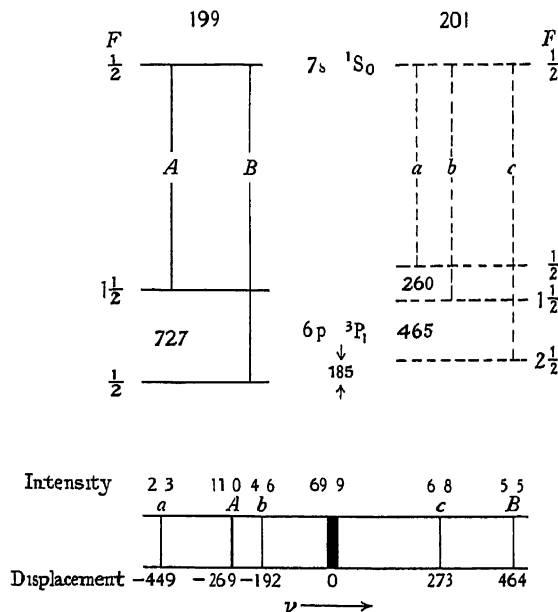


Fig. 20-20 Structure of the 4078 Å, $2 \ ^1S_0 \rightarrow 2 \ ^3P_1$, line of Hg I. The displacements are given in 10^{-3} cm^{-1} (After Schuler and Keyston, *ZP*, 1931, 72 428)

isotope abundance. Thus the even isotopes produce only one component each, but the isotopes 199 and 201, having nuclear moments of $\frac{1}{2}$ and $1 \frac{1}{2}$, produce two and three components respectively. These are shown as A, B and a, b, c in Figs. 20-20–21, and the theoretical intensity ratios are calculated as 2:1 and 1:2:3. But the mass spectrograph has shown that mercury contains 16.44 per cent of isotope 199, so that the absolute intensity of the A component should be $\frac{2}{3}$ of 16.44 or 10.96, and similarly since isotope 201 occurs to the extent of 13.68 per cent, the line b

should be of absolute intensity $\frac{1}{3}$ of 13 68 or 4 56 The intensities calculated in this way are summarised in Fig 20 22

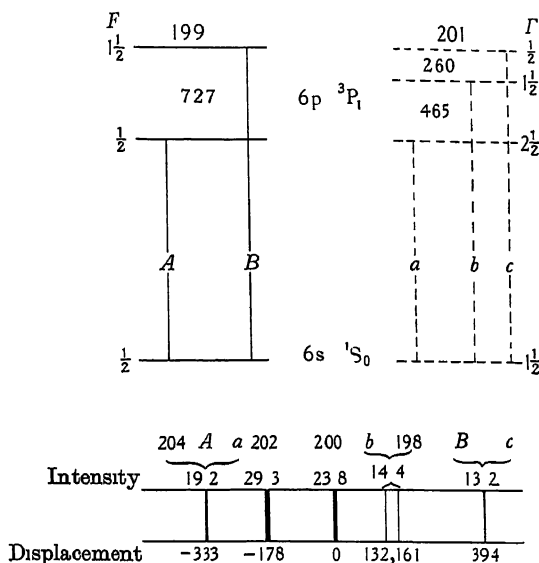


Fig 20 21 Structure of the 2536 Å, $2\ ^3P_1 \rightarrow 1\ ^1S_0$, line of Hg I The displacements are again in 10^{-3} cm^{-1} (After Schuler and Keyston, ZP, 1931, 72 434)

Isotope	Abundance ratio	Component	Relative intensity	Absolute intensity
199	16 44	A	2	10 96
		B	1	5 48
201	13 68	a	1	2 28
		b	2	4 56
		c	3	6 84
198	9 89	No hyperfine structure		9 89
200	23 77			23 77
202	29 27			29 27
204	6 85			6 85
	99 90			99 90

Fig 20 22 Intensities in a $3P_1 \rightarrow 1S_0$ or $1S_0 \rightarrow 3P_1$ line of Hg I

With these predictions in mind, the analysis of the structures shown in Figs 20 20 and 21 presents no great difficulty, provided one understands that components separated by less than $0\ 030\text{ cm}^{-1}$ are not resolved

This somewhat complicated analysis of the 2536 Å. line has

been beautifully confirmed by Mrozowski,* who has shown that when separately excited the various isotopes behave like a mixture of independent gases. If light from a mercury arc is sent through mercury vapour in a magnetic field, various hyperfine components can be filtered out. By varying the field strength Mrozowski in this way isolated three groups of lines, (a) the $394 \cdot 10^{-3} \text{ cm}^{-1}$ component, (b) the $+161$ and -333 components, (c) the $0 \cdot 0$ and -178 components. No matter which of these groups is used to excite cold mercury vapour, the resonance radiation contains only that group, when only the $+161$ and -333 components are incident, only these two components are emitted. Thus each hyperfine line is itself a resonance line and there is no fluorescence.

Fluorescence appears, however, when a little helium is added to the tube,† for the increased pressure means inelastic collisions for the mercury atoms. If the cold vapour mixed with helium is excited by the $+394$ line, the three lines $+394$, $+161$ and -333 are emitted, for the $+394$ line excites both the 199 and 201 isotopes and these after losing a little energy in colliding with a helium atom can return to the ground state while emitting the $+161$ or -333 line. The details are easily followed in Fig. 20.21. On the other hand, irradiate the vapour with the $0 \cdot 0$ and -178 lines, and only these lines are emitted, for these lines excite only the 200 and 202 isotopes, and as these have only one upper level they must radiate what they absorbed.

The analysis of a single line (Fig. 20.23) does not determine the absolute isotope displacements of either term, it determines only the displacements of the isotopes in one term relative to their displacements in the other, and what is true of the analysis of a single line is true of the analysis of any number of lines, so that apparently the absolute displacements can never be determined. Many lines, however, which arise between high levels of the mercury atom show no hyperfine structure at all, so that all the terms concerned must have identical isotope displacements, and

* Mrozowski, *Sci. Bull. Acad. Pol.* 1930, 464, 1931, 489. The lines with displacements of 132 and $161 \cdot 10^{-3} \text{ cm}^{-1}$ in Fig. 20.21 were not resolved in these experiments, they are referred to on this page as the 161 line.

† Mrozowski, *ZP.*, 1932, 78 826.

this is extremely improbable unless the displacements of all are zero. In support of this thesis, theory and experiment show that isotope displacements must and do decrease, as the chief quantum number increases (Figs 20 24-26)

The hyperfine levels thus obtained may be simplified by substituting for the odd components a single level at the centroid of the isotope, and when this is done an interesting regularity

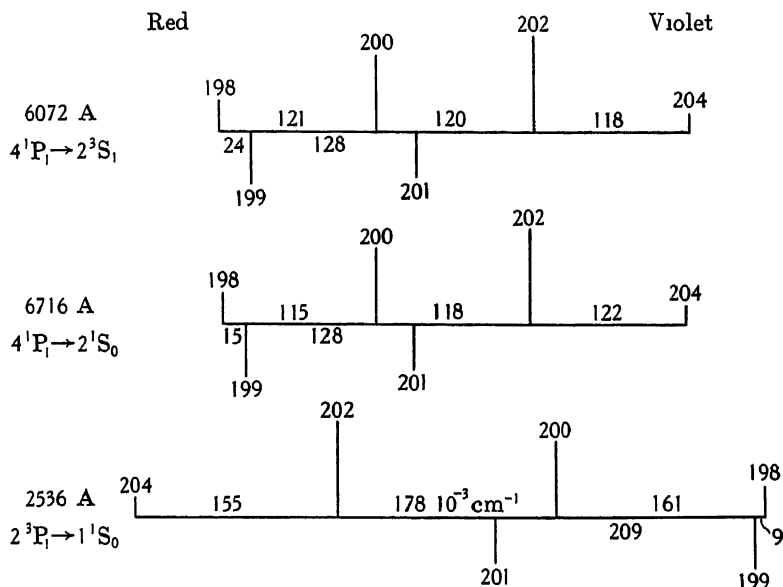


Fig 20 23 Centroids of the isotopes in three lines of Hg I (After Schuler and Keyston, *ZP*, 1931, 72 423)

emerges, the interval separating each even isotope from its neighbour is roughly constant and equal to the interval separating the odd isotopes, but the odd isotopes are not spaced mid-way between the even isotopes, this is clearly shown in Fig 20 23 and is true not only in Hg I, but also in Pb I and Pb II. As the isotopes are not displaced in the upper levels of many lines, this property may well be demonstrated in a single line

The relative energies of the isotopes also need to be related to their masses, for there has never been any doubt that the sequence

of the isotopes is the sequence of their masses, nor that in the arc and spark spectra of mercury the isotope of lowest atomic

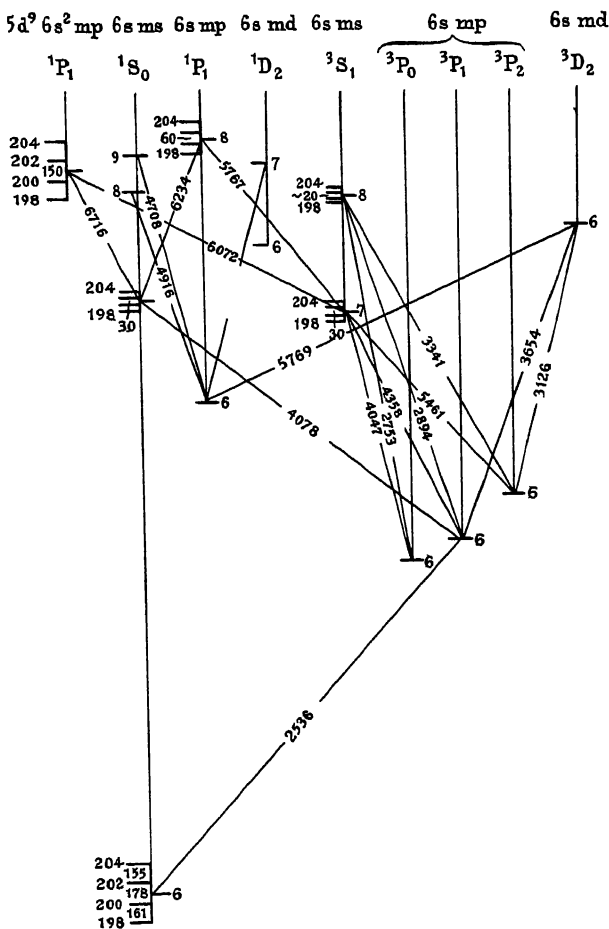


Fig 20 24 Isotope displacements in Hg I The wide splitting of the $8p\ ^1P_1$ term is anomalous (After Kallmann and Schuler, *EEN*, 1931, 11 165)

weight lies lowest In the light of recent theory this order seems to be general, but several spectra have been a little difficult to bring into line, thus the displacements in thallium seemed to be in different directions in the arc and spark spectra, but Breit* has

* Breit, *PR*, 1932, 42 348

since shown that this was due to a misinterpretation of certain lines of Tl II and that in fact the isotopes of thallium are arranged like the isotopes of mercury, the smallest at the bottom. The

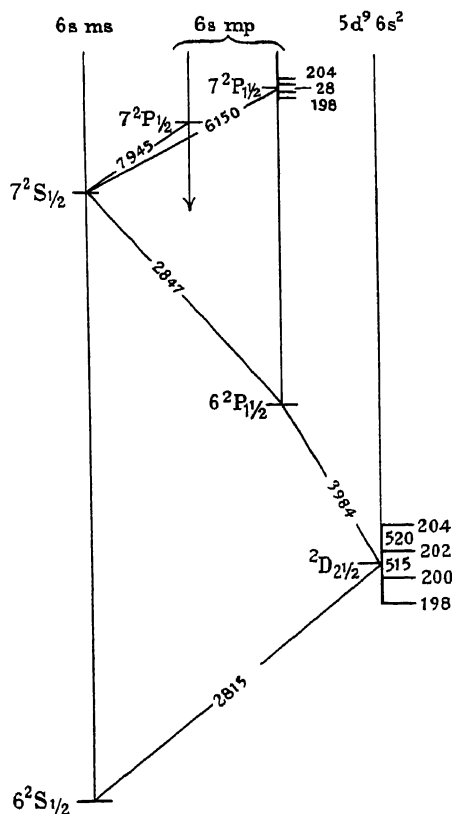


Fig 20 25 Isotope displacements in Hg II (After Kallmann and Schuler, *EEN*, 1932, 11 167)

theoretical explanation is interesting, isotope displacements are due to the increase in the volume of the nucleus resulting on the addition of a neutron,* this causes the same charge to occupy a larger volume and so reduces the electrostatic energy of the system, as the electrostatic energy of a condenser is negative,

* Schuler and Schmidt, *ZP*, 1935, 94 463

this means that the terms of the larger isotope lie higher than the smaller

Turning to the quantitative side, the displacement of one isotope relative to another seems to be the same for all terms of a

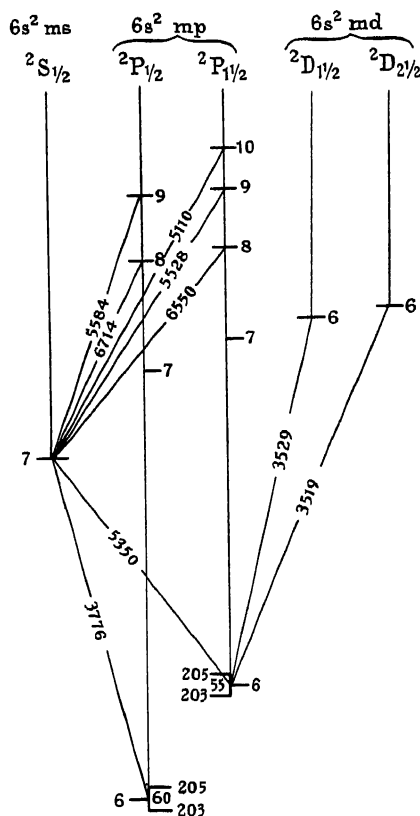


Fig 20 26 Isotope displacements in Tl I (After Kallmann and Schuler, *EEN*, 1932, 11 167)

configuration, Fig 20 27 shows this regularity in the $6s^2 6p^2$ configuration of Pb I Attempts actually to calculate the displacement have not yet met with complete success, because large corrections have to be made for the screening of one configuration by another, and these cannot be accurately estimated Of partial solutions the most important is that produced by Breit,* who has

* Breit, *PR*, 1932, 42 348

related the displacement to the nuclear radius, the latter being assumed proportional to the cube root of the mass, but there is little doubt that other influences contribute *

	1S_0	1D_2	3P_2	3P_1	3P_0	
204	65	84	?	?	?	204
206	72	88	90	85	83	206
208	—	—	—	—	—	208

Fig 20 27 Isotope displacements in the $6s^2 6p^2$ configuration of Pb I

7 Deviations from the interval rules

In general hyperfine multiplets obey the interval rule better than gross multiplets, but this very regularity makes exceptions the more striking. When two gross levels lie near one another and satisfy certain conditions they perturb one another, one may expect the same phenomenon among hyperfine levels, and in fact Schuler and Jones[|] have shown that when in Hg I the $6d^1D_2$ and $6d^3D_1$ levels lie at a distance apart comparable with their hyperfine intervals, the interval rule is no longer valid and the isotope displacements are anomalous. In the Hg^{201} isotope the hyperfine intervals of the $6d^1D_2$ term are 181, 301 and $313 \cdot 10^{-3} \text{ cm}^{-1}$, numbers which are in the ratio of 3 0 5 0 5 2 instead of in the normal ratio of 3 5 7. Moreover, in these terms the even isotopes are all coincident, showing that there is no isotope displacement, but the centroids of the odd isotopes instead of coinciding with the even isotopes lie farther apart. This suggests at once that the reason why both intervals and isotope displacements are abnormal, is that the two terms perturb one another, for the characteristic sign of perturbation is repulsion. Among gross multiplets two components can perturb one another only if they have the same J , and so among hyperfine multiplets we may expect components to perturb one another only if they have the same F , if this is true the even isotopes will not be displaced, for in them I is zero, and the two levels will have different values

* Breit, *PR*, 1933, **44** 418*a*, Bartlett and Gibbons, *PR*, 1933, **44** 538, Grace and More, *PR*, 1934, **45** 169

[|] Schuler and Jones, *ZP*, 1932, **77** 801

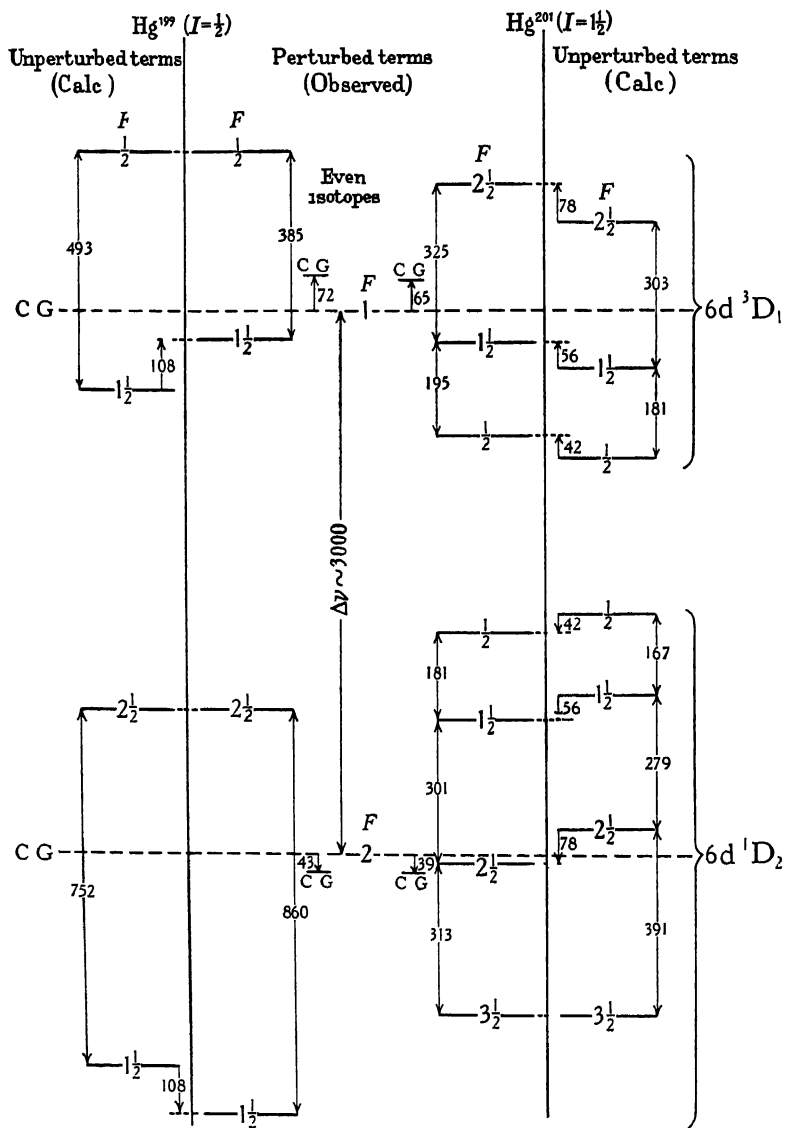


Fig 20 28 Two perturbed terms of mercury In the middle between the two uprights is the observed term system, while to left and right the terms are shown in the positions they would occupy if they were not perturbed The argument by which the various displacements are obtained is fully given by Schuler and Jones, ZP, 1932, 77 806

of F , namely 1 and 2. This makes the investigation much simpler than the analogous investigation of two gross multiplets, for the even isotopes provide a fixed scale of reference.

A bird's eye view of the changes produced by perturbation is provided by Fig. 20.28, in the 199 isotope system, the levels in which F is $1\frac{1}{2}$ perturb one another, while the levels in which F is $\frac{1}{2}$ above and $2\frac{1}{2}$ below remain undisturbed. As the two levels in which F is $1\frac{1}{2}$ repel one another, the 1D_2 term has an enlarged and the 3D_1 a reduced interval. In the isotope 201 on the other hand three components wander, for there are levels with F values of $\frac{1}{2}$, $1\frac{1}{2}$ and $2\frac{1}{2}$ in both gross terms, only the $F = 3\frac{1}{2}$ level of the 1D_2 term remains undisturbed. The repulsion of the levels with $F = \frac{1}{2}$, $1\frac{1}{2}$ and $2\frac{1}{2}$ leads to a great reduction of the $2\frac{1}{2}-3\frac{1}{2}$ interval, and a consequent departure from Landé's interval rule.

When the interval rule is not obeyed the interaction between the nucleus and the electron shells is no longer proportional to the scalar product of the two vectors, and the vector model fails,* the quantum mechanics shows, however, that two states lying close to one another perturb one another and gives a satisfactory account of the displacements.†

As perturbation is a phenomenon well known when two gross multiplets lie close to one another, physicists have naturally tried to use it to explain all deviations from the interval rule. But whereas two gross levels may perturb one another when 2000 cm^{-1} apart, two hyperfine levels must lie within a few units of one another, and so it is much less probable that an unknown level lies close enough to produce the observed distortion. And in fact Schuler and Schmidt have shown that some multiplets in Eu^{151} , Eu^{153} ,‡ Lu^{175} § and Hg^{201} || do not satisfy the interval rule, and that perturbation is an inadequate explanation. Instead of the displacements of the levels from the centroid being proportional simply to $\cos(\mathbf{IJ})$, a second term proportional to $\cos^2(\mathbf{IJ})$

* Casimir, *ZP*, 1932, 77 811

† Goudsmit and Bacher, *ZP*, 1933, 43 894

‡ Schuler and Schmidt, *ZP*, 1935, 94 457

§ Schuler and Schmidt, *ZP*, 1935, 95 265

|| Schuler and Schmidt, *ZP*, 1935, 98 239.

appears, this second term has a coefficient small compared with that of the normal $\cos(\mathbf{IJ})$ term, but though the deviations are small, they are greater than the experimental error. Whether this second term affects the extreme interval is doubtful, its obvious result is to make the levels crowd towards the levels of large or small F , the one when the term is positive the other when it is negative.

The appearance of a \cos^2 term means that the interaction of the nucleus and electrons cannot be wholly explained as due to the action of a magnetic nuclear dipole on the electrons, the form the deviation takes suggests that the new force is electrostatic in origin, and is due to a lack of spherical symmetry in the electric field of the nucleus.*

8. The spin of the nucleus

The spin of an atomic nucleus may be obtained from the hyperfine structure of a line, from the deflection of an atomic ray in a magnetic field and from the alternating intensities of band spectra. All three methods have been applied to sodium, and all give the same nuclear spin, and more generally where two methods have been used the results are consistent. Besides these three methods there are others which are permissible in theory, but which do not seem to have been used as yet, these are the specific heat at low temperatures† and scattering‡.

An atomic ray can be used only if the resolution is increased by including a magnetic velocity selector,§ this consists of an inhomogeneous magnetic field, which spreads the beam out into a velocity spectrum. A movable slit selects a part of the beam homogeneous in velocity to about 10 per cent, and this then passes through a weak inhomogeneous field of the Stern-Gerlach type, thereafter a third magnetic field is used to focus the beam on the detector wire. This technique has only recently been perfected, but already in Rabi's hands it has disclosed the spin of

* The equations for the interaction of such a core with the nucleus have been given by Casimir, *Physica*, 1935, 2 719.

† Dennison, *PRS*, 1927, 115 483.

‡ Sexl, *Nw*, 1934, 22 205.

§ Rabi and Cohen, *PR*, 1933, 43 582a.

potassium and in Stern's the magnetic moments of the proton and deuteron,* important quantities which could not have been easily measured by other methods When applied to sodium the

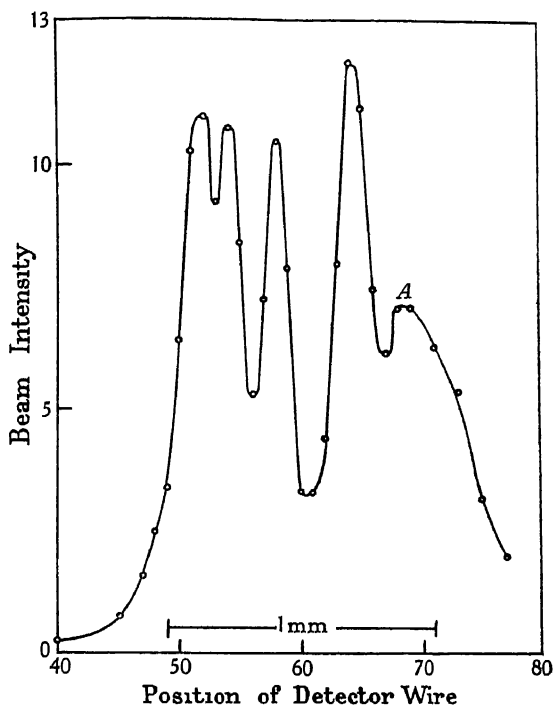


Fig 20 29 An atomic ray of sodium split into four components in passing through a magnetic field The high resolution is obtained by using an inhomogeneous magnetic field and a slit to select a beam homogeneous in velocity to 10 per cent The figure shows the points obtained in one particular run The peak marked *A* is due to atoms other than those selected (After Rabi and Cohen, *PR*, 1933, 43 582)

beam splits into four components as shown in Fig 20 29, the spin of sodium is therefore $1\frac{1}{2}$ †

In the application of band spectra‡ to this purpose, the relative intensities of the alternating bands of symmetrical molecules

* References to particular elements appear in the bibliography of h f s

† Broit and Rabi, *PR*, 1931, 38 2082*a*, Rabi and Cohen, *PR*, 1933, 43 582*a*, Rabi, Kellogg and Zacharias, *PR*, 1934, 46 157, Dickinson, *PR*, 1934, 46 598

‡ Jevons, *Band spectra of diatomic molecules*, 1932, 140

are measured, $\text{Cl}^{35}\text{Cl}^{35}$ is a symmetrical molecule, $\text{Cl}^{35}\text{Cl}^{37}$ is not, there are no alternating intensities in the bands of unsymmetrical molecules. The rotational levels of a symmetrical molecule divide into two systems, known as symmetrical or (*s*) and anti-symmetrical or (*a*), a level of one system never combines with a level of the other, (*s*) combines only with (*s*) and (*a*) only with (*a*). An (*s*) line defined by *J* lies between two (*a*) lines defined by (*J* - 1) and (*J* + 1), but its intensity is not a mean of its neighbours, the symmetrical lines are strong and the anti-symmetrical lines are weak. Theory ascribes this alternating intensity to the nuclear spin of the atoms, and shows that if the intensities are as I_s to I_a , then

$$\frac{I_s}{I_a} = \frac{I+1}{I},$$

or in tabular form

$I =$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
$I_s/I_a =$	∞	3	2	$1\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

When *I* is zero, the alternate lines are missing, so that the appearance of the anti-symmetrical lines is quite definite proof that *I* is not zero, but as intensities are not easily measured accurately, this method is not well adapted to measuring large nuclear moments, any estimate greater than $1\frac{1}{2}$ is certainly dubious. The method has been applied to show that the nuclei He^4 , C^{12} , O^{16} , S^{32} and Se^{80} have no nuclear spin, and to estimate the nuclear spin of ten other elements, in the table of values these are indicated by a *B*.

An examination of the spins obtained by hyperfine structure and band spectra (Figs 20-30-32) shows that they divide nuclei into two groups, in one of which the mass is odd and in the other even. Other evidence suggests that nuclei should be further divided according as the charge is odd or even. When the mass and charge are both even, the spin is always zero, this statement is supported by measurements on more than 30 nuclei. When mass is even and the charge odd, the spin is certainly 1 in H^2 and N^{14} , of this the alternating intensities of band spectra leave no doubt, in Li^6 the absence of hyperfine structure has been taken to indicate zero spin, but a narrow hyperfine structure can easily be

missed. Nuclei of this type are so unstable that only four examples are known. When the mass is odd, the spin is always half an odd integer, from time to time, zero moment has been ascribed to a nucleus of this type, but in view of the 42 nuclei which conform and the fact that hyperfine structure can always be missed if the lines are too blurred or the resolving power too low, they need not be further considered, no exception has been established.

How these regularities are related to be the structure of the nucleus can be most profitably considered after the magnetic moments have been discussed.

9 The magnetic moment of the nucleus

The simplest theory of hyperfine structure ascribes a magnetic moment to the nucleus, and argues that when this acts upon an electron in its orbit, the electronic term splits. If the coupling of the nucleus and electron obeys a cosine law, this hypothesis can be shown to predict the doublet formulae, the interval rule and the usual magnetic splitting.*

In particular the displacement of a particular level from the centroid will be given by the formula

$$\begin{aligned} \nu - \nu_G &= A I J \cos(IJ) \\ &= \frac{1}{2} A \{F(F+1) - J(J+1) - I(I+1)\}, \end{aligned}$$

where A is the hyperfine interval quotient.* When only one electron is active A is identical with a , the electronic interval quotient, which is determined by the formula

$$a = \frac{R\alpha^2 Z^3}{n^3(l + \frac{1}{2})j(j+1)} \frac{g(I)}{1838} \text{ cm}^{-1}$$

for a hydrogen-like orbit, or by

$$a = \frac{R\alpha^2 Z_i^2 Z_a^2}{n^{*3}(l + \frac{1}{2})j(j+1)} \frac{g(I)}{1838} \text{ cm}^{-1}$$

for a penetrating orbit. The latter is considered as composed of two parts, an inner in which the electron moves as if under the influence of a charge Z_i , and an outer where the effective charge is Z_a . n^* is the effective quantum number. In theory these

* Goudsmit, *P.R.*, 1931, 37 663, Fermi and Segrè, *ZP.*, 1933, 82 729

Element	Charge	Mass	Spin	Magnetic moments
H	1	1	$\frac{1}{2}B$	
Li	3	7	$1\frac{1}{2}H, B, R$	3 29 S, G, 3 20 F, 3 20
B	5	11		
N	7	15		
F	9	19	$\frac{1}{2}H, B$	3
Na	11	23	$1\frac{1}{2}H, B, R$	2 14 S, 2 09 F, 2 08
Al	13	27	$\frac{1}{2}H$	1 93 S, 2 1 G, 2 4
P	15	31	$\frac{1}{2}B$	
Cl	17	35	$2\frac{1}{2}B$	
		37		
K	19	39	$1\frac{1}{2}R$	0 38, 0 43, 397
		41	$>\frac{1}{2}$ and $<2\frac{1}{2}R$	
Sc	21	45	$3\frac{1}{2}H$	3 6
V	23	51	$3\frac{1}{2}H$	
Mn	25	55	$2\frac{1}{2}H$	
Co	27	59	$3\frac{1}{2}H$	2 5?
Cu	29	63	$1\frac{1}{2}H$	2 74 S
		65	$1\frac{1}{2}H$	2 74 S
Ga	31	69	$1\frac{1}{2}H$	2 14 S, F, 2 01 G
		71	$1\frac{1}{2}H$	2 74 S, 1 49 F, 2 55 G
As	33	75	$1\frac{1}{2}H$	0 90 S, G
Br	35	79	$1\frac{1}{2}H, B?$	
		81	$1\frac{1}{2}H, B?$	
Rb	37	85	$2\frac{1}{2}H$	1 49 S, 1 36 F, 1 3 G
		87	$1\frac{1}{2}H$	3 06 S, 2 79 F, 2 7 G
Y	39	89		
Cb	41	93	$4\frac{1}{2}H$	3 7
Ma	43	—		
Rh	45	—		
Ag	47	107		
		109		
In	49	115	$4\frac{1}{2}H$	$<0.24 S$
Sb	51	121	$2\frac{1}{2}H$	5 25 S, 5 3 F, 5 4 G
		123	$2\frac{1}{2}H$	2 70 S, 2 7 G, 2 75
I	53	127	$4\frac{1}{2}H, B?$	2 10 S, 2 1 G, 2 0
Cs	55	133	$3\frac{1}{2}H$	
La	57	139	$3\frac{1}{2}H$	2 91 S, 2 63 F, 2 50
Pr	59	141	$2\frac{1}{2}H$	2 5
II	61			
Eu	63			
Tb	65			
Ho	67	165		
Tu	69	169		
Lu	71	175	$3\frac{1}{2}H$	2 5
Ta	73	181	$3\frac{1}{2}H$	
Re	75	185	$2\frac{1}{2}H$	
		187	$2\frac{1}{2}H$	
Ir	77	191	$2\frac{1}{2}H$	
		193	$\frac{1}{2}H$	
Au	79	197	$1\frac{1}{2}H$	0 23 S, 1 67 F
Tl	81	203	$\frac{1}{2}H$	1 47 S, 1 42 F, 1 8 G
		205	$\frac{1}{2}H$	1 47 S, 1 42 F, 1 8 G
Bi	83	209	$4\frac{1}{2}H$	3 60 S, 3 54 F, 4 0 G

Fig 20 30 Protonic nuclei with charge and mass both odd In the spin column, the letters *H*, *B*, and *R* show the method used, hyperfine structure, band spectra or atomic rays In the column of magnetic moments the letters denote the authority, *S* for Schuler, *ZP*, 1933, 88 324, *F* for Fermi and Segrè, *ZP*, 1933, 82 748, and *G* for Goudsmit, *PR*, 1933, 43 638 Figures given without a letter are from recent papers which can be traced in the bibliography

Element	Charge	Mass	Spin	Magnetic moments
H	1	2	1 <i>B</i>	0 7-0 8 <i>R</i>
Li	3	6	0 <i>H</i> , ≥ 1 <i>R</i>	0 7-0 8 <i>R</i>
B	5	10		
N	7	14	1 <i>B</i>	≤ 0.2 <i>H</i>

Fig 20 31 Deutonic nuclei with charge odd and mass even For key to letters see Fig 20 30

Element	Charge	Mass	Spin	Magnetic moments
Be	4	9	$\frac{1}{2}^+ H$	
C	6	13		
O	8	17		
Ne	10	21		
Mg	12	25		
Si	14	29		
S	16	33		
Cr	24	53		
Zn	30	67	$1\frac{1}{2} H$	
Ge	32	73		
Se	34	77		
Kr	36	83	$\geq 3\frac{1}{2} H$	
Sr	38	87	$\geq 1\frac{1}{2} H$	-0 86 S
Mo	42	95		
		97		
Ru	44	99		
		101		
Pd	46	?		
Cd	48	111	$\frac{1}{2} H$	-0 63 S, -0 53 F, -0 67 G
		113	$\frac{1}{2} H$	-0 63 S, -0 53 F, -0 67 G
Sn	50	115		
		117	$\frac{1}{2} H$	-0 95 S, -0 89
		119	$\frac{1}{2} H$	-0 95 S, -0 89
		121		
Te	52	123		
		125		
Xe	54	129	$\frac{1}{2} H$	-ve } $\mu_{129}/\mu_{131} = -1.1$
		131	$1\frac{1}{2} H$	+ve }
Ba	56	135	$2\frac{1}{2} H$	
		137	$2\frac{1}{2} H$	0 94 S, 1 05 F
Sm	62	?		
Gd	64	?		
Dy	66	161		
		163		
Er	68	167		
Yb	70	171		
		173		
Hf	72	?		
W	74	183		
Os	76	187		
		189		
Pt	78	195	$\frac{1}{2} H$	
Hg	80	199	$\frac{1}{2} H$	0 55 S, 0 46 F, 0 55 G
		201	$1\frac{1}{2} H$	-0 62 S, -0 51 F, -0 62 G
Pb	82	207	$\frac{1}{2} H$	0 60 S, 0 67 F, 0 60 G
		209		

Fig 20 32 Neutronic nuclei with charge even and mass odd For key to letters see Fig 20 30

equations offer a simple method of determining the magnetic splitting factor $g(I)$, and hence the magnetic moment of the nucleus, but in practice a large relativity correction has to be made before these formulae can be applied * A more general formula is thus of greater value

When several electrons are active the interval quotient can be shown to have the value

$$A = \frac{\mu H}{IJ},$$

where μ is the magnetic moment of the nucleus, and H is the magnetic field produced by the electrons at the nucleus When the eigenfunctions of the electrons are known, the field H can be calculated, and then the experimental values of the interval quotient determine the magnetic moment of the nucleus

Whether this theory can be considered satisfactory depends on how well nuclear moments agree when calculated from different terms of the same spectrum, and from different spectra with the same nucleus That 20 terms distributed among the first three spectra of thallium do all give the same magnetic moment is therefore encouraging

The magnetic moments thus obtained are summarised in Figs 20 30–32 In these figures the nuclear moment is given in nuclear magnetons, μ_N , where

$$\mu_N = \frac{e\hbar}{4\pi Mc} = \frac{\mu_B}{1838},$$

M is the mass of the nucleus and μ_B is the Bohr magneton

Of the isotopes of even mass only two magnetic moments are known, H^2 and N^{14} , in all the other isotopes the mechanical moment is zero, so that there is no hyperfine structure and therefore no evidence An inspection of the table of isotopes of odd mass, however, shows a clear division between those of even and odd charge, among the former positive and negative moments are equally frequent, the latter being shown empirically by inverted hyperfine terms, moreover, the moment is always less

* Goudsmit, *PR*, 1933, 43 636 The precise form the formulae should take seems also in doubt Pauling and Goudsmit, *Structure of line spectra*, 1930, 60 and 209, note 1

than 1, on the other hand, when the charge is odd, the moment is always positive, and with two or three exceptions it is greater than 1 *

Though this last statement certainly indicates a tendency, it must not be taken quite at its face value, for there are many spectra in which spectroscopists have commonly failed to find any structure at all, among these are N I, P II, Cl II, K I, K II, Ag I and Cd II. Band spectra dispose of the hypothesis that the mechanical moments of these atoms are all zero, nitrogen, phosphorus and potassium all spin. A second alternative is that the magnetic field produced by the electrons at the nucleus is very weak, but as some of the atoms possess a penetrating s electron such as usually produces wide hyperfine intervals, this explanation is improbable. Better is the suggestion that the magnetic moments of these nuclei are small, in two elements certainly recent work supports this, thus Jones † has shown that in Cd II, though 12 out of the 13 lines which he examined have no structure, the thirteenth reveals the ground, 2S , term as double and indicates a nuclear magnetic moment of $0.625\mu_N$. In K I too Jackson and Kuhn ‡ have recently shown that if sufficient care is taken, the resonance lines may be resolved to a narrow doublet, the lines were obtained from a discharge tube containing neon at a pressure of a few mm. and potassium at a pressure of less than one two-thousandth of a mm., before entering the spectrograph, the light passed through a ray of potassium atoms travelling at right angles across its path. This atomic ray passed through a cool tube whose length was twenty times its diameter, so that the Doppler width of absorption lines was only 1/20 of that obtained with a random distribution of velocities. Resolved with a Fabry-Perot étalon, the lines appeared as a doublet with an interval of 0.015 cm^{-1} .

* The magnetic moments have been tabulated by Fermi and Segrè, *ZP* 1933, 82 748, Goudsmit, *PR*, 1933, 43 638, Schuler, *ZP*, 1933, 88 324. Schuler gives a critical survey of the data and recalculates the values.

† Jones, *Phys Soc Proc* 1933, 45 625.

‡ Jackson and Kuhn, *PRS*, 1934, 148 335.

10. The structure of the nucleus

At the present day the evidence of hyperfine structure suffices to divide the nuclei into four types, though history relates that Aston* recognised these types long before hyperfine structure had advanced far enough to be of much service. The nuclear type is most simply determined by the mass and charge, the crucial question being whether each of these is odd or even (Fig 20 33).

Nuclear mass	Even	Even	Odd	Odd
Nuclear charge	Even	Odd	Even	Odd
No of isotopes known	114	4	44	45
No of nuclear moments known	>30	3	12	30
Value of I	0	1 or $\frac{1}{2}$ 0	Half integral	Half integral
No of magnetic moments known	0	2	10	20
Values and sign of μ	No evidence, probably 0	—	As often -ve as +ve, $\mu < 1$	Always +ve, in all but 4, $\mu > 1$
Structure of nucleus suggested	Core with $I = \mu = 0$	Core + 1 deuteron	Core + 1 neutron	Core + 1 proton

Fig 20 33 Frequency of occurrence of the four nuclear types and their properties

When both mass and charge are even, the nuclear moment is always zero, nothing is then known of the magnetic moment, though it is commonly assumed zero. This structure is particularly stable, Aston's figures showing that more than half the known isotopes are built to this plan.

When the mass is even and the charge is odd, the nucleus is usually unstable, though four light isotopes do occur, the heaviest of these is N^{14} . Of the known mechanical moments, two are of magnitude 1 and a third is possibly zero.

When the mass is odd and the charge even, the mechanical moment is half an odd integer, the magnetic moment is as often negative as positive, and is always less than one. In terms of the number of isotopes known, this nuclear type is less than half as stable as the even-even type.

* Aston, *Mass spectra and isotopes*, 1933, 175

Finally, when the mass and charge are both odd, the mechanical moment is again half an odd integer, but the magnetic moment is always positive and usually greater than 1. This type of nucleus is about as stable as the preceding type. Further, of ten of these elements, which have two isotopes, nine pairs have the same spin, the one exception, rubidium, is exceptional also in being radioactive. When two isotopes are of even charge, they do not obey this law.*

These facts are independent of any theory of the structure of the nucleus, but to proceed further some simplifying assumptions are necessary. A very few years ago, the nucleus was supposed constructed of protons and electrons,† and there seemed no alternative to allotting the proton a spin of $\frac{1}{2}\hbar/2\pi$ and depriving the electron of the spin it has outside the nucleus. But to-day this awkward trick of giving the electron different properties in different places can be avoided by introducing the neutron, and supposing the nucleus built of three bricks only, the proton, the neutron and the α -particle. The odd-even rule of mechanical moments is then easily explained if both the proton and neutron have a spin of $\frac{1}{2}\hbar/2\pi$, while the α -particle has no spin at all. The band spectra of H^1 and He^4 confirm this assignment for they show that these nuclei have respectively moments of $\frac{1}{2}$ and 0, while the moment assigned to the neutron is that suggested by the quantum mechanics.‡ Becoming still more definite, if a nucleus of mass M and charge Z is composed of N neutrons, P protons and A α -particles, then obviously

$$M = 4A + N + P,$$

$$Z = 2A + P$$

As these equations are indeterminate, there are current at the present time two simplifying assumptions, the first due to Heisenberg§ eliminates the α -particles altogether and leaves the

* Schuler and Schmidt, *ZP*, 1935, 94 460

† Bryden, *PR*, 1931, 38 1989, White, *PR*, 1931, 38 2073a

‡ Temple, *PRS*, 1934, 145 344

§ Heisenberg, *ZP*, 1932, 78 156. This theory has been developed by Eastman, *PR*, 1934, 46 1

nucleus composed of neutrons and protons only, the second proposed by Iwanenko and used by Landé* does not allow the presence of more than one proton, thus one being present whenever the charge is odd.

For the present purpose, however, the respective merit of these two hypotheses are of little account, for both are consistent with the assumption that the even-even nucleus is the core from which the other three types are formed by the addition of a proton, a neutron or a deuteron. For convenience we may then describe a nucleus with odd mass and odd charge as 'protonic', one with odd mass and even charge as 'neutronic', and one with even mass and odd charge as 'deutonic'. This hypothesis in its simplest form has not thus far been stretched to cover all the facts of hyperfine structure, but it has succeeded so well that recent papers have agreed that some variation is indicated.

Consider first the protonic nucleus. If the mechanical and magnetic moments are to be ascribed to a single proton moving outside a core in which both moments are zero, the proton is best assumed to move in an orbit and to have on this account an angular momentum \mathbf{L} , which is always integral, the inherent spin of the proton is then written \mathbf{s} with value $\frac{1}{2}$, and this is made to set parallel or anti-parallel to the orbital vector, so that the resultant moment \mathbf{I} is half integral. That the magnetic moments are often large suggests that the magnetic moment of the proton is itself large; and in fact Stern† has arrived at the same conclusion from an entirely independent experiment. Working on the deflection of an atomic ray in a magnetic field, he concluded that the magnetic moment of the proton is $2\frac{1}{2}$ nuclear magneton, the experimental error being given as 10 per cent; Landé, however, in applying this observation to hyperfine structure has found that a g factor of 4 fits the facts better than one of 5.

If the proton revolves about a core, the magnetic moment may be calculated by the methods developed by Goudsmit for an

* Iwanenko, *Comptes Rendus*, 1932, 195 430; Landé, *PR*, 1933, 43 620, 1620 and Landé, 1931, 45 842a.

† Landé, *PR*, 1933, 44 1028a; Kallmann and Schuler, *ZP*, 1934, 88 240.

‡ Stern, *Helv. Phys. Acta*, 1933, 6 426.

electron, the moment will be gI , where the splitting factor g is given by the equation

$$g = \frac{I(I+1) + l(l+1) - s(s+1)}{2I(I+1)} g(l) + \frac{I(I+1) + s(s+1) - l(l+1)}{2I(I+1)} g(s)$$

As $g(l)$ is the splitting factor of a revolving particle, its value must be 1, $g(s)$ is assigned the value 4 for reasons already given. The nuclear moments resulting from this system are shown in Fig 20 34

The experimental moments do not agree very well with this theory, but as no observation can be trusted to less than 10 per cent and the calculations of different authors from the same data

$I \backslash l$	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$
0	2				
1	0	3			
2		$\frac{3}{2}$	4		
3			$\frac{1}{2}$	5	
4				$\frac{1}{2}$	6
5					$\frac{11}{2}$

Fig 20 34 Protonic nuclei Magnetic moments dictated by theory

show even wider discrepancies, comparison is not easy (Fig 20 35). The general tendency of the magnetic moments to increase with the spin is well covered by the theory, while if we concentrate attention on the nuclei with a spin of $1\frac{1}{2}$, the division into seven elements with magnetic moments greater than 2 and three elements with moments less than 1 is striking and in accord with theory, on the other hand the prediction that for any value of I there can be only two values of μ seems definitely at variance with experiment, the range from sodium with an estimated moment of 2.14 to lithium with a moment of 3.29 seems to exceed the possible error, moreover, there is no doubt that the two gallium isotopes have different moments. Ways of circumventing this difficulty will, however, be better taken up when the neutronic nuclei have been considered.

Landé* has pictured the neutronic nuclei built to a similar plan, the mechanical and magnetic moments of the nucleus are due wholly to a single neutron, moving with angular moment 1 and inherent spin s round an inert core, the inherent spin is again

Spin	Element	Mass	Magnetic moment	
			Obs	Theory
$\frac{1}{2}$	F	19	3.2	2 or 0
	Al	27	1.93	
	Tl	203	1.47	
		205	1.47	
$1\frac{1}{2}$	Li	7	3.29	3 or 0.60
	Na	23	2.14	
	K	39	0.38	
	Cu	63	2.74	
		65	2.74	
	Ga	69	2.14	
		71	2.74	
	As	75	0.90	
	Rb	87	3.06	
	Au	197	0.23	
$2\frac{1}{2}$	Rb	85	1.49	4 or 1.43
	Sb	121	2.70	
$3\frac{1}{2}$	Sb	123	2.10	5 or 2.33
	Cs	133	2.91	
$4\frac{1}{2}$	In	115	5.25	6 or 3.27
	Bi	209	3.60	

Fig 20.35 Protonic nuclei Observed magnetic moments compared with theory

$\frac{1}{2}h/2\pi$, but the magnetic splitting factors are different, an uncharged particle will create no magnetic field in its motion so that $g(l)$ will be zero, and the nuclear splitting factor g will be simply

$$g = \frac{I(I+1) + s(s+1) - l(l+1)}{2I(I+1)} g(s)$$

That the magnetic moment of a neutronic nucleus is as often negative as positive has been taken to mean that the inherent magnetic moment of a neutron is negative, but it would appear that equal numbers of positive and negative moments appear provided that $g(l)$ is equated to zero, the sign of $g(s)$ is irrelevant

* Inglis and Landé, *PR*, 1934, 45 842a

Thus if $g(s)$ is assigned the value 1, so that the magnetic moment of the neutron is $\frac{1}{2}$ a nuclear magneton, the permitted nuclear moments are those shown in Fig 20 36, while if $g(s)$ is given the value -1 , the numerical values will be unaltered, but the $+$ and $-$ signs must be interchanged.

$I \backslash l$	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$
0	$+\frac{1}{2}$		
1	$-\frac{1}{6}$	$+\frac{1}{2}$	
2		$-\frac{1}{10}$	$+\frac{1}{2}$
3			$-\frac{1}{14}$

Fig 20 36 Neutronic nuclei Magnetic moments dictated by theory

That the nuclear moments thus calculated are rather smaller than those observed is of no account, for the neutron may easily be assigned a larger magnetic moment, but the inequality of the positive and negative values is serious, for experiment suggests that positive and negative values occur over the same numerical range, indeed, all observed moments lie between $+\frac{1}{2}$ and $+1$ or between $-\frac{1}{2}$ and -1 . Clearly these figures are more in conformity with the suggestion that the magnetic moment of the neutron is -0.6μ and can set itself parallel or anti-parallel to the field, there being no orbital motion. Whether the observed moments cover too wide a range actually to invalidate this suggestion is probably still a matter of opinion.

In the hope of obtaining better agreement in both protonic and neutronic nuclei, Schuler* has proposed the introduction of another vector \mathbf{r} , which shall combine with the resultant of \mathbf{l} and \mathbf{s} to produce \mathbf{I} , while Tamm and Altschuler† have proposed that in some nuclei two neutrons uncouple from the core, and combine freely with the external proton or neutron. Both authors can claim that the magnetic moments thus calculated nowhere clash with experiment, but both theories give a choice of values, which must be considered rather wide when compared with the paucity

* Schuler, *ZP*, 1934, 88 323

† Tamm and Altschuler, *Comptes Rendus de l'Acad de Sci de l'U S S R*, 1934, 1 458

and inaccuracy of the experimental figures Inglis and Landé* have expressed their preference for the solution of Tamm and Altschuler, but where rapid advance is likely in the next few years, a detailed analysis of what is tentative seems hardly justified

Thus the deutonic nuclei alone remain to be considered. In these the mass is even and the charge odd, so that both a neutron and a proton must be added to the core of even mass and even charge. If these always combine in the same way to form the deuteron nucleus of heavy hydrogen, nuclei of this type should always have the unit spin of the deuteron itself, N^{14} actually has this spin, but Li^6 has always been assumed to have zero spin. Whereas the spins of H^2 and N^{14} were obtained from band spectra, however, the spin of Li^6 has been obtained only from hyperfine structure and signifies therefore only that if the nucleus has a spin moment its magnetic moment is very small. Indeed the lines of N^{14} are also narrow, so that all we know of its magnetic moment is that it is certainly less than 0.2 nuclear magneton. Clearly band spectra observations which will determine the nuclear moments of the two deutonic nuclei Li^6 and B^{10} are much to be desired.

BIBLIOGRAPHY

Much the most thorough article is one by Kallmann and Schuler in *Erg. d. Exact Naturwiss.* 1932, 11, 134. The treatment is exhaustive, but much has been learnt since 1932.

* Inglis and Landé, *PR*, 1934, 46, 76a.

CHAPTER XXI

QUADRIPOLE RADIATION

1 Forbidden lines

The transition producing a normal line must satisfy two selection rules, Laporte's rule forbids a jump from even term to even term and from odd to odd, and a second limits the changes in the angular momentum of the atom, J . On occasion, however, lines appear which violate these rules, in an electric field, for example both rules are violated, the phenomena being referred to as the 'completion of the multiplet'. As this has been known for twenty years the more recently discovered forbidden lines have been frequently ascribed to an unsuspected electric field, the precise conditions under which forbidden lines appear are therefore important.

For this purpose the forbidden lines may be classified in three divisions (1) the alkali $D \rightarrow S$ doublets, (2) the green aurora line 5577 Å and some nebular lines, (3) the mercury line $^3P_2 \rightarrow ^1S_0$, 2270 Å. Of these the last may be dismissed as probably due to the interaction between the electrons and the nucleus, the J selection rule applying rigidly only to the total angular momentum of the atom F . * The two other divisions merit fuller discussion.

The $D \rightarrow S$ lines of sodium and potassium seem to have been studied for the first time in 1922, when Datta† showed that they are absorbed by a tube of potassium vapour, the lines appeared at all pressures used from 2.5 up to 46 mm, while the potassium bands, which are known to be sensitive to electrostatic fields, did not blur until the pressure rose to 30 mm. At about the same time Foote, Mohler and Meggers‡ showed that the $D \rightarrow S$ lines are emitted in a space shielded from the applied electrostatic field, even when the field itself is very weak, though the presence of ions in the vapour may be an essential condition of the experi-

* Rayleigh, *PRS*, 1927, 117 294, Huff and Houston, *PR*, 1930, 36 842

† Datta, *PRS*, 1922, 101 539

‡ Foote, Mohler and Meggers, *PM*, 1922, 43 659

ment, it seems more probable that the lines are both radiated and absorbed in a field-free space. Later the lines were shown to occur also in a newly struck arc,* the arc was struck and run until the tips of the electrodes were red hot, when caesium carbonate was fused on to both electrodes, thereafter whenever the arc was struck, the $D \rightarrow S$ lines appeared strong for a few seconds and as they faded, the ordinary caesium lines shone forth.

The next forbidden line to attract attention was the auroral line 5577 Å, which McLennan† examined and traced to a forbidden transition in the $O\text{I}$ spectrum. As the spectrum of the night sky does not contain any nitrogen bands, the potential necessary to produce the green auroral line is presumably less than the 11.5 volts required to produce the most easily excited of these. Experiment in the laboratory confirms this hypothesis, in a discharge tube containing pure oxygen the 5577 Å line is swamped by the band spectrum, but if some neon or argon is introduced so that when the total pressure is 3 cm. of mercury the partial pressure of the oxygen is only 3 mm., the green line comes out strongly. Now the inert gases are known to have a very small potential drop throughout the discharge, so that the oxygen lines produced when the inert gas is in large excess will be limited to those of low excitation energy.

But if the green line is produced by a potential of less than 11.5 electron volts, the energy which has to be given to the oxygen atom must be less than 4.4 volts, for oxygen can be dissociated into neutral atoms by light of wave-length 1750 Å corresponding to a dissociation potential of 7.1 volts, and other evidence serves to confirm this value. The resonance potentials of atomic oxygen are, however, 9.11 volts for $^5S \leftarrow ^3P$ and 9.48 for $^3S \leftarrow ^3P$, so that an excitation energy of 4.4 volts can hardly cause the atom to emit any lines of the triplet or quintet systems. In fact with only this potential available, the terms which can be concerned in the production of the 5577 Å line seem to be limited to the five members of the p^4 configuration, $^3P_{2,1,0}$, 1D_2 and 1S_0 .

* Shrum, Carter and Fowler, H. W., *PM*, 1927, 3 27.

† McLennan and Shrum, *PRS*, 1925, 108 501, McLennan, *PRS*, 1928, 120 327

Of these low levels 1D_2 and 1S_0 are metastable, and if one of them is the initial state, the final state must be 1S_0 or one of the 3P components, but even this leaves seven possible transitions. To distinguish between them McLennan and his co-workers examined the longitudinal Zeeman effect, and found the two circularly polarised lines characteristic of a singlet line, a result which seems to reduce the seven alternatives to three, namely $^1S_0 \rightarrow ^1D_2$, $^1S_0 \rightarrow ^3P_0$, $^1D_2 \rightarrow ^3P_0$. Of these the last may be rejected because in nebular spectra which show the transitions $^1D_2 \rightarrow ^3P_{2,1}$ occurring strongly, the $^1D_2 \rightarrow ^3P_0$ line is always so weak that it has so far escaped detection, accordingly if the line 5577 Å line is $^1D_2 \rightarrow ^3P_0$, two other lines $^1D_2 \rightarrow ^3P_{2,1}$ should occur even more brightly in positions which can be calculated, but in fact these lines do not appear either in the aurora or in oxygen gas excited to produce the 5577 Å line. A similar argument applied to $^1S \rightarrow ^3P_0$ transition compels us to reject it too, so that the green auroral line may be taken as $^1D_2 \rightarrow ^1S_0$ without any reference to the theory of quadrupole radiation, history shows indeed that this evidence was accepted in 1928, though only later was the corner stone added in an observation of the transverse Zeeman effect, work which will be discussed in a later section.

While McLennan was developing this work on the auroral line, Bowen* showed that eight strong nebular lines could be ascribed to transitions between various metastable states of oxygen and nitrogen (Fig. 21.1). Whereas stellar spectra contain lines characteristic of almost all elements, nebulae emit only a few lines and of these all that have been assigned arise from the six elements H, He, C, O, N and A, thus of a list of 79 lines between 3300 and 7300 Å, 57 have been assigned to these elements.† The lines due to hydrogen and helium are of no particular interest, for they are simply the lines normally produced in the laboratory, but of the lines due to the other elements many are normally forbidden.

That lines forbidden in the laboratory should appear in nebulae

* Bowen, *N*, 1927, 120 473

† Becker and Grotrian, *EEN*, 1928, 7 56. A iv lines from Boyce, *PR*, 1935, 48 401

is not difficult to explain. The intensity of a line depends both on the transition probability and the concentration of atoms in the excited state, in nebulae a low probability may be balanced by a high concentration, but this is not usually possible in the labora-

Spectrum	Wave-length	Transition	Allowed or forbidden
O I	6302	$2p^4\ ^1D_2 \rightarrow 2p^4\ ^3P_2$	f
	6364	$2p^4\ ^1D_2 \rightarrow 2p^4\ ^3P_1$	f
O II	3726	$2p^3\ ^2D_{1/2} \rightarrow 2p^3\ ^4S_{1/2}$	f
	3729	$2p^3\ ^2D_{3/2} \rightarrow 2p^3\ ^4S_{1/2}$	f
	4076	$2p^2\ 3d\ ^4P_{1/2} \rightarrow 2p^2\ 3p\ ^4D_{3/2}?$	a
	4416 {	$2p^2\ 3p\ ^2D_{1/2} \rightarrow 2p^2\ 3s\ ^2P_{1/2}$	a
		$2p^2\ 3p\ ^2D_{3/2} \rightarrow 2p^2\ 3s\ ^2P_{1/2}$	a
		$2p^2\ 3p\ ^2D_{3/2} \rightarrow 2p^2\ 3s\ ^4P_{1/2}$	a
	4649	$2p^2\ 3p\ ^4D_{1/2} \rightarrow 2p^2\ 3s\ ^4P_{1/2}$	a
	7325	$2p^3\ ^2P \rightarrow 2p^3\ ^2D$	f
O III	3313	$2p\ 3p\ ^3S_1 \rightarrow 2p\ 3s\ ^3P_1$	a
	3342	$2p\ 3p\ ^3S_1 \rightarrow 2p\ 3s\ ^3P_2$	a
	3445	$2p\ 3p\ ^3P_2 \rightarrow 2p\ 3p\ ^3P_2$	a
	3759	$2p\ 3p\ ^3D_2 \rightarrow 2p\ 3s\ ^3P_2$	a
	4363	$2p^2\ ^1S_0 \rightarrow 2p^2\ ^1D_2$	f
	4959	$2p^2\ ^1D_2 \rightarrow 2p^2\ ^3P_1$	f
	5007	$2p^2\ ^1D_2 \rightarrow 2p^2\ ^3P_2$	f
N II	5755	$2p^2\ ^1S_0 \rightarrow 2p^2\ ^1D_2?$	f
	6548	$2p^2\ ^1D_2 \rightarrow 2p^2\ ^3P_1$	f
	6584	$2p^2\ ^1D_2 \rightarrow 2p^2\ ^3P_2$	f
N III	4097	$3p\ ^2P_{1/2} \rightarrow 3s\ ^2S_{1/2}$	a
	4102	$3p\ ^2P_{3/2} \rightarrow 3s\ ^2S_{1/2}$	a
	4634	$3d\ ^2D_{1/2} \rightarrow 3p\ ^2P_{1/2}$	a
	4641	$3d\ ^2D_{3/2} \rightarrow 3p\ ^2P_{1/2}$	a
A IV	4711	$3p^3\ ^2D_{3/2} \rightarrow 3p^3\ ^4S_{1/2}$	f
	4740	$3p^3\ ^2D_{1/2} \rightarrow 3p^3\ ^4S_{1/2}$	f

Fig 21.1 Nebular lines. This list is abstracted from one given by Becker and Grottrian, *EEN*, 1928, 7 56, the original gives lines of hydrogen and helium as well

tory, because atoms leave the excited state by collision. This general explanation has received interesting confirmation in a comparison of the relative intensities of the magnesium lines 2852 and 4571 Å *. The first is the line $2\ ^1P \rightarrow 1\ ^1S$, which is very intense in the arc, the second $2\ ^3P_1 \rightarrow 1\ ^1S$, which is very weak. The natural lives of the atoms in these initial states, calculated by the

* Frayne, *PR*, 1929, 34 590

methods of the quantum mechanics,* are $3 \cdot 10^{-9}$ and $4 \cdot 10^{-8}$ sec respectively, so that if the concentration of the atoms in the two states are at all comparable, the 2^1P line will appear much the more intense. In the arc the number of atoms in the 2^3P state is kept down only by frequent collisions, but if the pressure is sufficiently reduced and an inert gas is introduced to prevent the magnesium atoms diffusing to the walls, the 4571 Å line should increase in intensity. Kinetic theory shows that if the time between successive collisions is to be reduced to 10^{-8} sec, the vapour pressure must be reduced to 10^{-4} mm, and in fact the vapour pressure of magnesium is of this magnitude at $500^\circ C$. Working with an electrodeless discharge at this temperature Frayne found that the 4571 Å line was fairly prominent and that the introduction of 10 mm of various inert gases increased the intensity 50–100 times. In agreement with this work the auroral line 5577 Å appears only in the presence of an inert gas, and the intensity increases with increase in the diameter of the tube.

As a general explanation this hypothesis of Bowen's appears satisfactory enough, but recent theory shows that it may be made much more precise.

2. Quantum mechanics

The quantum mechanics shows that the ordinary spectral lines arise from a dipole oscillation and that, usually, this alone is important, but mathematically it is only the first term in the series which arises when the vector potential is developed in powers of the atomic radius divided by the wave-length of the emitted light, the second and third terms indicate quadrupole and octapole radiation of much lower intensity. Of these the quadrupole oscillation will be shown responsible for the forbidden lines described in the last section, no line has yet been attributed to an octapole oscillation.

In agreement with this theory is the low intensity of the quadrupole lines. Thus experiments in absorption suggest that 12,000 times as many atoms absorb the $3^2P \leftarrow 1^2S$ lines of K and Rb as absorb the $3^2D \leftarrow 1^2S$ lines, the ratio being roughly the

* Houston, *PR*, 1929, 33 297

same in the two metals,* while Rasetti,† using a more sensitive method depending on anomalous dispersion, found the intensity ratio of the emitted lines of K to be $1.1 \cdot 10^{-6}$, and as Rasetti estimates his accuracy as only 50 per cent this may be considered in satisfactory agreement with the theoretical value of $1.5 \cdot 10^{-6}$ ‡ That the absorption measurements show rather a large error is of little account, for the authors themselves insist that their estimate was intended to be little more than qualitative

The selection rules observed in quadrupole lines can be predicted from the simple hypothesis that a quadrupole line results from two simultaneous dipole transitions, though this rough analogy is of course a poor substitute for the rigid argument of the quantum mechanics. Thus in a dipole jump the sum of the orbital vectors $\Sigma \mathbf{L}_1$ changes always from odd to even or even to odd, and accordingly in a quadrupole jump the same quantity changes from even to even or odd to odd, in particular a quadrupole line may be emitted when the jump is between two terms of the same configuration. Again in a dipole line

$$\Delta J = 0 \quad \text{or} \quad \pm 1,$$

so that in a quadrupole line we expect

$$\Delta J = 0 \quad \text{or} \quad \pm 1 \quad \text{or} \quad \pm 2$$

And in fact all known lines do satisfy this condition

In practice the quadrupole selection rules are not used like the dipole rules to predict which lines will appear, since so few of the lines allowed have yet been produced, rather do they serve to determine whether an observed line arises from a dipole or a quadrupole transition, and for this in a field-free space the Laporte rule is crucial, if a transition is known to start from one term and end in another, then it can be written down at once as a dipole or a quadrupole. But so often one cannot be certain whether there is a stray electric field or not, and then the Laporte rule cannot be trusted, instead one has to examine the Zeeman effect and let it decide

* Sowerby and Barratt, *PRS*, 1926, 110 190

† Rasetti, *Accad. Lincei, Atti*, 1927, 6 54

‡ Stevenson, *PRS*, 1930, 128 591

3 Zeeman effect

In the Zeeman splitting of quadrupole, as of dipole, lines, the term displacements are simple fractions of the Lorentz unit, $\sigma_m = \frac{eH}{4\pi m_e c}$, so that the change in energy ΔE of any state may be written

$$\Delta E/hc = M g \sigma_m,$$

where g has the normal Landé values given by

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}.$$

But whereas in a dipole line the transitions are governed by the selection rule

$$\Delta M \leq 1,$$

in quadrupole lines

$$\Delta M \leq 2$$

And the polarisation rules too are different, as the summary given in Fig 21 2 shows, in particular, when the pattern is viewed at an angle of 45° to the field, components appear which are invisible when the line of sight is parallel or perpendicular to the field

Transition	Polarisation when viewed		
	Across field	Diagonal at 45°	Parallel to field
$\Delta M = -2$	σ	l ell	0
$\Delta M = +2$	σ	r ell	0
$\Delta M = -1$	π	σ	l circ
$\Delta M = +1$	π	σ	r circ
$\Delta M = 0$	0	π	0

Fig 21 2 Quadrupole radiation Transitions permitted in a magnetic field and the polarisation of the line produced

Applied to a singlet line, such as the green auroral line, this theory shows that the Zeeman pattern should be that shown in Fig 21 3, thus when viewed parallel to the field the Zeeman pattern of a dipole and a quadrupole line are identical, a fact which explains why McLennan* was able to confirm the identification of the 5577 Å line as due to $^1D_2 \rightarrow ^1S_0$ before the quadrupole theory was developed, though later Frerichs and Campbell† showed that

* McLennan, McLeod and Ruedy, *PM*, 1928, 6 558, Sommer, *ZP*, 1928, 51 451

† Frerichs and Campbell, *PR*, 1930, 36 1460

it arises in a quadrupole transition, for McLennan viewed the pattern along the field axis, while Frerichs and Campbell viewed it transversely. The latter obtained a beautiful confirmation of theory, finding two π components at a distance of one, and two σ components at a distance of two, Lorentz units, all four lines being of equal intensity (Fig 21 3)

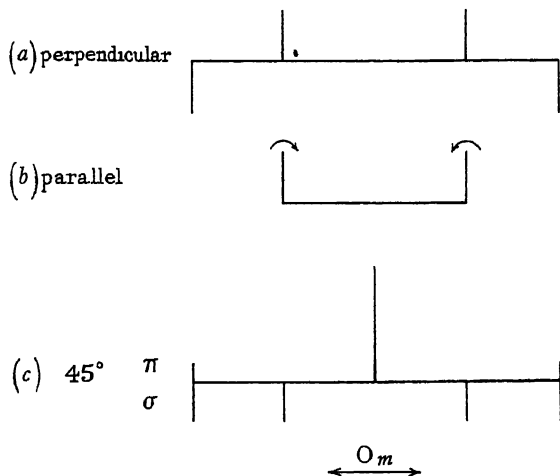
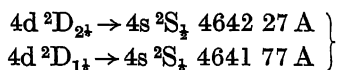


Fig 21 3 Zeeman pattern of a quadrupole singlet line, when viewed (a) perpendicular, (b) parallel, (c) at 45°, to the field. The intensities are proportional to the lengths of the verticals

A more rigorous because more complex confirmation of theory has been obtained by Segre and Bakker, who measured both the Zeeman and Paschen-Back splitting of SD alkali doublets,* and contrasted these with the splitting of the mercury line 3680 Å, this line arises as $7p\ ^3P_2 \rightarrow 6p\ ^3P_2$ and is normally forbidden, but it appears in an electric field †

The Zeeman effect of the SD doublet was demonstrated with a field of 7500 gauss applied to the potassium lines



with an interval of $2\ 325\text{ cm}^{-1}$

* Segre and Bakker, *ZP*, 1931, **72** 724

† Bakker and Segre, *ZP*, 1932, **79** 655

For this doublet the theoretical patterns are

(1) When observed parallel to the field,

$$^2D_{2\frac{1}{2}} \rightarrow ^2S_{\frac{1}{2}} \quad 4 \quad 8/5$$

$$^2D_{1\frac{1}{2}} \rightarrow ^2S_{\frac{1}{2}} \quad 1 \quad 7/5$$

all components being circularly polarised

(2) When observed transverse to the field,

$$^2D_{2\frac{1}{2}} \rightarrow ^2S_{\frac{1}{2}} \quad (4) \quad (8) \quad 10 \quad 14/5$$

$$^2D_{1\frac{1}{2}} \rightarrow ^2S_{\frac{1}{2}} \quad (1) \quad (7) \quad 11/5$$

The figures in brackets measure π components, the other figures σ components

(3) When observed at an angle of $\pi/4$ to the field,

$$^2D_{2\frac{1}{2}} \rightarrow ^2S_{\frac{1}{2}} \quad (2) \quad 4 \quad 8 \quad 10 \quad 14/5$$

$$^2D_{1\frac{1}{2}} \rightarrow ^2S_{\frac{1}{2}} \quad 1 \quad (3) \quad 7 \quad 11/5$$

The figures in *italics* are elliptically polarised with an axis ratio of $\sqrt{2}$ 1 between the σ and π axes. The intensities in these simple transitions may be obtained from the polarisation rule and the sum rule of Burger and Dorgelo (Fig 21 4) Fig 21 5

		$^2D_{1\frac{1}{2}} \rightarrow ^2S$		$^2D_{2\frac{1}{2}} \rightarrow ^2S$		
	M	$\frac{1}{2}$	$1\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$
M	qM	$\frac{2}{5}$	$\frac{9}{5}$	$\frac{2}{5}$	$\frac{9}{5}$	3
$\frac{1}{2}$	1	$-\frac{3}{5} (2)$	$\frac{1}{5} (1)$	$-\frac{2}{5} (3)$	$\frac{4}{5} (4)$	$2 (5)$
$-\frac{1}{2}$	-1	$\frac{7}{5} (3)$	$\frac{1}{5} (4)$	$\frac{3}{5} (2)$	$\frac{1}{5} (1)$	

Fig 21 4 The displacements and in brackets the intensities of the Zeeman components of a D \rightarrow S doublet

shows that the experimental results are in complete agreement with theory

The Zeeman effect thus satisfactorily confirmed, Segrè and Bakker changed from potassium to sodium, in which the two lines lie so close together that they were not resolved in the instrument used. The pattern should then be (1) 2/1, when observed transverse to the field. Experiment confirmed this Paschen-Back effect (Fig 21 6) though the photographs are somewhat

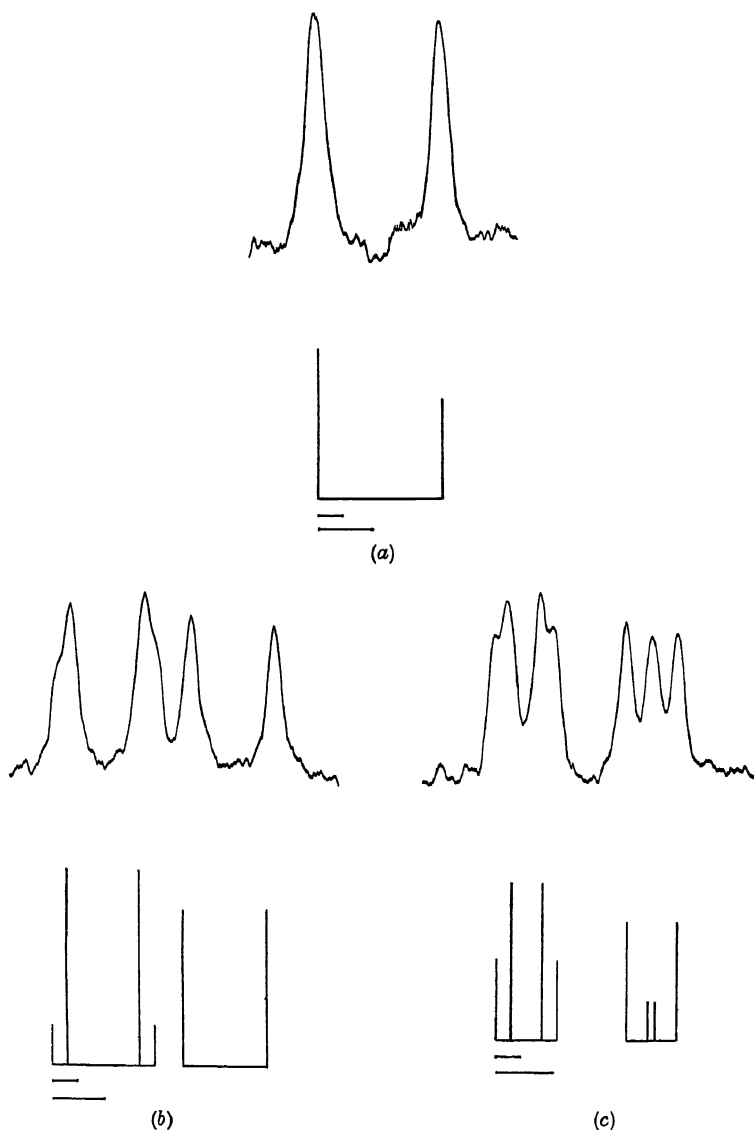


Fig 21.5 The 4642-41 Å, D \rightarrow S, doublet of potassium. In each figure the photometer curve appears above and the theoretical splitting below. The intensity scale is arbitrary, but the same in all figures. The $^2D_{3/2} \rightarrow ^2S_{1/2}$ line is on the left, and the $^2D_{5/2} \rightarrow ^2S_{1/2}$ line on the right. At the foot the scale is shown by two lines of length 0.1 Å and 1 cm⁻¹. The three figures show respectively (a) the doublet in the absence of a magnetic field, (b) the σ components viewed perpendicular to a field of 7500 gauss, and (c) the π components viewed from the same direction. (After Segre and Bakker, *ZP*, 1931, 72 728.)

distorted by the appearance of bands due to the sodium molecule, but if the intensities observed in the absence of a field are compared with those obtained with a field, the lines can be distinguished. Not content with these successes the authors measured

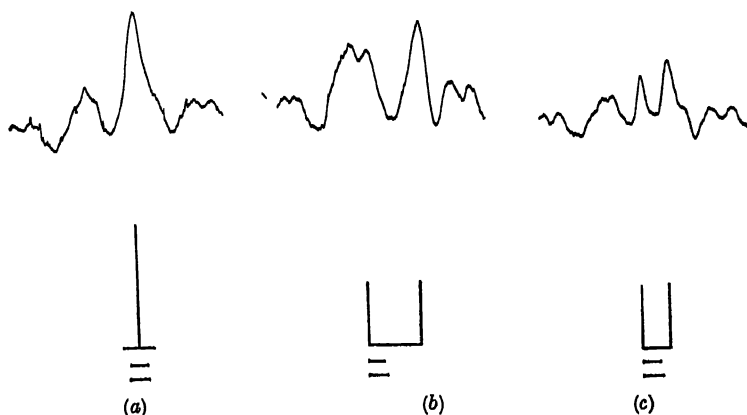


Fig 21.6 Paschen-Back effect in the 3427 Å $3^2D \rightarrow 1^2S$, line of sodium. Figure *a* is taken in the absence of a magnetic field, *b* and *c* in a field of 16,100 gauss, *b* shows the σ components viewed perpendicular to the field, *c* the π components viewed in the same direction. The appearance of the Na_2 bands confuses the pictures a little, but it is the changes which occur between *a* on the one hand and *b* and *c* on the other which are of importance. (After Segre and Bakker, *ZP*, 1931, 72 731.)

the potassium doublet in an 'intermediate field', the splitting and intensity having been calculated by Milhanczuk* with the help of Darwin's work on the gross Paschen-Back effect. A field of 17,800 gauss was used giving the ratio $\sigma_m/\Delta\nu$ a value of 0.35, and again satisfactory agreement was obtained.

Finally Segre and Bakker† examined the $7p^3P_2^0 \rightarrow 6p^3P_2^0$ line of mercury in order to show that it arises, not as a quadrupole transition, but as a dipole conditioned by the electric field. The quantum mechanics makes this probable, for a rough calculation shows that the quadrupole term in the radiation of a forbidden line is usually larger than the dipole produced by an electric field, but this is not true when a state with which both levels may com-

* Milhanczuk, *ZP*, 1932, 74 825.

† Segre and Bakker, *ZP*, 1932, 79 655.

bine lies close to one of them,* and in mercury the $7p\ ^3P_2^o$ term lies at a distance of only 188 cm^{-1} from the $6d\ ^3D_2$ term

An electric field modifies the ordinary selection rule, when the pattern is viewed transverse to the magnetic field, π components

M	-2	-1	0	1	2
Mg of $7p\ ^3P_2$	-3	$-1\frac{1}{2}$	0	$1\frac{1}{2}$	3
Mg of $6p\ ^3P_2$	-3	$-1\frac{1}{2}$	0	$1\frac{1}{2}$	3
Pattern if a dipole	(0) (3) 0 3 6/2				
Pattern if a quadrupole	(3) 6/2				

Fig 21 7 Theoretical Zeeman pattern of the 3680 Å, $7p\ ^3P_2 \rightarrow 6p\ ^3P_2$, line of Hg I, when viewed perpendicular to the magnetic field, if it arises as (a) a dipole transition conditioned by an electric field, (b) a quadrupole transition

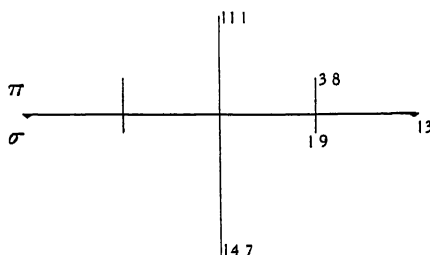


Fig 21 8 Theoretical intensities of the Zeeman components of the 3680 Å line of Hg I, if it arises as a dipole in an electric field. The line is viewed transverse to the field. (After Segrè and Bakker, *ZP*, 1932, 79 655)

arise from transitions in which ΔM is 0 or ± 1 and σ components when ΔM is 0, ± 1 or ± 2 , so that the pattern should be (0) (3) 0 3 6/2 (Fig 21 7) and intensity calculations show that of these components the undisplaced line is far the strongest (Fig 21 8). In contrast the selection rules of a quadrupole line allow π components to arise only when ΔM is ± 1 and σ components only when ΔM is ± 2 , so that the pattern predicted is (3) 6/2. The photometer curve reveals simply a strong central line, but as there seems no doubt that the resolving power was sufficient to have separated the side components of a quadrupole line, this may be taken as proof that the transition is a dipole.

* Huff and Houston, *PR*, 1930, 36 842.

4 Intensities

Theory has been able to predict the intensities of normal quadrupole multiplets,* but very little has been confirmed by experiment. In part this is due to the difficulty of producing quadrupole lines in the laboratory, and in part to the restriction of theory to multiplets arising from Russell-Saunders coupling, a restriction which implies the exclusion of all inter-system lines, no general theory of the latter has yet been developed, so that the intensity of each separate line has to be calculated.

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There are two important articles, the first by Becker and Grottrian "Über die galaktischen Nebel und den Ursprung der Nebellinien" in *Erg d Exact Naturwiss* 1928, 7 8, and the second by Rubinowicz and Blaton on "Die Quadrupolstrahlung" in *Erg d Exact Naturwiss* 1932, 11 170. The second of these deals with the theory more thoroughly than here.

* Rubinowicz and Blaton, *EEN*, 1932, 11 176

CHAPTER XXII

FLUORESCENT CRYSTALS

1 The energy levels of crystals

Though the energy levels of many gases and vapours are now well known, the energy levels of solids are the hills of an unexplored land, for only two routes have yet led to quantitative results, and both of these are of limited value. X-ray levels do change from one chemical compound to another,* but as 15,000 calories correspond to a shift of between 0.1 and 0.01 X unit or 10^{-4} of the energy involved, the measurements have to be very accurate. The light scattered in the Raman effect,† on the other hand, is accurate enough, but it reveals only certain energy states out of the many which exist.

A third route runs through the absorption and fluorescent spectra of crystals, though as yet this has given hardly any quantitative results. At room temperature the lines are blurred by the agitation of the molecules, for the levels of one ion are split by the electric field of the ions which surround it, but the lines sharpen as the temperature is reduced. Much valuable work has been done in liquid air, but Spedding and his co-workers in California are rapidly making this of little more than historic interest by their work in liquid hydrogen.

Many absorption lines are of course due to molecular vibrations, but in the last five years evidence has accumulated to show that in the rare earths and the phosphors of chromium at least, many lines arise as electronic transitions within the atom. This evidence is both simpler and more conclusive in chromium.

2. Fluorescence and phosphorescence

When a solution of chlorophyll is illuminated with a beam of violet or ultra-violet light, a green band can be seen from all

* Siegbahn, *Spektroskopie der Röntgenstrahlen*, 1931, 278

† Kohlrausch, *Der Smekal-Raman-Effekt*, 1931. Placzek, *Rayleigh Streuung und Raman-Effekt*, 1934. Symposium at Faraday Congress by Raman, Wood, Cabannes and others, *Trans. Far. Soc.* 1929, 25, 781. Dadiou and Kohlrausch, "Raman effect in organic chemistry", *JOSA*, 1931, 21, 286.

sides, this is an example of fluorescence. Again, in time past luminous paints consisted of impure barium sulphide, if first exposed to sunlight, this remains visible in the dark for some hours, it is said to phosphoresce. Fluorescence is distinguished from phosphorescence by the persistence of the latter after the exciting rays have been extinguished. To-day, however, this distinction does not seem of great importance, it is probably related to the fact that only solids phosphoresce, though solids, liquids and gases may all fluoresce.

In general fluorescence and phosphorescence may both be excited by cathode rays and X-rays as well as by light. When light is used, however, the wave-length of the exciting beam must be shorter than the wave-length of the light emitted, this law, discovered in 1853 by Sir George Stokes,* is simply explained by the quantum theory, energy may be lost between absorption and emission, it cannot be gained. This must not be taken to mean, however, that all wave-lengths shorter than those emitted are equally effective, on the contrary, a fluorescent solution or crystal absorbs only certain bands, and these are characteristic of the substance. The emission spectrum too is typically independent of the exciting source.

Many organic substances, such as quinine sulphate and eosin, fluoresce, but in these the cause is undoubtedly molecular. Of more interest here are the solid phosphors, a group of substances which are conveniently divided under five heads: the phosphors of chromium, the phosphors of the rare earths, the uranyl salts, the Lenard phosphors and the platino-cyanides. The phosphorescence of the first two has recently been shown to originate within the atom, and the evidence deserves detailed consideration. The phosphorescence of the other three may be atomic in origin, but no evidence is yet forthcoming, accordingly they are treated much more briefly.

3 Chromium phosphors

Many naturally occurring stones, such as the ruby, sapphire, red spinell, alexandrite and topaz, emit a red fluorescence. This

* Stokes, *Roy Soc, Phil Trans*, 1853, 142 463, 1853, 143 385

spectrum was thoroughly studied at a time when a ruby was supposed to consist of almost pure aluminium oxide, the odd 1 per cent of chromium oxide being regarded as an unimportant impurity. Of recent years however the ruby has been imitated in the laboratory, and the chromium oxide, though present in such a small proportion, has been proved an essential constituent, for without it the aluminium oxide is not fluorescent, while with it the artificial ruby exhibits a spectrum identical with that of the natural stone.

In preparing one of the chromium phosphors in the laboratory a few drops of a chromium salt solution are added to the salt of some other metal, such as calcium nitrate, and heated twice to a temperature of about 1400°C . The spectrum of this phosphor does not depend on the amount of chromium, though it is brightest with a few parts per thousand present. But the oxide must be an intrusion in the regular lattice of the bedding, mixed crystals of $\text{KAl}(\text{SO}_4)_2 \cdot 12\text{H}_2\text{O}$ and $\text{KCr}(\text{SO}_4)_2 \cdot 12\text{H}_2\text{O}$ show no phosphorescence, whatever the proportions of chromium and aluminium,* moreover, the Cr_2O_3 , or some molecule which contains it such as MgCr_2O_4 , must be isomorphous with the material in which it is bedded, for Deutschbein has shown that chromium fluoresces in:

$\alpha\text{Al}_2\text{O}_3$	which is isomorphous with	Cr_2O_3 ,	both trigonal
MgAl_2O_4	„ „	MgCr_2O_4 ,	„ regular
ZnGa_2O_4	„ „	ZnCr_2O_4 ,	„ regular
BeAl_2O_4	„ „	BeCr_2O_4 ,	„ rhombic
MgO because it forms mixed crystals with MgCr_2O_4 ,			
„ regular			

On the other hand, chromium does not fluoresce in

$\gamma\text{Al}_2\text{O}_3$ because it is not isomorphous with Cr_2O_3 , the one being regular and the other trigonal,

BeO because it forms no mixed crystals with BeCr_2O_4 , the former being hexagonal and the latter rhombic †

That Cr_2O_3 must be isomorphous with the bedding means presumably that the outer electrons must not be much distorted,

* Deutschbein, *AP*, 1932, 14 713

† Deutschbein, *PZ*, 1932, 33 874

and in fact the spectrum of the ruby is very sharp and bright, for the Cr_2O_3 molecule fits easily into the $\alpha\text{Al}_2\text{O}_3$ lattice. On the other hand, in Mg_2TiO_4 when the distortion is greater, the spectrum is more diffuse, moreover, the unaided eye notices that the phosphorescent light is much weaker

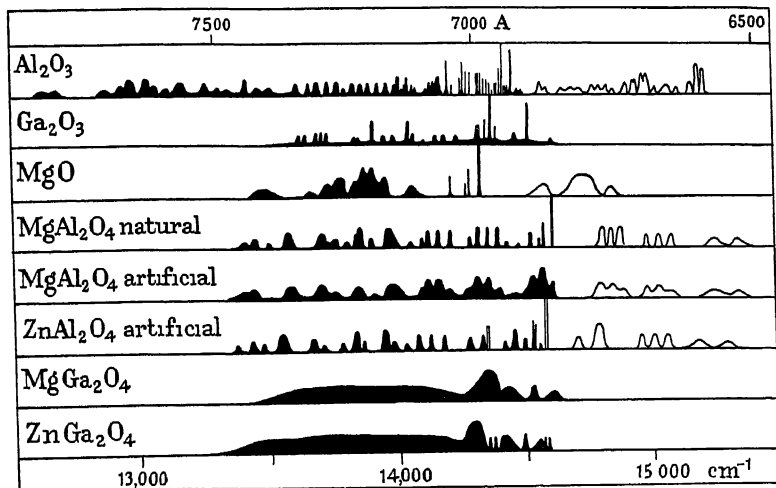


Fig. 22.1 Spectra of some chromium phosphors. The lines shown black are those emitted at -186°C , those shown in outline appear in absorption at low temperatures, but in emission only at high temperatures. Comparison of the spectra in Al_2O_3 and Ga_2O_3 , crystals belonging to the same system, shows that the principal doublet appears in both, but with fivefold greater interval in Ga_2O_3 , the two strongest subsidiary lines are also more widely separated in Ga_2O_3 . The spectra in the regular crystal system— MgO , MgAl_2O_4 , ZnAl_2O_4 —show greater symmetry and a much smaller doublet interval, the doublet of MgO being still unresolved (After Deutschbein, *PZ*, 1932, 33 876)

Whether the chromium phosphors are excited by cathode rays, X-rays or ultra-violet light, the spectrum produced is always the same, moreover, the emission spectrum is very like that produced in absorption. In both the lines are easily divided into three groups, first, principal lines, which are intense, sharp and only some half-dozen in number, second, subsidiary lines, which are weaker and more numerous, and third, bands, which are weak and often lie near the lattice bands of the pure bedding. Plate V shows these three in the emission spectrum of chromium bedded

in aluminium oxide, at the short wave-length end of the spectrum lies the principal doublet, which is here some thousand times over-exposed. Then follow on the long wave-length side a few weaker but very sharp subsidiary lines, while beyond again lie some diffuse bands. That this type is general Fig. 22.1 shows.

All the chrome phosphors have one or two principal lines in the red, while several exhibit also a group of three lines in the blue (Fig. 22.2). Moreover, examination of a series of alums of the

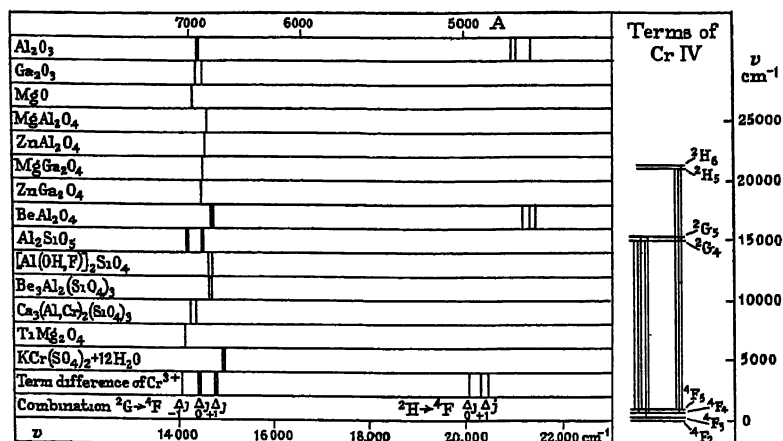


Fig. 22.2 Principal emission and absorption lines of some chrome phosphors compared with the term differences of Cr IV (After Deutschbein, *PZ*, 1932, 33 877.)

types $\text{RCr}(\text{SO}_4)_2 \cdot 12\text{H}_2\text{O}$ and $\text{RCr}(\text{SeO}_4)_2 \cdot 12\text{H}_2\text{O}$, where R is variously K, Rb, Tl and (NH_4) , shows that in liquid air all absorb a strong doublet near 6700 Å. (Fig. 22.3) * Other chromium compounds are not of course so regular as these, but most show line absorption † The frequencies of these lines, and especially of the phosphorescent lines, are comparable with term differences found in an analysis of the Cr IV spectrum, in this the $3d^3$ configuration produces a 4F ground term and above it metastable 2G and 3H

* Sauer, *AP*, 1928, 87 197

† Snow and Rawlins, *Camb Phil Soc Proc* 1932, 28 522, Joos and Schnetzler, *Z Phys Chem* 1933, B, 20 1 A paper on $\text{KCr}(\text{SO}_4)_2 \cdot 12\text{H}_2\text{O}$ by Spedding and Nutting, *J Chem Phys* 1934, 2 421, appeared after this section was written

terms, in the vapour these terms do not combine, but in the phosphors the mechanism is probably not a pure dipole, so that

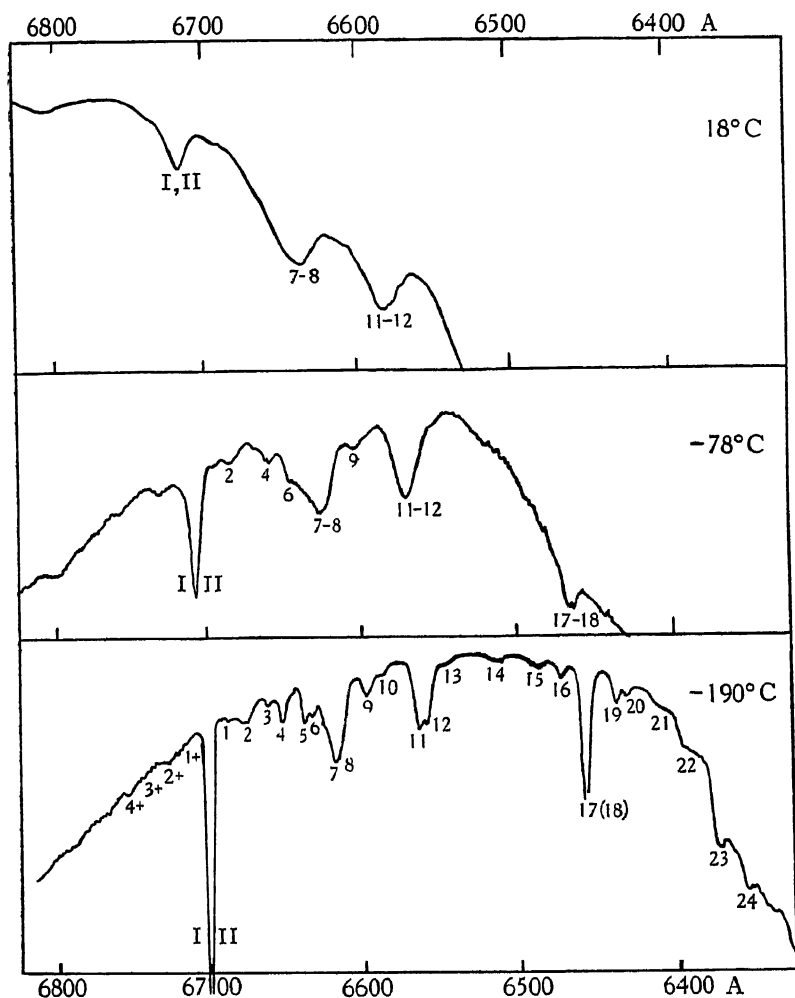


Fig 22.3 Absorption curves of $\text{RbCr}(\text{SeO}_4)_2 \cdot 12\text{H}_2\text{O}$ at 18, -78 and -190°C. Crystal thickness about 4 mm (After Sauer, *AP*, 1928, 87-219)

the transitions may reasonably be allowed. At any rate, Fig 22.2 shows that the frequencies agree so remarkably that the identification can hardly be doubted, the red principal lines are there

shown to arise in the ${}^2G \rightarrow {}^4F$ transition and the blue lines in ${}^2H \rightarrow {}^4F$,* the precise transitions to which the individual lines are to be assigned are not yet clear, but as the doublet of chrome alum is still strong at -195° and is not fading out, the two highest levels of the ground term, ${}^4F_{4\frac{1}{2}}$ and ${}^4F_{3\frac{1}{2}}$, lying at 950 and 550 cm^{-1} above ${}^4F_{1\frac{1}{2}}$, may be reasonably excluded. The Boltzmann distribution would allow very few atoms in these states at so low temperature, and the law so admirably explains the fading of certain lines in samarium that it can hardly be in error †

Like the principal lines, the subsidiary do not change greatly when the temperature is reduced from 20° to -195°C , they grow rather sharper, and the whole spectrum has its wave-length reduced by a few angstroms,‡ but they do not otherwise change much in position or intensity. They appear both in phosphorescent and absorption spectra, but they occupy very different positions in different beddings, the positions, however, are characteristic of the crystal lattice, not of the anion, indeed with practice the lattice type can be recognised from the look of the spectrum. This would seem to suggest that the subsidiary lines are Stark components produced by the electric fields of the ions in the crystal lattice, but if the atomic states are split by a strong electric field all components should be Stark components, and it is not easy to see why the half-dozen lines should be so much stronger than any others. Again, is it mere chance that nearly all subsidiary lines are of shorter wave-length than the principal lines? One might assume that some Cr_2O_3 molecules are lumped together, and that these produced different lines to those spread evenly through the bedding of say Al_2O_3 , but if the fields of Cr_2O_3 and Al_2O_3 are different, as is probable, we should expect all fields from that characteristic of Cr_2O_3 to that characteristic of Al_2O_3 , and then the lines would be diffuse not sharp, accordingly one must suppose the Cr_2O_3 molecules spread evenly through the Al_2O_3 lattice like currants in a cake. The electric field is then effectively that of the Al_2O_3 lattice

* Deutschbein, *ZP*, 1932, 33 877

† Spedding, *PR*, 1933, 43 143a

‡ Deutschbein, *AP*, 1932, 14 720

The bands are considerably more complicated than either the principal or the subsidiary lines, for they are not the same in emission and absorption, and grow more complex if the temperature is raised. At low temperatures, however, the emission spectrum consists only of a long wave-length band lying in the red and infra-red, while the absorption spectrum consists of a short wave-length band lying in the blue. If the principal lines, which lie between these two bands, are unaltered electron transitions, and the bands conform to Stokes's law, these facts are easily explained, the absorption bands must lie on the high-frequency side of the principal lines, and the emission bands on the low-frequency side *. At room temperature emission still occurs chiefly in the red and infra-red, but some diffuse bands appear on the violet side of the principal lines, these short-wave or 'anti-Stokes' bands must be emitted by molecules passing from an excited state to one of low energy, so that their intensity should be proportional to the number of molecules in the excited state, this number is determined by the Boltzmann law and decreases rapidly with decrease of temperature. Again, in the absorption spectrum there appear at room temperature bands of wave-length longer than the principal lines, these anti-Stokes absorption bands can be explained in the same way as the anti-Stokes emission bands, they are due to absorption by molecules which are already in an excited state †.

The frequencies of some of the stronger bands are also of interest. The Raman spectrum of pure $\alpha\text{Al}_2\text{O}_3$ reveals a lattice frequency of 417 cm^{-1} . Now in the emission spectrum of a ruby, the principal doublet and a strong band lie at $14,416$ and $14,006\text{ cm}^{-1}$ respectively, thus in the phosphor a band occurs with a frequency difference slightly less than that of pure Al_2O_3 , 410 instead of 417 cm^{-1} . This relationship is not uncommon in the chrome phosphors, and is accounted due to a decrease in the lattice frequency caused by the introduction of the chromium oxide molecules *.

* Dörschbeim, *ZP*, 1932, 77 490

† For the effect of a magnetic field on the lines of a ruby Du Bois and Elias, *AP*, 1908, 27 233, 1911, 35 617, Du Bois, *PZ*, 1912, 13 128.

4 Rare earth phosphors

As long ago as the eighties of last century Crookes* showed, during the course of his pioneer work on high vacua, that when rare earth minerals are irradiated with cathode rays, they emit a strong phosphorescent spectrum, but his work is only of historic interest, because very few of his rare earth preparations were pure. More recent work shows that salts, which are colourless like those of lanthanum, gadolinium and ytterbium, exhibit no after-glow, but the smallest trace of active impurity makes the substance phosphorescent, $4 \cdot 10^{-6}$ gm of samarium in 1 gm of calcium oxide is sufficient to produce a red after-glow, while the spectrum is brightest if the active earth is present in a proportion of only 1 per cent, thus the smallest trace of dysprosium in yttrium, or of terbium in gadolinium, can easily be detected †

Naturally occurring fluorites too have long been known to exhibit blue, yellow and green fluorescence, when excited with radium, but little progress was made until synthetic calcium fluoride was used and 1/10 per cent of a rare earth added. Then it was shown that the blue band appears only when europium is present, and the yellow-green band only when ytterbium. The active agent of the red band has not yet been traced, all other rare earths are inactive. As europium and ytterbium are the two rare earths which most readily become divalent,‡ and the fluorescence may be excited by heating the activated fluorite in a reducing flame, the transition from tri- to divalent form is clearly linked with the fluorescence §

Of all the rare earth phosphors, samarium has been most thoroughly studied (Fig 22 4), as elsewhere the spectrum is much the same if excited by ultra-violet light instead of by cathode rays. The lines are somewhat sharper than those of the chromium phosphors, especially at room temperature, while at low temperatures even naturally occurring fluorspar emits lines which are as sharp as the D lines of a flame poor in sodium || Again rare

* Crookes, *Chem Soc J* 1889, 55 255

† Urbain, *Chem Rev* 1924, 1 167

‡ Jantsch and Klemm, *Z f anorg und allgem Chemie*, 1933, 216 80

§ Haberlandt, Karlik and Przibram, *Akad Wiss Wien, Ber* 1934, 143 151

|| Tomaschek, *AP*, 1927, 84 329, 1047

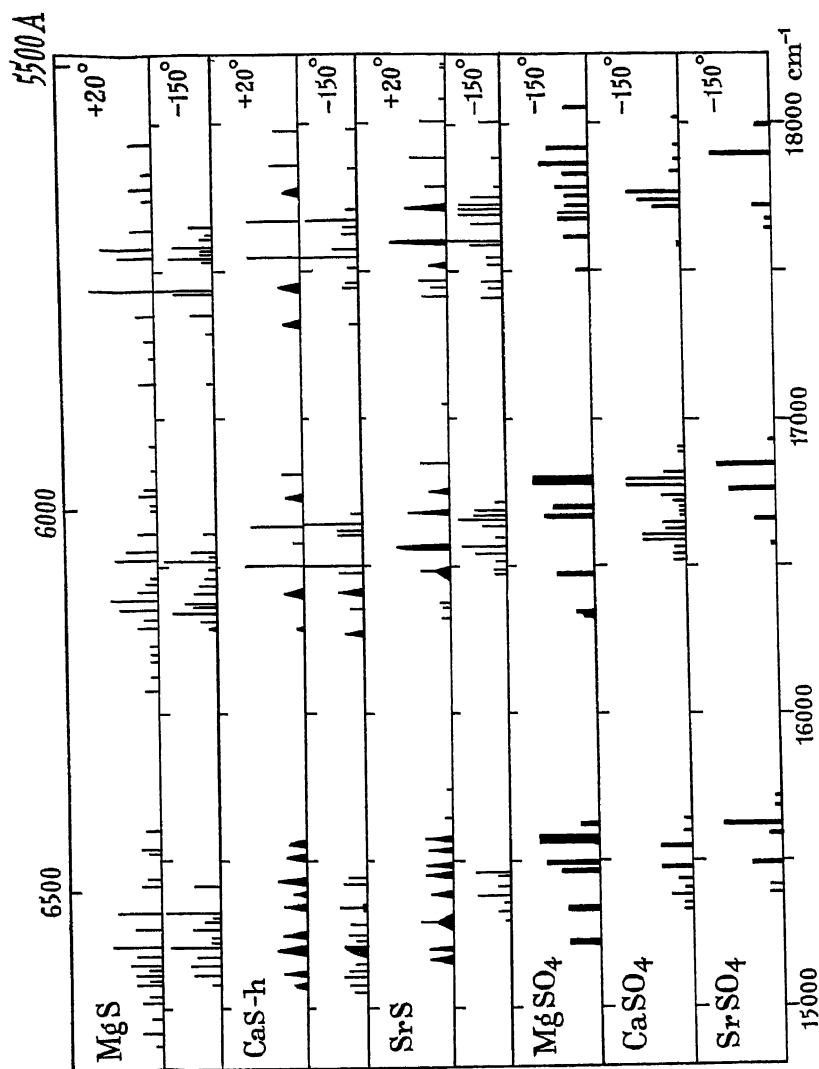


Fig 22.4 Phosphorescent spectra of samarium bedded in various sulphides and sulphates (After Tomaschek, *AP*, 1928, 84 374, 382)

earth compounds need not be isomorphous with the bedding, and this can very probably be related to the sharpness of the lines. In contrast to the chromium phosphors, however, a phosphorescent spectrum of the rare earths is usually much more complicated than the absorption spectrum *. This may be ascribed to two causes, first absorption lines all arise in levels less than 500 cm^{-1} above the ground term, but emission lines may end in much higher levels, and second all absorption lines arise in atomic transitions,

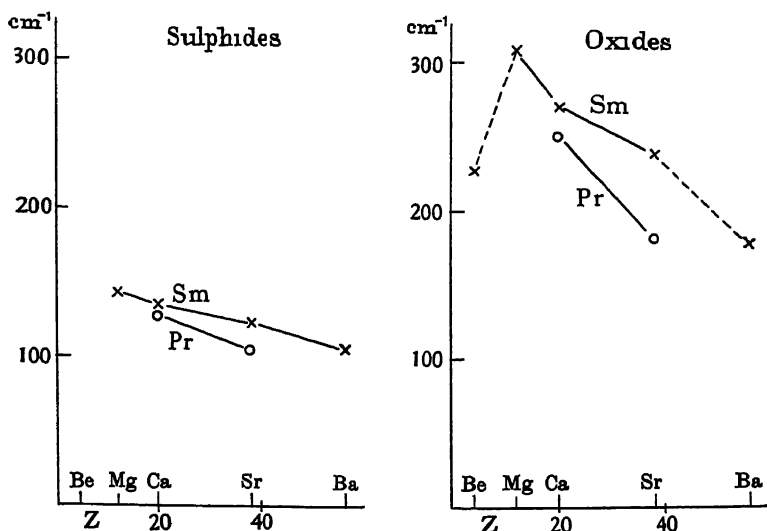


Fig 22.5 Term displacements of samarium and praseodymium phosphors, when bedded in the sulphides and oxides of the alkaline earths. The heavier the rare earth and the smaller the diameter of the alkaline earth atom, the greater the displacement. (After Tomaschek, *PZ*, 1932, 33 878.)

while some emission lines arise in the rare earth molecule and others in the lattice of the bedding.

In the rare earth phosphors the vibrations of the bedding lattice are rather slower than those calculated for the pure crystal, and this is significant for the rare earth molecules may be supposed to act as inert loads. If this explanation is correct, the change of frequency should be greater the larger the rare earth and the smaller the bedding molecule, these predictions experi-

* Spedding, *PR*, 1933, 43 143a

ment confirms The praseodymium ion has a diameter 3 per cent greater than the samarium ion, and the change of frequency is slightly greater, barium oxide has a larger diameter than calcium

cm ⁻¹	16,000	17,000	18,000	α	A-X	Type
MgO				4 21	2 10	Sodium chloride
CaO				4 80	2 38	
SrO				5 15	2 59	
BaO				5 50	2 77	
MgS				5 19	2 52	
CaS				5 69	2 80	
SrS				6 01	3 00	
BaS				6 37	3 18	
CaF ₂				5 45	2 36	Fluorite
SrF ₂				5 78	2 50	
BaF ₂				6 19	2 68	
ZrO ₂				5 10	2 21	
CeO ₂				5 40	2 34	
ThO ₂				5 57	2 42	
Al ₂ O ₃				5 15	1 84 1 99	Corundum
Ga ₂ O ₃				5 28		
Li ₂ SO ₄						
CaSO ₄						
SrSO ₄						
BaSO ₄						
λ	6200	6000	5800	5600	A	

Fig 22 6 The two short wave line groups of some samarium phosphors arranged by the crystal type of bedding α is the lattice constant and A-X the distance between the electropositive and electronegative centres in the lattice, both are in angstroms (After Tomaschek, *PZ*, 1932, 33 880)

oxide, and the change of frequency produced by bedding samarium in it is much less (Fig 22 5) *

Though in their general features the spectra of samarium in

* Tomaschek, *PZ*, 1932, 33 878

different beddings are similar, a closer examination reveals many differences. Thus in Fig. 22.6 a series of spectra are arranged according to their crystal types, the figures alongside being the lattice constant a and the distance between the positive and negative ions, both in angstroms. The crystal type is here revealed as an important influence, but in fact this is clearer still in the original photographs,* for in them a practised eye can recognise the crystal type at a glance †

With so few principal lines in the samarium emission spectrum, one naturally asks whether they may not be interpreted, like the principal lines of the chromium phosphors, as electronic transitions within the ion. As the spectrum of samarium vapour has not been analysed, the way is by no means clear, but the fact that the rare earths do not have to be isomorphous with their bedding, suggests that the outer electrons can hardly be concerned, rather are the transitions likely to occur between different terms of the $4f^n$ configuration, the brightest lines arising between terms of the same multiplicity. If transitions between the $4f^n$ and $4f^{n-1} 5d$ configurations are responsible for some lines, as Laporte has suggested, they are likely to be much more diffuse than the lines arising within the $4f^n$ configuration, indeed it is tempting to identify a diffuse spectrum, photographed beside the sharp one by Fagerberg in neodymium and by Tomaschek‡ in samarium, with these predicted lines.

Assuming that the principal lines arise within the $4f^n$ configuration, their positions can be calculated by the method which Goudsmit developed §. This method gives the extreme intervals of the various terms, if the coupling is Russell-Saunders, and the individual terms can then be interpolated with the interval rule. The calculations are laborious, but the results are eminently satisfactory, especially in praseodymium, whose emission spectrum has been recently measured by Evert ||. The lines shown in

* Photographs, *AP*, 1927 84 Taf IX-XI Cf *Z. Elect.* 1930, 36 737

† Tomaschek, *PZ*, 1932, 33 879.

‡ Tomaschek, *PZ*, 1932, 33 882

§ Goudsmit, *PR*, 1928, 31 948, and chapter XVIII of this book

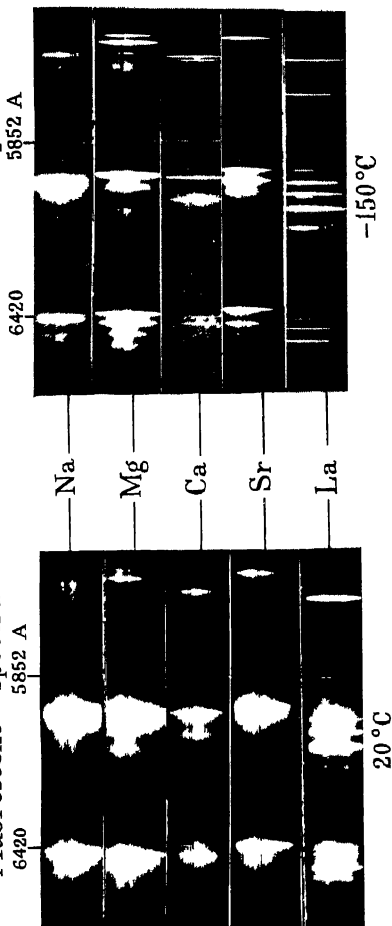
|| Evert, *AP*, 1932, 12 144

PLATE VIII

1. Fluorescent spectra of samarium bedded in various sulphates at 20 and -150°C . At room temperature the lines are so blurred that the photographs do little more than show that the positions of the multiplets are independent of the bedding, but they sharpen as the temperature is reduced. The samarium lines are much sharper in $\text{La}_2(\text{SO}_4)_3$ than in any other bedding, probably because lanthanum like samarium is trivalent (After Tomaschek, *AP*, 1927, **84**, Taf X, XI)

2. DF quintet from the iron arc. This multiplet arises as $3d^6\ 4s(4P)\ 4p\ ^5F^{\circ} \rightarrow 3d^6\ 4s^2\ ^5D$ (Lent by Prof H Dingle)

Fluorescent spectra of Samarium in various Sulphates



$5D\ 5F$ multiplet from Iron Arc

$3d^6\ 4s\ (4p)\ 4p\ 5p^o \rightarrow 3d^6\ 4s^2\ 5D$

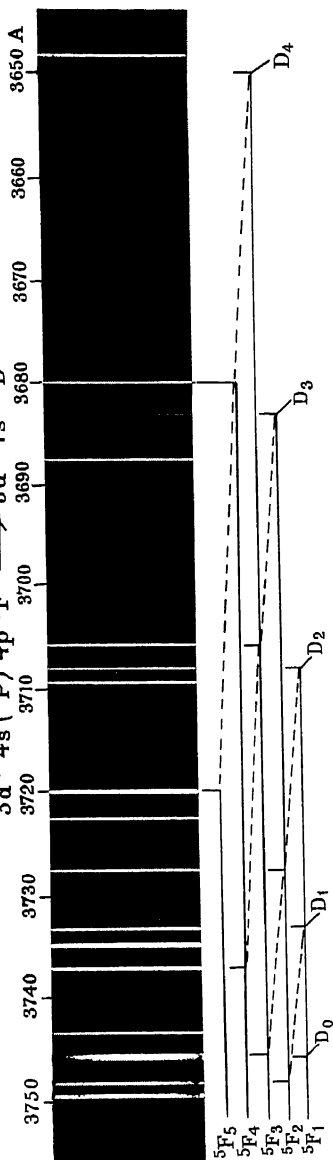


Fig 22 7, taken from left to right, arise in transitions between the following values of J

for $^3F \rightarrow ^3H$ 4-6, 3-5, 2-4, 4-5, 3-4, 4-4,

for $^1G \rightarrow ^3H$ 4-5, 4-4

Since quadrupole transitions are probable, J has been allowed to change by two units. These predictions are compared with the phosphorescent spectrum of the metal plotted above the line, and the absorption spectrum plotted below

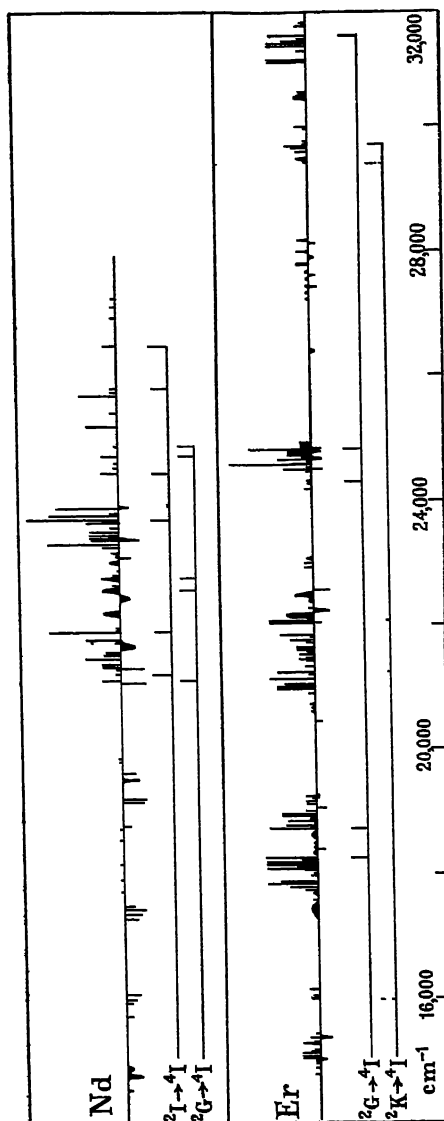
Besides praseodymium satisfactory agreement is obtained in neodymium and erbium, spectra arising from 3 and (14-3) electrons respectively. In samarium only the terms of highest multiplicity have yet been calculated, for when there are five electrons the work becomes very heavy, but the $^6F \rightarrow ^6H$ transition accounts apparently for three strong lines in the infra-red

5 Uranyl salts*

Like the rare earths the uranyl salts are fluorescent in their own right, and are not dependent on the crystal in which they happen to be embedded, but fluorescence is a property of the uranyl radical UO_2 , not of the uranium atom, for salts in which uranium is quadrivalent do not fluoresce. On the other hand the absorption spectra of all uranium compounds are so similar, that the uranyl radical cannot be considered peculiar in the energy it absorbs, but only in re-emitting some of this energy as visible light.

At room temperature and viewed in a spectroscope of low resolving power, both the fluorescent and absorption spectra of uranyl salts consist of bands. In the emission spectra some seven or eight bands appear in the yellow and red, while in absorption a smaller number appear in the green and blue, the emission band of shortest wave-length coincides with the longest absorption band. The emission bands are equally spaced in the scale of frequency, the interval varies a little from salt to salt, but is never far from 830 cm^{-1} . Again, all emission bands show the same variation in intensity, but the intensity distribution in absorption bands is quite different (Fig 22 8).

* Pringsheim, *Fluorescenz und Phosphorescenz*, 1928, 238, Nichols and Howes, *The fluorescence of the uranyl salts*, Carnegie Institute Publication, 1919, No 298



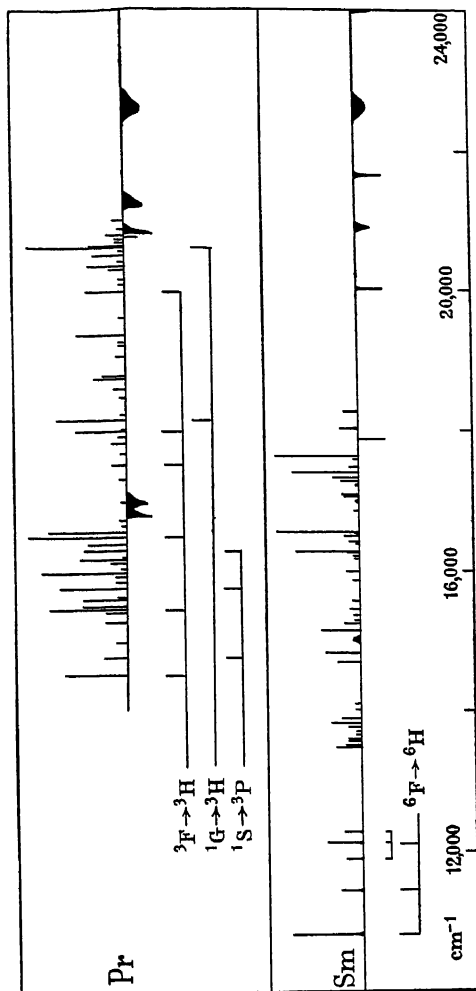


Fig 22.7 Multiplets of some rare earth phosphors compared with the lines predicted by theory. Above the line is the phosphorescent spectrum when bedded in CaO, below, the absorption of salts in solution (After Tomaschek, *PZ*, 1932, 33 883)

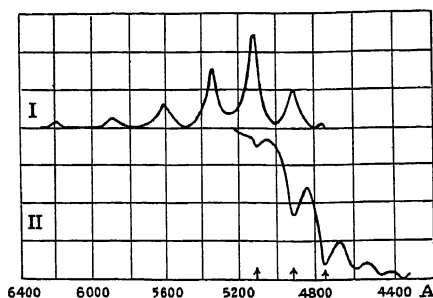


Fig 22 8 Spectra of uranyl potassium sulphate at 25°C I, the fluorescent spectrum showing the intensity increasing towards the blue, II, the absorption spectrum showing the intensity decreasing towards the blue. The three marked with arrows appear in both emission and absorption (After Pringsheim, *Fluoreszenz und Phosphoreszenz*, 1928, 241)

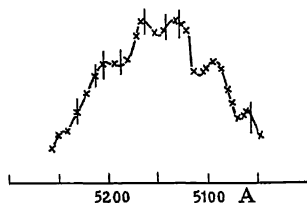


Fig 22 9 Structure of a single fluorescent band of uranyl sulphate The curve gives the intensity at room temperature, the lines are those observed at -185°C (After Pringsheim, *Fluoreszenz und Phosphoreszenz*, 1928, 243, and Wick, *PR*, 1918, 11 126)

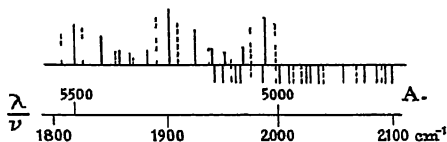


Fig 22 10 Spectra of uranyl potassium chloride at -186°C The fluorescent lines are shown above the horizontal, the absorption lines below The lines are polarised parallel and perpendicular to the principal axis of the crystal, the ordinary spectrum is shown by full and the extraordinary by dotted lines. The lengths of the lines indicate intensities (After Pringsheim, *Fluoreszenz und Phosphoreszenz*, 1928, 243)

At low temperatures high resolution splits these bands into a number of components, indeed in liquid hydrogen, at 20°A , many lines are as fine as the lines of a spark spectrum (Fig 22 9) But rather curiously even these sharp lines show no trace of broadening in a magnetic field of 2500 gauss In both emission and absorption spectra many lines are polarised parallel or perpendicular to the principal axis, in Fig 22 10 these lines are represented by continuous and dotted lines respectively, and their different positions show that when the phosphorescent spectrum is viewed through a nicol prism, two entirely different spectra appear as the nicol is rotated

The spectra of the uranyl salts resemble those of chromium sufficiently closely to make one wonder whether the frequencies of the strong lines are not equally significant, but as yet none of the higher spark spectra of uranium has been analysed

6 Lenard phosphors

Many phosphorescent compounds, whether occurring naturally as minerals or artificially produced, consist of a crystalline salt carrying a trace of some other substance which will not fit into the crystal lattice, the intruding substance is commonly a metal, and the greater number of these phosphors may be regarded as members of a family of which the alkaline earth sulphides activated by a heavy metal may be considered the prototype Having been exhaustively studied by Lenard and his school, they are commonly known as Lenard phosphors

As the intruding metal seems responsible for the fluorescence, one might have hoped for a line spectrum, instead of the broad bands actually observed, though as many workers have used spectroscopes of low resolving power combined with a slit 1 mm wide the evidence is not so extensive as could be desired When a substance exhibits several bands these often behave independently when a change is made in the wave-length of the exciting light or in the temperature, some bands fade and others grow brighter Further, the positions of the bands change when either the active element or the bedding is changed *

* Pringsheim, *Hb d Phys* 1929, 21 600

Like the Lenard phosphors the platino-cyanides phosphoresce only when bedded in some inert material, neither the pure salts nor their solutions are themselves phosphorescent, but the phosphorescence appears so regularly that it has been attributed to the platino-cyanide molecule in the absence of other evidence. The details of the emission spectra depend largely on the crystal type, and change when water of crystallisation is added. The polarisation depends on the polarisation of the incident light.

At room temperature the emitted light lasts a very short time, but if a body at -250°C is illuminated, it phosphoresces only when allowed to warm up. In this the platino-cyanides are like the Lenard phosphors.

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The newer work of Spedding, Tomaschek and Deutschbein, with which this chapter is chiefly concerned, does not seem to have been written up.

APPENDIX V

KEY TO REFERENCES

The periodicals, in which most papers appear, I have cited by the capitals introduced by Gibbs, the less common by the usual abbreviations. A key to the former is given here, a key to the latter may be found in *Science Abstracts*

<i>AJ</i>	Astrophysical Journal
<i>AP</i>	Annalen der Physik
<i>BSJ</i>	Bureau of Standards, Journal of Research
<i>EEN</i>	Ergebnisse der Exakten Naturwissenschaften
<i>JOSA</i>	Journal of the Optical Society of America
<i>N</i>	Nature
<i>Nw</i>	Naturwissenschaften
<i>PM</i>	Philosophical Magazine
<i>PR</i>	Physical Review
<i>PRS</i>	Proceedings of the Royal Society, London, series A
<i>PZ</i>	Physikalische Zeitschrift
<i>ZP</i>	Zeitschrift für Physik

An author's initials are not given in the text unless two authors of the same name occur, but all authors are given their initials in the index

APPENDIX VI

BIBLIOGRAPHY

The bibliography is divided into three sections

- A Books of reference
- B Spectra of the elements
- C Hyperfine structure of the elements

In the last two sections I follow Bacher and Goudsmit in arranging the elements in the alphabetical order of their chemical symbols

A Books of reference

The following books are frequently needed for reference, because they contain experimental data conveniently arranged. Books dealing with a particular subject are discussed at the end of the chapter on that subject

1 Gibbs, J. C. 'A complete bibliography of individual spectra for the years 1920-1931' *Rev. Mod. Phys.* 1932, 4, 278

2 Bacher, R. F. and Goudsmit, S. *Atomic energy states*, 1932. The terms of all known spectra, but no wave-lengths

3 Kayser, H. and Konen, H. *Handbuch der Spektroskopie*, vol. VIII, 1932. A complete review of the first nineteen elements in alphabetical order, that is A, Ag, Al, As, Au, B, Ba, Be, Bi, Br, C, Ca, Cd, Ce, Cl, Co, Cr, Cs, Cu. Wave-lengths, terms, magnetic splitting factors and hyperfine structure are all given with a bibliography

4 Grottrian, S. *Graphische Darstellung der Spektren von Atomen mit ein, zwei und drei Valenzelektronen*, 1928. The energy diagrams of vol. II show the transitions which produce all the strong lines

5 Fowler, A. *Report on series in line spectra*, 1922. Paschen, F. and Gotze, R. *Seriengesetze der Linienspektren*, 1922. Both books contain lists of terms and lines for the elements of the first three columns

B The spectra of the elements

These lists are select, where one paper provides a satisfactory summary, earlier papers are not cited. The papers deal chiefly with the analysis into systems of terms, but papers on intensities and the Zeeman effect have been included when they deal with a particular spectrum

When one of the books of reference, Kayser and Konen, Grottrian and Fowler, deals with a spectrum, I have included the author's name in the bibliography, as these sources are too easily overlooked.

The abbreviations are those used in the text.

A Argon 18

A Kayser and Konen

- A I Meissner, *ZP*, 1926, **39** 172, **40** 839 Terms and lines
 Dorgelo and Abbink, *ZP*, 1927, **41** 753 Extension in ultra-violet
 Gremmer, *ZP*, 1928, **50** 716 Extension in infra red
 Rasmussen, *Nw*, 1930, **18** 1112 Bergmann series
 Terrien and Dijkstra, *J de Phys* 1934, **5** 443 Zeeman effect
 Pogány, *ZP*, 1935, **93** 364 Zeeman effect
 Boyce, *PR*, 1935, **48** 396 Extreme ultra-violet
- A II Compton, Boyce and Russell, *PR*, 1928, **32** 179 Extreme ultra-violet
 Bakker, de Bruin and Zeeman, *ZP*, 1928, **51** 114, **52** 299, *K Akad Amsterdam*, 1928, **31** 780 Zeeman effect
 de Bruin, *ZP*, 1930, **61** 307, *K Akad Amsterdam*, 1930, **33**, 198
 Complete term scheme and lines
 Boyce, *PR*, 1935, **48** 396 Extreme ultra-violet
- A III Keussler, *ZP*, 1933, **84** 42 Lines and terms
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 Boyce, *PR*, 1936, **49** 351 Intersystem lines
- A IV Boyce and Compton, *Proc Nat Acad Sci* 1929, **15** 656 Lines and terms
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Ag Silver 47

Ag Kayser and Konen

- Ag I Fowler Grottrian
 Blair, *PR*, 1930, **36** 1531 Extension of series
- Ag II Shenstone, *PR*, 1928, **31** 317 Terms and lines
 Blair, *PR*, 1930, **36** 173 Two new terms
 Gilbert, *PR*, 1935, **47** 847 High terms
- Ag III Gilbert, *PR*, 1935, **47** 847 Lines and terms

Al Aluminium 13

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- Al I Fowler Grottrian
- Al II Sawyer and Paschen, *AP*, 1927, **84** 1 Terms and classified lines
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- Al III Paschen, *AP*, 1923, **71** 152 Terms and lines
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- Al IV Edlén and Ericson, *Comptes Rendus* 1930, **190** 116, 173 Resonance lines
- Al V *Ibid*
- Al VI *Ibid*

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- As I Meggers and de Brum, *BSJ*, 1929, **3** 765 Lines and terms
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- As II Rao, A S, *Proc Phys Soc* 1932, **44** 343
- As III Lang, *PR*, 1928, **32** 737 Series
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- As IV Sawyer and Humphreys, *PR*, 1928, **32** 583
- As V *Ibid*

As Arsenic 33 (*cont*)As vi Borg and Mack, *PR*, 1931, 37 470 Series**Au Gold 79**

Au Kayser and Konen

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B Kayser and Konen

B i Bowen, *PR*, 1927, 29 231 Terms and linesSmith and Sawyer, *JOSA*, 1927, 14 287 Series

B ii Bachor and Goudsmit, 1932 Terms from unpublished material

B iii *Ibid*Edlén, *ZP*, 1931, 72 763 Ground termB iv Edlén, *N*, 1931, 127 405**Ba Barium 56**

Ba Kayser and Konen

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Be Kayser and Konen

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Bi i Thorsen, *ZP*, 1926, 40 642 Terms and linesToshniwal, *PM*, 1927, 4 774 Terms and linesZeeman, Back and Goudsmit, *ZP*, 1930, 66 1 Interpretation of low terms Zeeman effectBi ii McLennan, McLay and Crawford, *PRS*, 1930, 129 579 SeriesBi iii Lang, *PR*, 1928, 32 737 SeriesBi vi Mack and Fromer, *PR*, 1935, 48 357 Pt i isoelectronic sequence**Br Bromine 35**

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Br Bromine 35 (*cont*)Br IV Deb, *PRS*, 1930, 127 197 SeriesBr V *Ibid***C Carbon 6**

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Bacher and Goudsmit, using unpublished material**Ca Calcium 20**

Ca Kayser and Konen

Ca I Fowler Grottrian
Russell and Saunders, *AJ*, 1925, 61 38 Displaced terms
Back, *ZP*, 1925, 33 579 Zeeman effect
Russell, *AJ*, 1927, 66 191 New termsCa II Fowler Grottrian
Saunders and Russell, *AJ*, 1925, 62 1 Terms and lines
Russell, *AJ*, 1927, 66 283Ca III Bowen, *PR*, 1928, 31 497Ca IV *Ibid*Ca V *Ibid***Cb Columbiu 41**Cb I Meggers, *J Wash Acad Sci* 1924, 14 442 Zeeman effect
Meggers and Kiess, C C, *JOSA*, 1926, 12 417 Lines and terms
King and Meggers, *PR*, 1931, 37 226a Furnace spectrum
Meggers and Scribner, *BSJ*, 1935, 14 629 New termsCb II Meggers and Kiess, C C, *JOSA*, 1926, 12 417 Multiplets and Zeeman effect
King and Meggers, *PR*, 1931, 37 226a Furnace spectrum
Meggers and Scribner, *BSJ*, 1935, 14 629 New termsCb III Gibbs and White, *PR*, 1928, 31 520 Multiplets
Elhason, *PR*, 1933, 43 745 New multipletsCb IV Gibbs and White, *PR*, 1928, 31 520 Multiplets**Cd Cadmium 48**

Cd Kayser and Konen

Cd I Fowler Grottrian
Ruark, *JOSA*, 1925, 11 199 Higher terms addedCd II McLennan, McLay and Crawford, *T Roy Soc Canada*, 1928, 22 45
Takahashi, *AP*, 1929, 3 27 SeriesCd III Gibbs and White, *PR*, 1928, 31 776 Multiplets

Ce Cernum 58

- Ce Kayser and Konen
 Ce I Karlson, *ZP*, 1933, **85** 482 Terms and lines
 Ce II Haspas, *ZP*, 1935, **96** 410 Terms and Zeeman effect
 Ce III Kaha, *Indian J Phys* 1933, **8** 137 Terms and lines
 Ce IV Gibbs and White, 1929, **33** 157 Doublets of stripped atoms
 Badami, *Proc Phys Soc* 1931, **43** 53 Lines and terms
 Lang, *Can J Research*, 1935, **A 13** 1 New terms
 Lang, *PR*, 1936, **49** 552 a Ground term

Cl Chlorine 17

- Cl Kayser and Konen
 Cl I Kiess, C C and de Bruin, *BSJ*, 1929, **2** 1117 Lines and terms
 Cl II Bowen, *PR*, 1928, **31** 34, 1934, **45** 401 New lines and terms.
 Murakawa, *ZP*, 1931, **69** 507 Terms and lines
 Murakawa, *ZP*, 1935, **96** 117 New terms
 Cl III Bowen, *PR*, 1928, **31** 34, 1934, **45** 401 Lines and terms
 Cl IV *Ibid*
 Cl V *Ibid*
 Cl VI Bowen and Millikan, *PR*, 1925, **25** 591 Lines and terms
 Cl VII Bowen and Millikan, *PR*, 1925, **25** 295 Lines and terms

Co Cobalt 27

- Co Kayser and Konen
 Co I Catalan, *ZP*, 1928, **47** 89 Terms and classified lines
 Catalan, *An Soc fis y quim (Madrid)*, 1929, **27** 832
 Co II Findlay, *PR*, 1930, **36** 5 Lines, terms and Zeeman effect
 Co V Gilroy, *PR*, 1931, **38** 2217 V I isoelectronic sequence

Cr Chromium 24

- Cr Kayser and Konen
 Cr I Kiess, C C, *BSJ*, 1930, **5** 775 Terms and lines
 Allen and Hesthal, *PR*, 1935, **47** 926 Intensities
 Cr II Kromer, *ZP*, 1928, **52** 531 Zeeman effect
 Kiess, C C, *BSJ*, 1930, **5** 775 Terms and lines
 Gilroy, *PR*, 1931, **38** 2217 V I isoelectronic sequence
 Cr III White, *PR*, 1929, **33** 914 Th I isoelectronic sequence
 Cr IV White, *PR*, 1929, **33** 676 Sc I isoelectronic sequence
 Cr V White, *PR*, 1929, **33** 543 Ca I isoelectronic sequence
 Cr VI Gibbs and White, *PR*, 1929, **33** 157 Doublets of stripped atoms

Cs Caesium 55

- Cs Kayser and Konen
 Cs I Fowler Grotrian
 Cs II Bacher and Goudsmit, 1932, using unpublished material
 Laporte, Miller and Sawyer, *PR*, 1931, **37** 845, 1932, **39** 458.
 Olthoff and Sawyer, *PR*, 1932, **42** 766 Analysis extended

Cu Copper 29

- Cu Kayser and Konen
 Cu I Sommer, *ZP*, 1926, **39** 711 Zeeman effect
 Bacher and Goudsmit, 1932, using unpublished material
 Cu II Shenstone, *PR*, 1927, **29** 380 Terms and lines
 Bacher and Goudsmit, 1932, using unpublished material

Eu Europium 63

- Eu I Russell and King, *PR*, 1934, 46 1023 Low terms
 Eu II Albertson, *PR*, 1934, 45 499a Low terms

F Fluorine 9

- F I Bowen, *PR*, 1927, 29 231 Ground state from ultra-violet lines
 Dingle, *PRS*, 1928, 117 407 Terms and lines
 Edlén, *ZP*, 1935, 98 445 New terms
 F II Dingle, *PRS*, 1930, 128 600 Series.
 F III Dingle, *PRS*, 1929, 122 144 Series
 F IV Bowen, *PR*, 1927, 29 231
 F VI Edlén, *ZP*, 1934, 89 179
 F VII Edlén, *ZP*, 1934, 89 179

Fe Iron 26

- Fe I Laporte, *ZP*, 1924, 23 135, 26 1 Classification and Zeeman effect
 Laporte, *Proc Nat Acad Sci* 1926, 12 496 Series
 Moore and Russell, *AJ*, 1928, 68 151 Terms
 Burns and Walters, *Alleghany Observatory Publications*, 1929, 6 159
 Fe II Russell, *AJ*, 1926, 64 194 Terms and classified lines
 Meggers and Walters, *Bur of Standards, Sci Pap* 1927, 22 205 Low terms
 Dobbie, *PRS*, 1935, 151 703 New terms
 Fe IV Gilroy, *PR*, 1931, 38 2217 V I isoelectronic sequences
 Fe v White, *PR*, 1929, 33 914 Ti I isoelectronic sequences
 Fe VI Bowen, *PR*, 1935, 47 924

Ga Gallium 31

- Ga I Grotrian
 Sawyer and Lang, *PR*, 1929, 34 718 Lines and terms
 Ga II Sawyer and Lang, *PR*, 1929, 34 712 Series
 Ga III Lang, *PR*, 1927, 30 762
 Ga IV Mack, Laporte and Lang, *PR*, 1928, 31 748

Gd Gadolinium 64

- Gd I Albertson, *PR*, 1935, 47 370 Ground term

Ge Germanium 32

- Ge I Gartlein, *PR*, 1928, 31 782 Lines and terms
 Rao, K R, *PRS*, 1929, 124 465 Lines and terms
 Ge II Lang, *PR*, 1929, 34 697 Lines and terms
 Gartlein, *PR*, 1931, 37 1704a Series
 Ge III Lang, *PR*, 1929, 34 697 Series
 Ge IV Lang, *PR*, 1929, 34 697 Series
 Ge v Mack, Laporte and Lang, *PR*, 1928, 31 748 Lines and terms

H Hydrogen 1

- H I Fowler Grotrian
 Bracket, *AJ*, 1922, 56 154 A new series
 Pfund, *JOSA*, 1924, 9 139 A new series
 Hansen, *AP*, 1925, 78 558 Fine structure
 Houston, *AJ*, 1926, 64 81 Fine structure
 Kent, Taylor and Pearson, *PR*, 1927, 30 266 Fine structure

H Hydrogen 1 (*cont*)

- H I Houston and Hsieh, *PR*, 1934, **45** 263 Intervals of Balmer lines
 Williams and Gibbs, *PR*, 1934, **45** 491*a* Correction to R_H

He Helium 2

- He I Fowler Grottrian
 Burger, *ZP*, 1929, **54** 643 Intensity
 Hopfield, *AJ*, 1930, **72** 133 Ultra-violet series extended
 Kruger, *PR*, 1930, **36** 855 New lines classified
 Gibbs and Kruger, *PR*, 1931, **37** 1559 Structure of 3888 Å line
- He II Fowler Grottrian
 Paschen, *AP*, 1927, **82** 689 Fine structure
 Kruger, *PR*, 1930, **36** 855 New lines classified

Hf Hafnium 72

- Hf I Meggers and Scribner, *BSJ*, 1930, **4** 169 Terms and lines
- Hf II Meggers and Scribner, *JOSA*, 1928, **17** 83 Terms
 Meggers and Scribner, *BSJ*, 1934, **13** 625 Terms, intensities and Zeeman effect

Hg Mercury 80

- Hg I Fowler Grottrian
 Takamine and Suga, *Inst Phys and Chem Tokyo*, 1930, **13** 1 New series in infra-red
- Hg II Paschen, *Berlin Akad Sitz* 1928, **32** 536 Terms and lines
 McLennan, McLay and Crawford, *PRS*, 1931, **134** 41 Terms and lines
- Hg III McLennan, McLay and Crawford, *T Roy Soc Canada*, 1928, **22** 247 Terms and lines
 Mack and Fromer, *PR*, 1935, **48** 357 Pt I isoelectronic sequence

I Iodine

- I I Evans, *PRS*, 1931, **133** 417 Lines and terms
 Deb, *PRS*, 1933, **139** 380 Lines and terms
- I IV Krishnamurty, *Proc Phys Soc* 1936, **48** 277 Lines and terms

In Indium 49

- In I Fowler Grottrian
 Sawyer and Lang, *PR*, 1929, **34** 718 New terms
 Lansing, *PR*, 1929, **34** 597 New terms
 Lang, *PR*, 1930, **35** 126*a* Ionisation potentials
- In II Bacher and Goudsmit, 1932, using unpublished data
- In III Lang, *Proc Nat Acad Sci* 1927, **13** 341, 1929, **15** 414
 Rao, K R, Narayan and Rao, A S, *Indian J Phys* 1928, **2** 482
- In IV Gibbs and White, *PR*, 1928, **31** 776 Some multiplets

Ir Iridium 77

- Ir I Meggers and Laporte, *PR*, 1928, **28** 642 Low levels
 Albertson, *PR*, 1932, **42** 443*a* Low terms

K Potassium 19

- K I Fowler Grottrian
 Ferschman and Frisch, *ZP*, 1929, **53** 326 Doublet structure of D terms
 Edlen, *ZP*, 1935, **98** 445 Doublets resolved

K Potassium 19 (*cont*)

- K II Bowen, *PR*, 1928, 31 497 Terms and lines
Whitford, *PR*, 1932, 39 898 Zeeman effect
- K III de Bruin, *ZP*, 1929, 53 658
Ram, *Indian J Phys* 1933, 8 151
- K IV Bowen, *PR*, 1928, 31 497 One multiplet
Ram, *Indian J Phys* 1933, 8 151

Kr Krypton 36

- Kr I Gremmer, *ZP*, 1929, 54 215 Lines and terms
Meggers, de Bruin and Humphreys, *BSJ*, 1929, 3 129, 1931, 7 643
Lines and terms
Pogány, *ZP*, 1933, 86 729 Zeeman effect
- Kr II Bakker and de Bruin, *ZP*, 1931, 69 36 Zeeman effect New terms
de Bruin, Humphreys and Meggers, *BSJ*, 1933, 11 409
- Kr III Deb and Dutt, *ZP*, 1931, 67 138 Series
Humphreys, *PR*, 1935, 47 712 Lines and terms

La Lanthanum 57

- La I Meggers, *BSJ*, 1932, 9 239 Lines and Zeeman effect
Russell and Meggers, *BSJ*, 1932, 9 625 Lines and terms
- La II Russell and Meggers, *BSJ*, 1932, 9 625 Lines and terms
- La III Russell and Meggers, *BSJ*, 1932, 9 625 Lines and terms

Li Lithium 3

- Li I Fowler Grottrian
- Li II Werner, *N*, 1925, 116 574, 1926, 118 154 Series
Schuler, *ZP*, 1926, 37 568, 1927 42 487 Fine structure
Ericson and Edlén, *ZP*, 1930, 59 656 Ground term
- Li III Edlén and Ericson, *ZP*, 1930, 59 656 Two lines
Gale and Hoag, *PR*, 1931, 37 1703*a* New lines

Lu Lutecium 71

- Lu I Meggers and Scribner, *BSJ*, 1930, 5 73 Lines and terms
- Lu II *Ibid*
- Lu III *Ibid*

Mg Magnesium 12

- Mg I Fowler Grottrian
Bowen and Millikan, *PR*, 1925, 26 150 PP' multiplets
- Mg II Fowler Grottrian
- Mg III Mack and Sawyer, *Science*, 1928, 68 306, 1761 Series
Edlén and Ericson, *Comptes Rendus*, 1930, 190 116 Doublet
- Mg IV Edlén and Ericson, *Comptes Rendus*, 1930, 190 173 Extreme ultra-violet
Mack and Sawyer, *PR*, 1930, 35 299 Screening doublets
- Mg v *Ibid*

Mn Manganese 25

- Mn I McLennan and McLay, *T Roy Soc Canada*, 1926, 20 89 Lines and terms
Russell, *AJ*, 1927, 66 184, 347 Configurations assigned
Seward, *PR*, 1931, 37 344 Intensities

Mn Manganese 25 (cont)

- Mn II Russell, *AJ*, 1927, **66** 233
 Duffendack and Black, *PR*, 1929, **34** 42 Lines and terms
 Seward, *PR*, 1931, **37** 344 Intensities
- Mn III Gibbs and White, *Proc Nat Acad Sci* 1927, **13** 525 Multiplets
 Gilroy, *PR*, 1931, **38** 2217 Lines and terms
- Mn IV White, *PR*, 1930, **33** 914 Tri isoelectronic sequence
- Mn V White, *PR*, 1929, **33** 678 Sc I isoelectronic sequence
 Bowen, *PR*, 1935, **47** 924 New terms
- Mn VII Gibbs and White, *Proc Nat Acad Sci* 1926, **12** 676

Mo Molybdenum 42

- Mo I Catalan, *An Soc fis y quim (Madrid)*, 1923, **21** 213 Lines and terms
 Wilhelmy, *AP*, 1926, **80** 305 Zeeman effect
 Meggers and Kiess, *JOSA*, 1926, **12** 417 Lines and terms
- Mo II Meggers and Kiess, *JOSA*, 1926, **12** 417 Lines and terms
 Wilhelmy, *AP*, 1926, **80** 305 Zeeman effect
- Mo IV Elhason, *PR*, 1933, **43** 745 Multiplets
- Mo V Trawick, *PR*, 1935, **48** 223 Sr I isoelectronic sequence

N Nitrogen 7

- N I Ingram, *PR*, 1929, **34** 421 Terms and lines
 Hopfield, *PR*, 1930, **36** 789 Adds some higher terms
 Ekefors, *ZP*, 1930, **63** 437 Lines and terms
 Stucklen and Carr, *PR*, 1933, **43** 944a One term
- N II Fowler, A and Freeman, *PRS*, 1927, **114** 662 Lines and terms
 Bowen, *PR*, 1927, **29** 231 Lines and terms
 Freeman, *PRS*, 1929, **124** 654 New terms
 Bowen, *PR*, 1929, **34** 534 Series
- N III Freeman, *PRS*, 1928, **121** 318 Lines and terms
- N IV Freeman, *PRS*, 1930, **127** 330 Lines and terms
 Edlen, *N*, 1931, **127** 744 Singlets
 Bacher and Goudsmit, using unpublished material
- N V Edlen and Ericson, *ZP*, 1930, **64** 64 Series

Na Sodium 11

- Na I Fowler Grotrian
 Ferschman and Frisch, *ZP*, 1929, **53** 326 Doublets structure of D terms
- Na II Bowen, *PR*, 1928, **31** 967 Lines and terms
 Frisch, *ZP*, 1931, **70** 498 Lines and terms
 Vance, *PR*, 1932, **41** 480 Lines and terms
- Na III Edlen and Ericson, *Comptes Rendus*, 1930, **190** 173 Extreme ultra violet
 Mack and Sawyer, *PR*, 1930, **35** 299 New lines and levels
 Vance, *PR*, 1932, **41** 480 Lines and terms
 Soderqvist, *ZP*, 1932, **76** 316 Lines and terms
- Na IV Vance, *PR*, 1932, **41** 480

Ne Neon 10

- Ne I Paschen, *AP*, 1918, **60** 405
 Paschen and Gotze

Ne Neon 10 (*cont*)

- Ne I Back, *AP*, 1925, **76 317** Zeeman effect
 Lyman and Saunders, *Proc Nat Acad Sci* 1926, **12 92** Low levels
 Gremmer, *ZP*, 1928, **50 716** New levels from infra-red lines
 Murakawa and Iwama, *Inst Phys and Chem Tokio*, 1930, **13 283**
 Zeeman effect
- Ne II Russell, Compton and Boyce, *Proc Nat Acad Sci* 1928, **14 280** Lines
 and terms
 Frisch, *ZP*, 1930, **64 499** Two new terms
 de Bruin and Bakker, *ZP*, 1931, **69 19** Zeeman effect
- Ne III Boyce and Compton, *Proc Nat Acad Sci* 1929, **15 656** Series
 de Bruin, *ZP*, 1932, **77 489** New lines and terms
 Keussler, *ZP*, 1933, **85 1** Ground term
- Ne IV Boyce and Compton, *Proc Nat Acad Sci* 1929, **15 656** Series

Ni Nickel 28

- Ni I Russell, *PR*, 1929, **34 821** Lines and terms
 Ornstein and Buoma, *PR*, 1930, **36 679** Intensities
 Marvin and Baragar, *PR*, 1933, **43 973** Zeeman effect
- Ni II Shenstone, *PR*, 1927, **30 255** Lines, terms and Zeeman effect
 Menzies, *PRS*, 1929, **122 134** Ground term
 Ornstein and Buoma, *PR*, 1930, **36 679** Intensities
- Ni VI Gilroy, *PR*, 1931, **38 2217** V isoelectronic sequence

O Oxygen 8

- O I Hopfield, *PR*, 1931, **37 160** Lines and terms
 de Bruin, *N*, 1932, **129 469**
- O II Russell, *PR*, 1928, **31 27** Terms and lines
- O III Fowler, A, *PRS*, 1928, **117 317** Lines and terms
 Freeman, *PRS*, 1929, **124 654** Series
- O IV Bowen, *PR*, 1927, **29 231**
 Freeman, *PRS*, 1930, **127 330**
- O V Edlén, *N*, 1931, **127 744** Singlets
 Bacher and Goudsmit, 1932, using unpublished material.
- O VI Edlén and Ericson, *ZP*, 1930, **64 64** Li-like spectra

Os Osmium 76

- Os I Meggers and Laporte, *PR*, 1926, **28 642** Six low levels

P Phosphorus 15

- P I Kiess, C C, *BSJ*, 1932, **8 393** Lines and terms
 Robinson, *PR*, 1936, **49 297** New terms
- P II Bowen, *PR*, 1927, **29 510** Lines and terms
 Robinson, *PR*, 1936, **49 297** New terms
- P III Millikan and Bowen, *PR*, 1925, **25 600** Lines and terms
 Saltmarsh, *PRS*, 1925, **108 332** Higher terms
 Bowen, *PR*, 1928, **31 34** New lines
 Bowen, *PR*, 1932, **39 8** New terms
- P IV Bowen and Millikan, *PR*, 1925, **25 591** Terms and lines
 Bowen, *PR*, 1932, **39 8** New terms
- P V Millikan and Bowen, *PR*, 1925, **25 295** Lines and terms

Pb Lead 82

- Pb i Gieseler and Grottrian, *ZP*, 1925, **34** 374, 1926, **39** 377 Series
Back, *ZP*, 1926, **37** 193 Zeeman effect
Bacher and Goudsmit, 1932, using unpublished material
- Pb ii Gieseler, *ZP*, 1927, **42** 265 Lines and terms
- Pb iii Smith, *PR*, 1929, **34** 393, 1930, **36** 1 Terms and lines
Green and Loring, *PR*, 1932, **41** 389 Zeeman effect
- Pb iv Smith, *PR*, 1930, **36** 1 Lines and terms
- Pb v Mack, *PR*, 1929, **34** 17 Lines and terms
Mack and Fromer, *PR*, 1935, **48** 357 Pt i isoelectronic sequence

Pd Palladium 46

- Pd i Shenstone, *PR*, 1930, **36** 669 Lines, terms and Zeeman effect
- Pd ii Shenstone, *PR*, 1928, **32** 30 Lines and terms
Blair, *PR*, 1930, **36** 173 New terms

Pt Platinum 78

- Pt i Haussmann, *AJ*, 1927, **66** 333, *PR*, 1928, **31** 152 Zeeman effect and terms
Livingood, *PR*, 1929, **34** 185 Lines and terms

Ra Radium 88

- Ra i Rasmussen, *ZP*, 1934, **87** 607
- Ra ii Fowler
Rasmussen, *ZP*, 1933, **86** 24

Rb Rubidium 37

- Rb i Fowler Grottrian
Ramb, *AP*, 1931, **10** 311 Doublets resolved
- Rb ii Laporte, Miller and Sawyer, *PR*, 1931, **38** 843 Series

Re Rhenium 75

- Re i Meggers, *BSJ*, 1931, **6** 1027 Lines and terms

Rh Rhodium 45

- Rh i Sommer, *ZP*, 1927, **45** 147 Lines, terms and Zeeman effect
- Rh ii Bacher and Goudsmit, 1932, using unpublished material

Rn Radon 86

- Rn i Rasmussen, *ZP*, 1933, **80** 726 Lines and terms

Ru Ruthenium 44

- Ru i Sommer, *ZP*, 1926, **37** 1 Lines, terms and Zeeman effect
- Ru ii Meggers and Shenstone, *PR*, 1930, **35** 868
Bacher and Goudsmit, 1932, using unpublished material

S Sulphur 16

- S i Frerichs, *ZP*, 1933, **80** 150 Lines and terms
Meissner, Bartelt and Eckstein, *ZP*, 1933, **86** 54 Additional terms
Ruedy, *PR*, 1933, **44** 757 New terms
- S ii Ingram, *PR*, 1928, **32** 172 Lines and terms
Gilles, *Ann de Phys* 1931, **15** 267 Terms
Bartelt and Eckstein, *ZP*, 1933 **86** 77 New terms
- S iii Ingram, *PR*, 1929, **33** 907 Lines and terms
- S iv Millikan and Bowen, *PR*, 1925, **25** 602 Lines and terms

S Sulphur 16 (cont)

S iv Bowen, *PR*, 1928, 31 38 New terms

S v Millikan and Bowen, *PR*, 1925, 25 591, 26 150 Lines and terms

S vi Millikan and Bowen, *PR*, 1925, 25 295 Doublet Laws

Sb Antimony 51

Sb i McLennan and McLay, *T Roy Soc Canada*, 1927, 21 63 Lines and terms

Lowenthal, *ZP*, 1929, 57 822 Zeeman effect and new terms.

Sb ii Lang and Vestine, *PR*, 1932, 42 233 Terms

Sb iii Lang, *PR*, 1930, 35 445 Terms and lines

Sb iv Green and Lang, *Proc Nat Acad Sci* 1928, 14 706 Lines and terms

Badami, *Proc Phys Soc* 1931, 43 538 Series

Sb v Lang, *Proc Nat Acad Sci* 1927, 13 341 Series.

Badami, *Proc Phys Soc* 1931, 43 538 Series

Sc Scandium 21

Sc i Russell and Meggers, *Bur Stand, Sci Papers*, 1927, 22 329 Lines and terms

Sc ii Russell and Meggers, *Bur Stand, Sci Papers*, 1927, 22 329 Lines and terms

Russell and Meggers, *BSJ*, 1929, 2 733 Comparison with Y

Sc iii Russell and Lang, *AJ*, 1927, 66 13 Lines and terms

Smith, *Proc Nat Acad Sci* 1927, 13 65 Extension of series

Sc iv Majumdar and Toshniwal, *N*, 1928, 121 828 Screening doublets

Se Selenium 34

Se i McLennan, McLay and McLeod, *PM*, 1927, 4 486 Lines and terms

Gibbs and Ruedy, *PR*, 1932, 40 204 New terms

Se ii Badami and Rao, K. R., *PRS*, 1933, 140 387 Lines and terms

Martin, *PR*, 1935, 48 938 Lines and terms

Se iv Pattabhiramayya and Rao, A. S., *Indian J Phys* 1929, 3 531 Series

Se v Sawyer and Humphreys, *PR*, 1928, 32 583 Series

Se vi Sawyer and Humphreys, *PR*, 1928, 32 583 Series

Si Silicon 14

Si i Fowler, A., *PRS*, 1929, 123 422 Lines and terms

Kiess, C. C., *BSJ*, 1933, 11 775 New terms

Si ii Fowler, A., *Phil Trans R S* 1925, 225 1 Series

Bowen and Millikan, *PR*, 1925, 26 150 PP° multiplets.

Bowen, *PR*, 1928, 31 34 Series

Si iii Fowler, A., *Phil Trans R S* 1925, 225 1 Series

Sawyer and Paschen, *AP*, 1927, 84 1 Comparison with Al ii

Si iv Fowler, A., *Phil Trans R S* 1925, 225 1 Series

Si v Edlén and Ericson, *Comptes Rendus*, 1930, 190 116 Doublets.

Sm Samarium 62

Sm i Albertson, *PR*, 1935, 47 370 Ground term

Sn Tin 50

Sn i Back, *ZP*, 1927, 43 309 Lines, terms and Zeeman effect.

Green and Loring, *PR*, 1927, 30 575 Terms, lines and Zeeman effect

Bacher and Goudsmit, 1932, using unpublished material.

Sn Tin 50 (cont)

- Sn II Grotrian
Green and Loring, *PR*, 1927, 30 574 Lines, terms and Zeeman effect
Narayan and Rao, K R, *JP*, 1927, 45 350 Terms
Lang, *PR*, 1930, 35 445 Lines and terms
- Sn III Gibbs and Vieweg, *PR*, 1929, 34 400 Cd I isoelectronic sequence
Green and Loring, *PR*, 1931, 38 1289 Multiplets and Zeeman effect
- Sn IV Rao, K R, Narayan and Rao, A S, *Indran J Phys* 1928, 2 476 Series
- Sn V Gibbs and White, *Proc Nat Acad Sci* 1928, 14 345 Series

Sr Strontium 38

- Sr I Fowler Grotrian
Russell and Saunders, *AJ*, 1925, 61 39 Lines and terms
- Sr II Fowler Grotrian

Ta Tantalum 73

- Ta I McLennan and Durnford, *PRS*, 1928, 120 502 Zeeman effect
Kieess, C C and Kieess, H K, *BSJ*, 1933, 11 277 Lines and terms

Te Tellurium 52

- Te I Bartelt, *JP*, 1934, 38 522 Lines and terms
- Te III Krishnamurthy, *PRS*, 1935, 151 178 Lines and terms
- Te IV Rao, K R, *PRS*, 1931, 133 220 Lines and terms
- Te V Gibbs and Vieweg, *PR*, 1929, 34 400 Cd I isoelectronic sequence
- Te VI Lang, *Proc Nat Acad Sci* 1927, 13 341 Lines and terms
Rao, K R, *PRS*, 1931, 133 220 Lines and terms

Ti Titanium 22

- Ti I Russell, *AJ*, 1927, 66 347 Lines and terms
Harrison, *JOSA*, 1928, 17 389 Intensities
White, *PR*, 1929, 33 914 Ti I isoelectronic sequence
- Ti II Russell, *AJ*, 1927, 66 283 Lines and terms
- Ti III Russell and Lang, *AJ*, 1927, 66 13 Terms
- Ti IV Russell and Lang, *AJ*, 1927, 66 13 Terms

Tl Thallium 81

- Tl I Fowler Grotrian
- Tl II McLennan, McLay and Crawford, *PRS*, 1929, 125 570 Lines and terms
Smith, *PR*, 1930, 35 235 Extension of series
Ellis and Sawyer, *PR*, 1936, 49 145 New lines and terms
- Tl III McLennan, McLay and Crawford, *PRS*, 1929, 125 50 Lines and terms
- Tl IV Rao, K R, *Proc Phys Soc* 1929, 41 361 Terms
Mack and Fromer, *PR*, 1935, 48 357 Pt I isoelectronic sequence

V. Vanadium 23

- V I Bacher and Goudsmit, 1932, using unpublished material
Gilroy, *PR*, 1931, 38 2217 New lines and terms
- V II Meggers, *JP*, 1925, 33 509, 39 114 Lines, terms and Zeeman effect
Russell, *AJ*, 1927, 66 184, 194 Terms named
White, *PR*, 1929, 33 914 New terms and lines
- V III White, *PR*, 1929, 33 672 Sc I isoelectronic sequence

V Vanadium 23 (*cont*)

- V IV White, *PR*, 1929, **33** 542 Ca I isoelectronic sequence
 V V Gibbs and White, *PR*, 1929, **33** 157 Doublets of stripped atoms

W Tungsten 74

- W I Beiming, *ZP*, 1927, **42** 146 Zeeman effect
 Bacher and Goudsmit, 1932, using unpublished material
 Laun, *PR*, 1935, **43** 572*a* New terms
 W II Beiming, *ZP*, 1927, **42** 146 Zeeman effect

Xe Xenon 54

- Xe I Rasmussen, *ZP*, 1932, **72** 779
 Humphreys and Meggers, *BSJ*, 1933, **10** 139 Lines and terms
 Pogány, *ZP*, 1935, **93** 364 Zeeman effect
 Xe II Humphreys, de Bruin and Meggers, *BSJ*, 1931, **6** 287 Lines and terms
 Xe III Deb and Dutt, *ZP*, 1931, **67** 138

Y Yttrium 39

- Y I Meggers and Russell, *BSJ*, 1929, **2** 733 Lines and terms
 Y II Meggers and Russell, *BSJ*, 1929, **2** 733 Lines and terms
 Y III Meggers and Russell, *BSJ*, 1929, **2** 733 Lines and terms

Zn Zinc 30

- Zn I Fowler Grottrian
 Sawyer, *JOSA*, 1926, **13** 431 PP° multiplets
 Zn II Lang, *Proc Nat Acad Sci* 1929, **15** 414 Lines and terms
 Takahashi, *AP*, 1929, **3** 27 Lines and terms
 Zn III Laporte and Lang, *PR*, 1927, **30** 378 Series

Zr Zirconium 40

- Zr I Kiess, C C and Kiess, H K, *BSJ*, 1931, **6** 621 Lines, terms and Zeeman effect
 Zr II Kiess, C C and Kiess, H K, *BSJ*, 1930, **5** 1205 Lines, terms and Zeeman effect
 Zr III Kiess, C C and Lang, *BSJ*, 1930, **5** 305 Lines and terms
 Zr IV Kiess, C C and Lang, *BSJ*, 1930, **5** 305 Lines and terms

C The hyperfine structure of the elements

- Ag** Tolansky, *PPS*, 1933, **45** 559 No hyperfine structure in Ag I
 Williams and Middleton, *N*, 1933, **131** 691 No hyperfine structure found
 Jackson, *N*, 1933, **131** 691 No hyperfine structure found
 Hill, *PR*, 1934, **46** 536 No hyperfine structure found
Al Gibbs and Kruger, *PR*, 1931, **37** 656*a* No hyperfine structure in 4 lines
 White, *PR*, 1931, **37** 1175*a* Interpretation of Gibbs and Kruger
 Tolansky, *ZP*, 1932, **74** 336 Narrow lines suggests $g(I)$ small
 Williams and Sabine, *PR*, 1933, **43** 362*a* No hyperfine structure in Al III
 Paschen and Rutschl, *AP*, 1933, **18** 867 Hyperfine structure of Al II shows $I = \frac{1}{2}$
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 Rao, A S, *JP*, 1933, 84 236 I is $1\frac{1}{2}$ in As π
 Crawford and Crooker, *N*, 1933, 131 655 As ν confirms
 Tolansky and Heard, *PRS*, 1934, 146 818 Intensities of As π confirm
- Ba** Ritschl and Sawyer, *JP*, 1931, 72 36 Resonance lines of Ba π
 Kruger, Gibbs and Williams, *PR*, 1932, 41 322 $I = 2\frac{1}{2}$ in 135 and 137
- Be** Kruger and Wagner, *PR*, 1932, 41 373a Three lines of Be ν and two lines of Be π appear sharp
 Parker, *PR*, 1933, 43 1035a I is probably $\frac{1}{2}$
- B ν** Goudsmit and Back, *JP*, 1927, 43 321, 1928, 47 174 Hyperfine structure in magnetic field shows $I = 4\frac{1}{2}$
 McLennan, McLay and Crawford, *PRS*, 1930, 129 579 B ν π and B ν π
 Zeeman, Back and Goudsmit, *JP*, 1931, 66 1 Magnetic analysis of B ν ν
 Fisher and Goudsmit, *PR*, 1931, 37 1057 Analysis of partially resolved patterns
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 Tolansky, *PRS*, 1932, 136 585 Confirms de Bruin
 Brown, *PR*, 1932, 39 777 Band spectrum shows $I = 1\frac{1}{2}$ or $2\frac{1}{2}$ in Br ν^0 and Br ν^1
- Cb** Ballard, *PR*, 1934, 46 806 $I = 4\frac{1}{2}$, $\mu = 3$ 7
- Cd** Schuler and Bruck, *JP*, 1929, 56 291 Hyperfine structure shows $I = 0$ in even and $I = \frac{1}{2}$ in odd isotopes
 Schuler and Keyston, *JP*, 1931, 71 413 Abnormal intensities
 Jones, *Proc Phys Soc*, 1933, 45 625 In Cd π μ is -0 625
 Schuler and Westmeyer, *JP*, 1933, 82 685 Isotope displacement in Cd π
- Cl** Elliott, *PRS*, 1930, 127 638 Band spectrum shows $I = 2\frac{1}{2}$
 Tolansky, *JP*, 1931, 73 470, 1932, 74 336 Hyperfine structure shows μ is small, isotope displacement
- Co** Grace, *PR*, 1933, 43 762a I is probably $3\frac{1}{2}$
 More, *PR*, 1934, 46 470 $I = 3\frac{1}{2}$, μ is between 2 and 3
 Kopfermann and Rasmussen, *JP*, 1935, 94 58 $I = 3\frac{1}{2}$ confirmed
- Cs** Jackson, *PRS*, 1934, 143 455 Hyperfine intensity ratio shows $I = 3\frac{1}{2}$
 Granath and Stranathan, *PR*, 1934, 46 317 μ is 2 45
 Kopfermann, *JP*, 1932, 73 437 $I = 3\frac{1}{2}$ in Cs π
 Cohen, *PR*, 1934, 46 713 Atomic ray shows $I = 3\frac{1}{2}$
 Heydenburg, *PR*, 1934, 46 802 Polarisation of resonance radiation shows μ between 2 40 and 2 52
 Jackson, *JP*, 1935, 93 809 Intensity ratios in 4555 A line
 Granath and Stranathan, *PR*, 1935, 48 725 Hyperfine structure shows μ between 2 4 and 3 0
- Cu** Ritschl, *JP*, 1932, 79 1 I is $1\frac{1}{2}$ in both isotopes Cu ν Isotope displacement
 Schuler and Schmidt, *JP*, 1936, 100 113 $\mu = 2$ 5 in 63 and $\mu = 2$ 6 in 65
 Electric quadrupole moment
- Eu** Schuler and Schmidt, *JP*, 1935, 94 457 $I = 2\frac{1}{2}$ in 151 and 153, both μ 's are positive, μ_{151} $\mu_{153} = 2$ 2 Abnormal intervals

- F** Gale and Monk, *PR*, 1929, **33** 114 Band spectrum shows $I = \frac{1}{2}$
 Campbell, *JP*, 1933, **84** 393 In F I , $I = \frac{1}{2}$
 Brown and Bartlett, *PR*, 1934, **45** 527 μ is about $3\mu_N$
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- H²** Urey, Brickwedde and Murphy, *PR*, 1932, **40** 1 Mass effect in Balmer lines
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- Hg** Schuler and Keyston, *JP*, 1931, **72** 423 Hg I Isotope displacement
 Schuler and Jones, *JP*, 1932, **74** 631 Hg II Isotope displacement
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 Mrozowski, *Acad Pol Sci et Lettres*, 1931, **9** 464 2537 A line
 Inglis, *JP*, 1933, **84** 466 Magnetic wandering of 2537 A line
 Schuler and Schmidt, *JP*, 1935, **98** 239 Asymmetry of electric field of Hg^{201}
 Venkatesachar and Sibaiya, *Indian Acad of Sci Proc* 1934, **1** 8
 Hyperfine structure of Hg II
- I** Loomis, *PR*, 1927, **29** 112 Band spectrum shows I is large
 Tolansky, *PRS*, 1935, **149** 269 Hyperfine structure of I II shows $I = 2\frac{1}{2}$
 Tolansky, *PRS*, 1935, **152** 663 I I confirms A perturbed term
 Strait and Jenkins, *PR*, 1936, **49** 635a Alternating intensities confirm $I = 2\frac{1}{2}$
- In** Jackson, *JP*, 1933, **80** 59 Hyperfine intensity ratio shows $I = 4\frac{1}{2}$
 Paschen, *Sitz d Preuss Akad d Wiss Phys-Math* 1934, 456 Departures from interval rule in In II
- Ir** Schuler and Schmidt, *JP*, 1935, **94** 460 I is probably $\frac{1}{2}$ in 191 and 193
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 Schuler, *JP*, 1932, **76** 14 Isotope displacement
 Fermi and Segré, *JP*, 1933, **82** 749 Hyperfine structure cannot be resolved
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 Jackson and Kuhn, *PRS*, 1934, **148** 335 Hyperfine structure of resonance lines shows $I \geq 2\frac{1}{2}$
 Millman, Fox and Rabi, *PR*, 1934, **46** 320 Splitting of atomic rays gives $I = 1\frac{1}{2}$ and $\mu = 0.38\mu_N$
 Millman, *PR*, 1935, **47** 739 $I > \frac{1}{2}$ in K^{41}
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- Kr** Kopfermann and Wieth-Knudsen, *JP*, 1933, **85** 353 Work on Kr I shows in Kr^{83} I is $\geq 3\frac{1}{2}$ and μ probably negative

- La** Anderson, *PR*, 1934, **45** 685, **46** 473 Hyperfine structure shows $I=3\frac{1}{2}$ and $\mu=2.5$
 Crawford, *PR*, 1935, **47** 768 Hyperfine structure of La I confirms Anderson
- Li** Schuler, *AP*, 1925, **76** 292, *ZP*, 1927, **42** 487 Hyperfine structure of Li II
 Harvey and Jenkins, *PR*, 1930, **35** 789 Band spectrum shows $I=1\frac{1}{2}$ in Li⁷
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 Goudsmit and Inglis, *PR*, 1931, **37** 328 *a* Hyperfine structure of Li II
 Granath, *PR*, 1932, **42** 44 From 5485 Å of Li II, $I=1\frac{1}{2}$ and $\mu=3.29\mu_N$
 Gray, *PR*, 1933, **44** 570 $I=1\frac{1}{2}$ confirmed
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 Fock and Petrashen, *P Z Sowjetunion*, 1935, **8** 547 Theory suggests $\mu=4.57$
 Bartlett and Gibbons, *PR*, 1936, **49** 552 *a* Comment on Fock and Petrashen
 Schuler and Schmidt, *ZP*, 1936, **99** 285 If $I=1$ in Li⁶, then $\mu=0.6$
- Lu** Schuler and Schmidt, *ZP*, 1935, **95** 265 $I=3\frac{1}{2}$, deviations from interval rule
- Mg** Murakawa, *ZP*, 1931, **72** 793 Absence of hyperfine structure in Mg I suggests $I=0$
 Jackson and Kuhn, *PRS*, 1936, **154** 679 Isotope displacement in resonance line
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- N** Ornstein and van Wijk, *ZP*, 1928, **49** 315 Band spectrum shows $I=1$
 Bacher, *PR*, 1933, **43** 1001 Magnetic moment of nucleus $\geq 0.2\mu_N$ since lines very fine
- Na** Frisch, *P Z Sowjetunion*, 1933, **4** 559 Lines of Na II show $I=1\frac{1}{2}$
 Rabi and Cohen, *PR*, 1933, **43** 582 Atomic rays show $I=1\frac{1}{2}$
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 Joffe, *PR*, 1934, **45** 468 Band spectrum shows $I=1\frac{1}{2}$
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 Larrick, *PR*, 1934, **46** 581 Polarisation of resonance radiation explained by $I=1\frac{1}{2}$
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- Ne** Nagaoka and Mishima, *P Imp Acad Tokio*, 1929, **5** 200, 1930, **6** 143 Mass effect and magnetic splitting
 Bartlett and Gibbons, *PR*, 1933, **44** 538 Isotope displacement theory

- P** Herzberg, *PR*, 1932, **40** 313 Band spectrum
 Ashley, *PR*, 1933, **44** 919 Band spectrum shows $I = \frac{1}{2}$
 Jenkins, *PR*, 1935, **47** 783a Alternating intensities confirm Ashley
- Pb** Murakawa, *ZP*, 1931, **72** 793 $I = 0$ for 206 and 208, but $\frac{1}{2}$ for 207
 Kopfermann, *ZP*, 1932, **75** 363 Isotope displacements
 Schuler and Jones, *ZP*, 1932, **75** 563 Hyperfine structure shows new isotope
 Rose and Granath, *PR*, 1932, **40** 760 Isotope displacements
 Dickinson, *PR*, 1934, **46** 498 Isotope displacements
 Rose, *PR*, 1935, **47** 122 Isotope displacements reviewed
- Pr** White, *PR*, 1929, **34** 1397 In Pr II , $I = 2\frac{1}{2}$
- Pt** Fuchs and Kopfermann, *Nw*, 1935, **23** 372
 Jaeckel and Kopfermann, *ZP*, 1936, **99** 492 $I = \frac{1}{2}$ in Pt^{195} , isotope displacements
 Jaeckel, *ZP*, 1936, **100** 513 Confirms $I = \frac{1}{2}$
 Kopfermann and Krebs, *ZP*, 1936, **101** 193 Isotope proportions present
- Rb** Kopfermann, *ZP*, 1933, **83** 417 Rb II shows $I = 1\frac{1}{2}$ in 87 and $2\frac{1}{2}$ in 85
 μ_{87} is twice μ_{85} No isotope displacement
 Jackson, *ZP*, 1933, **86** 131 Confirms Kopfermann
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 Meggers, King and Bacher, *PR*, 1931, **38** 1258 Hyperfine structure shows $I = 2\frac{1}{2}$ in both isotopes
 Sommer and Karlson, *Nw*, 1931, **19** 1021 Confirms Meggers
- Sb** Tolansky, *PRS*, 1934, **146** 182 $I = 2\frac{1}{2}$ in 121 and 123, $\mu_{121} = 2.75$, $\mu_{123} = 2.0$
 Crawford and Batson, *Can J Research*, 1934, **10** 693 Sb III shows $I = 2\frac{1}{2}$ in 121 and $3\frac{1}{2}$ in 123, $\mu_{121} = 4.0$, $\mu_{123} = 3.2$
- Sc** Kopfermann and Rasmussen, *ZP*, 1934, **92** 82 $I = 3\frac{1}{2}$ and $\mu = 3.6$
- Se** Rafalowski, *Acta Phys Polonica*, 1933, **2** 119 Lines are single
 Olsson, *ZP*, 1934, **90** 134 Band spectrum shows $I = 0$ in Se^{80}
- Sm** Schuler and Schmidt, *ZP*, 1934, **92** 148 Isotope displacements
- Sn** Tolansky, *PRS*, 1934, **144** 574 Sn II shows $I = \frac{1}{2}$ in 117 and 119, μ is 0.89μ in both One line of Sn I confirms I Isotope displacement
- Sr** Benson and Sawyer, *PR*, 1933, **43** 766a No hyperfine structure wider than 0.05 cm^{-1}
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 McMillan and Grace, *PR*, 1933, **44** 949 Hyperfine structure shows $I = 3\frac{1}{2}$
- Te** Rafalowski, *Acta Phys Polonica*, 1933, **2** 119 Lines are single
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 Crooker, *PM*, 1933, **16** 994 Paschen-Back effect
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 μ_{129} is negative, μ_{131} is positive μ_{129}/μ_{131} is 1.1 No isotope displacement
 Gwynne-Jones, *PRS*, 1934, **144** 587 Xe I confirms
- Y** Kruger and Challacombe, *PR*, 1935, **48** 111 *a* Earlier report incorrect
- Zn** Schuler and Bruck, *ZP*, 1929, **56** 291 No hyperfine structure found Zn I
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 suggests $I = 0$
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 In Zn 67, $I = 1\frac{1}{2}$
 Billeter, *Helv Phys Acta*, 1934, **7** 413 No hyperfine structure found in
 resonance line

SUBJECT INDEX

Numbers in Clarendon refer to the second volume

- Absorption spectra, 68
 - rare earths, 81
 - X-rays, 193
- Alkaline earths, arc and spark spectra, 46
 - inter system lines, 57
 - interval rule, 63
 - isoelectronic spectra, 212
 - JJ coupling, 142
 - singlets and triplets, 47
- Alkalis, doublet series, 23, 31-2
 - doublet intensities, 107
 - forbidden lines, 215
 - isoelectronic spectra, 212
- Alternating intensities, 200-2
- Aluminium, 22-4
 - absorption spectrum, 68
 - displaced terms, 7
 - displacement law, Al III, 177
 - Landé's doublet formula, 215
 - perturbed series, 151
- Analysis, hyperfine, 189
 - multiplet, 48
 - series, 18
 - Zeeman, 99
- Angstrom unit, 5
- Anomalous Zeeman effect, 85, 144
- Antimony, 28-32
 - fluorescence, 80
- Arc spectra, 46
- Argon, 39-44
 - ionized, 38, 145
 - level diagram, 40
 - nebular lines, 218
- Arsenic, 28-32
- Atomic magnetism, atomic rays, 129
 - electronic theory, 129
 - paramagnetic ions, 135
- Atomic rays, 129
 - nuclear spin, 201
 - velocity selector, 200
- Atomic volumes, 161
- Auroral line, 35, 215-17
- Balmer series, 2, 5, 7, 9
 - fine structure, 41
 - reversal, 69
- Band spectra, 172
 - nuclear spin, 201
- Bands, anti-Stokes, 233
 - in phosphores, 233
- Barium, 46-58, 50
 - abnormal intensities, 135
 - Bergmann series, 22
- Beryllium, 46-58
 - displaced terms, 6
 - Moseley diagram, 204
- Bismuth, 28-32
 - hyperfine structure, 169-70, 181-3, 185
 - magnetic moment, 133
 - X-ray spectrum, 197-8
- Bohr magneton, 87, 128, 130
- Boltzmann distribution law, 70, 85, 234
- Boron, 22-4
- Bromine, 36-9
- Cadmium, 46-58
 - displaced terms, 7
 - hyperfine structure, 170-1, 207
 - ionised, 67
 - magnetic moment, 132
- Caesium, 18-39, 50
 - doublet structure, 30
 - intensities, 107-8
 - level diagram, 39
 - series, 39
- Calcium, 46-58, 50
 - diffuse triplets, 56
 - displaced terms, 1
 - doublet limit, 158
 - effective quantum numbers, 51
 - intensities, 102
 - ionised, 178, 205
 - level diagrams, 50, 52, 2
 - perturbed series, 151
 - series, 48
- Carbon, 24-8
- Catalysts, 188
- Centroid of a multiplet, 65
- Cerium, 53, 78
- Chlorine, 36-9
 - hyperfine structure, 207
 - level diagram, 37
- Chromium, 57-8
 - g factors, 125
 - intensities, 106
 - intervals, 148-9

- Chromium (*cont*)
 phosphores, 229-32
 Zeeman patterns, 117
- Cobalt, 61, 63
 intensities, 106
 magnetic moment, 134
- Coloured ions, 188
- Columbium, 54, 57
 Zeeman patterns, 103
- Column III, 22-4
 „ IV, 24-8, 140-2
 „ V, 28-32
 „ VI, 33-4
- Combination of electrons, energy rules,
 15
 equivalent electrons, 11
 inverted terms, 17
 unlike electrons, 9
- Combination principle, 21, 24
- Configurations, 179-93
 elements tabulated, 194-5
- Copper, 66-9
g factors, 155
 level diagram, 69
 magnetic moment, 132
 perturbed series, 151-4
 Zeeman patterns, 103-4, 118
- Correspondence principle, 13, 36, 93
- Coupling, deviations from Russell-Saunders, 43-4, 122
 JJ, 28, 136-43
 Russell Saunders, 3-5, 22
- Covalency, 167, 172
- Crystals, energy levels, 228
- Deutonic nuclei, 205, 214
- Diamagnetism, 127
- Diffuse series, 18
 doublets, 31, 34, 37
 formula, 23
 Stark effect, 158
 triplets, 54-6
 Zeeman types, 83
- Displaced terms, alkaline earths, 1
 Be and Mg, 6
 Zn, Cd and Hg, 7
- Displacement law for spectra, 175-8
- Displacements, atomic, 19, 128, 133
 electronic, 19, 128-33, 147-50
 in electric field, 145, 157-60
 in magnetic field, 112, 124
 sum rule, 124, 126
- Distribution law, Boltzmann's, 70, 85, 234
- Doublet formulae, irregular, 206
 Lande's, 212-16
 regular, 208
- Doublet formulae (*cont*)
 screening, 206
 Sommerfeld's, 210-11
 spin, 208
- Doublet series, 23, 31-2
- Earth metals, 22-4
- Effective nuclear charge, 203
- Effective quantum number, 22
- Electric field, see Stark effect, 144
- Electron, in nucleus, 209-10
 magnetic moment of, 96
 mass of, 14, 44, 84
- Electron impact, 76
- Electron orbits, 10-13, 26-8, 207
- Electron theory of magnetism, 127
- Electronic angular momentum, see *J*, 34
- Electronic displacements, 147-50
- Electronic structures, 179-95
- Electrovalency, 167
- Energy of interaction of two vectors,
 orbital and spin vectors, 65
 in strong magnetic field, 112
 in weak magnetic field, 97
- Energy rules, multiplet, 15
- Enhanced lines, 46
- Equivalent electrons, 11
- Erbium phosphores, 242
- Erect terms, 35
- Europium, hyperfine structure, 199
 phosphores, 236
 valency, 78
- Excitation, by electron impact, 76
 by monochromatic light, 78
 potentials, 73
- Exclusion principle, periodic system,
 201
 combination of electrons, 11, 17
- Fine structure, constant, 210
 of helium, 44-5, 60-1
 of hydrogen, 40-4
- Fluorescence, in crystals, 228
 hyperfine, 192
 in vapours, 80
- Fluorescent crystals, chromium phosphores, 229
 energy levels, 228
 Lenard phosphores, 245
 platino cyanides, 246
 rare earth phosphores, 236
 uranyl salts, 241
- Fluorine, 36-9
- Fluorite, 236
- Forbidden lines, 27, 151, 215
- Frame elements, 184

- Fundamental series, 22
 doublets, 32, 37
 formula, 23
 triplets, 57

g, see Magnetic splitting factor, 90
 Gadolinium, 83
 Gallium, 22-4
 nucleus, 211
 Γ , see Displacement
 Germanium, 24-8
 Gold, 66-7
 magnetic moment, 132
 Ground terms, 68
 elements tabulated, 194
 long periods, 187, 45-7
 rare earths, 192, 80
 short periods, 183

 Hafnium, 53-4
 Halogens, 36
 Heavy hydrogen, 15, 167
 nucleus, 206, 214
 Helium, 59-62
 ionised, 14, 44-5
 level diagram, 59
 Stark effect, 151-4
 Hydrogen atom, Bohr's theory, 10
 electron orbits, 28
 magnetic moment, 132
 mechanical moment, 11
 wave mechanics, 15
 Hydrogen spectrum, Balmer's series, 2,
 5, 13
 fine structure, 40-3
 series, 5
 Stark effect, 145-51
 Hyperfine structure, empirical, 166
 intensities, 183
 isotope displacement, 187
 nuclear mass, 167
 Paschen Back effect, 173
 vector model, 168
 Zeeman effect, 173

 Indium, 22-4
 absorption spectrum, 69-70
 Inert gases, 36-41
 diamagnetism, 139
 g factors, 146-7
 intervals, 127
 Inner quantum number, see *J*, 34
 Intensities, alkali doublets, 107
 experimental, 80
 hyperfine, 183-7
 iron frame elements, 104
 jj coupling, 150

 Intensities (*cont*)
 normal multiplet, 35, 91
 quadrupole, 227
 races ultimes, 120
 sum law, 83, 135
 super multiplet, 101
 tables, 95-9
 X ray, 196
 Zeeman, 98, 108
 Zeeman tables, 112-15
 Intermediate magnetic fields, 116
 Inter system lines, 57
 jj coupling, 28
 Zeeman effect, 116
 Interval quotient, 65
 Interval rule, 63
 hyperfine, 173
 hyperfine perturbed, 197-9
 perturbation, 155
 Invariance, of *g* sum, 123, 123-6
 of Γ sum, 124, 126
 Inverted terms, 35, 16-20
 Iodine, 36-9
 jj coupling, 142
 Ionisation potentials, from electron im-
 pact, 72
 from series limit, 20-1
 long periods, 187, 189
 rare earths, 192, 74
 short periods, 182-3
 Iron, 59-61
 magnetic moment, 132
 Iron frame elements, ground terms,
 46
 compared with other frames, 70-4
 paramagnetic ions, 140-3
 Irregular doublet law, 206
 Isoelectronic sequences, Moseley law
 203-5
 screening doublets, 206-8
 spin doublets, 211-12
 Isotope displacement, 187-97

J, definition, 34
 orientation in magnetic field, 88, 129-
 31
 series limit, 159
jj coupling, 28, 136-43
 intensities, 150

 Krypton, 39-44

 Landé's doublet formula, 212
 Lanthanum, 50, 53
 g factors, 155
 Lead, 24-8
 fluorescence, 79

- Lead (*cont*)
 isotope displacements, 197
 magnetic moment, 133
 Lenard phosphores, 245
 Level diagrams, optical, 11, 24
 X-ray, 197-8
 Limit, calculation of series, 20
 displaced, 3
 multiplet, 31, 42, 159
 Lithium, 18-39
 hyperfine structure, 166, 168
 level diagram, 25
 Moseley diagram, 204
 nucleus, 214
 series, 19
 Zeeman patterns, 105
 Long periods, analysis, 48
 configurations, 48
 ground terms, 45
 individual spectra, 49-70
 in periodic system, 183-7
 three rows compared, 70-4
 Lorentz unit, 84
 Lutecium, 50, 53
 hyperfine structure, 199
 Magnesium, 46-58
 displaced terms, 6
 intensities, 218
 intervals, 150
 ionised, 177
 Paschen Back effect, 115, 117
 Zeeman effect, 108, 119
 Magnetic field, see Zeeman effect, 82
 Magnetic moment, of atoms, 132-4
 of electrons, 96
 of ions, 135-43
 of nucleus, 203
 Magnetic splitting factor, atomic rays,
 131
 definition, 90
 sum law, 123
 tabulated, 92-3, 100-1
 Magnetism, 187
 Manganese, 58-9
 intensities, 100, 106
 Zeeman effect, 116
 Mass effect in hyperfine structure, 167
 Matching strong and weak terms, 118
 Mercury, 46-58
 absorption spectrum, 70-2
 displaced terms, 7
 excitation potentials, 75
 forbidden line, 215
 hyperfine intervals, 197-9
 hyperfine structure, 190-2
 intensities, 101-3
 Mercury (*cont*)
 ionised, 67
 isotope displacements, 193-5
 level diagram, 53
 magnetic moment, 132
 quadrupole radiation, 222
 resonance radiation, 79
 Zeeman effect, 118
 Molecules, symmetrical, 201-2
 Molybdenum, 57-8
 intervals, 148-9
 Moseley's law, optical, 199
 X ray, 203-5
 Multiplets, 62
 analysis, 48
 intensities, 94-9
 Multiplicities in iron row, 175
 Nebular lines, 217
 Neodymium phosphores, 242
 Neon, 39-43
 absorption spectrum, 69
 hyperfine structure, 168
 intervals of s terms, 134
 j coupling, 141
 series limits, 43, 159-60
 Stark effect, 155
 Neutronic nuclei, 205, 212-18
 Neutrons in nucleus, 209-10
 Nickel, 63-5
 g factors, 155
 intensities, 105-6
 magnetic moment, 134
 Stark effect, 158-60
 Nitrogen, 28-32
 inverted terms, 20
 level diagram, 29
 nebular lines, 218
 nucleus, 206-7, 214
 triplet limit, 32
 Normal Zeeman triplet, 134
 Notation in complex spectra, 49
 Nucleus, electric field of, 200
 magnetic moment of, 203-6
 spin of, 200
 structure, 208
 types of, 204-5, 208
 One line spectra, 76
 Orbits, exclusion principle, 201
 general atom, 26-8
 hydrogen, 10-13
 Orbital vector, 26
 Oxygen, 33-5
 auroral line, 216-17
 intensities, 101
 level diagram, 33

- Oxygen (*cont*)
 magnetic moment, 134
 nebular lines, 218
 Paschen-Back effect, 106, 115
- Palladium, 63-5
 Palladium frame elements, 188
 ground terms, 47
 paramagnetism, 143
- Paramagnetism, classical theory, 136-8
 frame elements, 140-3
 gases, 138
 Langevin's theory, 127
 rare earths, 138-40
- Partial Paschen Back effect, 115
 intensities, 118-19
- Paschen Back effect, empirical, 105
 hyperfine structure, 173
 intermediate fields, 117
 invariance of g sum, 123
 invariance of Γ sum, 125
 matching strong and weak terms, 119
 partial effect, 115
 quadrupole, 225
 vector model, 111
- Penetrating orbits, 27
- Periodic system, 161-202
 displacement law, 175
 electronic structures, 179
 exclusion principle, 201
 rare earths, 75
 valency, 166
- Periodic table, 163, 165
- Perturbation, 150-7
 hyperfine, 197-8
- Phosphores, chromium, 229
 Lenard, 245
 rare earth, 236
- Phosphorescence, 228-9
- Phosphorus, 28-32
 hyperfine structure, 207
- Photometry, 90
- Platino-cyanides, 246
- Platinum, 66
- Platinum frame elements, 189
 ground terms, 47
 paramagnetism, 143
- Polarisation in, Paschen Back effect, 112-13
 quadrupole radiation, 220
 Stark effect, 148, 154
 Zeeman effect, 112, 109
- Potassium, 18-39, 50
 displacement law, 178
 electron impact, 78
 forbidden lines, 215
- Potassium (*cont*)
 g sum, K II, 126
 hyperfine structure, 207
 magnetic moment, 132, 138
 Moseley diagram, 205
 quadrupole line, 222-5
 Stark effect, 178
- Praseodymium phosphores, 238, 240, 243
- Principal series, 18
 doublets, 31, 36
 formula, 23
 triplets, 51, 55
 Zeeman types, 83
- Protonic nuclei, 204, 210-12
- Protons in nucleus, 209-10
- Quadrupole radiation, forbidden lines, 215
 intensities, 227
 polarisation rule, 221
 quantum mechanics, 219
 selection rules, 220
 Zeeman effect, 221
- Quantum mechanics, hyperfine perturbation, 199
 Paschen-Back effect, 117
 quadrupole radiation, 219
 valency, 172
 Zeeman effect, 97
- Quantum of action, 10
- Radium, 46-58, 50
Raves ultimes, 120
- Raman spectra, 228, 235
- Rare earths, 75-89
 absorption spectra, 81
 arc and spark spectra, 79
 configurations, 190-2
 in periodic system, 75
 ionisation potentials, 192
 paramagnetism, 138
 phosphores, 236, 242-3
 valency, 77
- Regular doublets, 208
- Resonance potentials, 72
- Resonance radiation, 78
- Rhenium, 59
- Rhodium, 63
- Rubidium, 18-39, 50
 electron impact, 78
 g sum, Rb II, 126
 nucleus, 209
- Russell-Saunders coupling, 3-5, 22
 deviations from, 122
- Ruthenium, 59, 61
 Γ sum, 132

- Rydberg constant, 2
 for helium, 14
 for hydrogen, 12
 for other atoms, 19
- Samarium, 85-9
 phosphores, 236-40, 243
 valency, 78
- Scandium, 50-3
 ionised, 178, 205
- Screening, constant, 203
 doublet, 206
- Selection rules, displaced terms, 4
 for J , 57
 for L , 26
 Paschen Back, 112-13
 quadripole, 220
 Stark, 148, 154
 X ray, 196
 Zeeman, 89
- Selenium, 33-5
- Self-reversal, 18, 69
- Serial number, 19, 23, 49
- Series, 5, 18
 formula, 18-21
 intensities, 104
 limits, 10-11, 21, 153-65
- Sharp series, 18
 doublet, 31, 36
 formula, 21-3
 Stark effect, 158
 triplet, 54-5
 Zeeman types, 83
- Short periods, 181-3
 deep terms, 14
 elements, 21
 ground terms, 16-18
 irregularities, 22
- Silicon, 24-8
 doublet limit, 160
 ionised, 177
 level diagram, 25
- Silver, 66-7
 hyperfine structure, 207
 magnetic moment, 130-2
- Singlet spectra, 47
- Sodium, 18-39
 absorption spectrum, 68
 controlled electron impact, 77
 D lines in magnetic field, 106-10, 112-13
 displacement law, 177
 fluorescence, 79-81
 forbidden lines, 215
 Landé's doublet formula, 215
 level diagram, 33
 magnetic moment, 132
- Sodium (*cont*)
 nuclear spin, 201
 Paschen-Back effect, 115
 quadripole lines, 223-5
 Stark effect, 155
 Zeeman patterns, 85
- Spark spectra, 46
- Spectroscopic terms, 62
- Spin doublets, 208
 X-ray, 130
- Spin of nucleus, 200
- Spinning electron, doublet series, 32
 Zeeman effect, 94
- Stark effect, 144-60
 experimental, 144
 forbidden lines, 215
 in crystals, 234
 in hydrogen, 145
 in other elements, 151
- Stationary states, 3, 10
- Statistical weight of a term, 66
- Stripped atoms, 207
- Strong field, 106
- Strontium, 46-58, 50
 intensities, 102
 perturbation, 157
- Sulphur, 33-5
- Sum rules, g sum, 123
 Γ sum, 126
 intensity, 135
- Super multiplet, intensities, 101
- Tellurium, 33-5
- Temperature class, 49
- Terbium, valency, 78
- Terms, 3, 9, 11
- Thallium, 22-4
 absorption, 69-70
 fluorescence, 79
 hyperfine structure, 172-89
 intensities, 108
 isotope displacement, 194
 level diagram, 23
 magnetic moment, 132
 nucleus, 206
- Tin, 24-8
 magnetic moment, 133
- Titanium, 53-4
 g factors, 155
 Γ sum, 132
 intensities, 101, 106
 ionised, 178, 205
- Transition elements, 184
- Triplet terms, 49
- Tungsten, 58
- Uranyl salts, 241

- Valency, 166-75
 rare earths, 77
Vanadium, 54, 57
Vector model, alkali doublets, 29
 combination of several electrons, 9
 displaced terms, 3
 general coupling, 135
 hyperfine intervals, 199
 hyperfine structure, 168, 172
 hyperfine Zeeman effect, 173
 interval rule, 65
 jj coupling, 27-8
 Paschen-Back effect, 111-12
 Stark effect, 157
 Zeeman effect, 88-9

Wave mechanics, 3
 hydrogen atom, 15
 hyperfine structure, 176, 179
Wave numbers, 5
Weak field, 106

X rays, 193-200, 228
 spin doublets, 209-11, 130
Xenon, 39-43
 Stark effect, 156-7

Ytterbium, phosphores, 236
 valency, 78
Yttrium, 50, 53

Zeeman effect, 82-104
 analysis, 99
 anomalous types, 85
 hyperfine, 173
 intensities, 98, 108-20
 magnetic splitting factors, 92-3,
 100-1
 normal triplet, 82
 quadrupole, 221
 quantum theory, 86
 spinning electron, 94
 unresolved patterns, 106
Zinc, 46-58, 67
 displaced terms, 7
 Lande's doublet formula, 215
 magnetic moment, 132
Zirconium, 53-4
 doublet limit, 161
 g factors, 154, 156
 intensities, 106
 .

AUTHOR INDEX

Numbers in Clarendon refer to the second volume

- Albertson, W, 79
 Allen, J S V, 106
 Altschuler, S, 213
 Andrade, E N da C, 69, 78, 81
 Arnot, F L, 81
 Aston, F W, 188, 208
- Bacher, R F, short periods, 44, long periods, 74, hyperfine structure, 173, 199
 Back, E, Paschen-Back effect, 104-6, 115-18, 126, *g* factors, 147, hyperfine structure, 169, 178, 180, 187
 Bakker, C J, halogens, 36, *g* sum, 125, quadrupole Zeeman effect, 222-6
 Balmer, J J, 2, 5
 Barratt, S, 220
 Bartlett, J H, 150
 Bear, R S, 85, 88-9
 Bechert, K, 150
 Becker, F, 217, 227
 Becquerel, J, 82-3
 Blair, H A, 104, 125
 Blaton, J, 227
 Bohr, N, 3, H atom 10f, He ion, 14, selection rule, 89, Stark effect, 157, periodic table, 163-4, 180-91, rare earths, 75, 79
 Boltzmann, L, 2, 70, 85, 234
 Born, M, 175, 202
 Bourland, L T, 143
 Bowen, I S, doublet laws, 204, 207, 212, nebular lines, 217-19
 Boyce, J C, nitrogen, 20, nebular lines, 217-19
 Brackett, F S, 7
 Bragg, W H, 202
 Bragg, W L, 202
 Breit, G, matching terms, 120, isotope displacements, 194-7, nuclear spin, 201
 Brickwedde, F G, 167
 Bruck, H, 170-1
 Bryden, S D, 209
 Buoma, T, 106
 Burger, H C, helium, 60, intensities, 101-4, Zeeman intensities, 108-10, photometry, 121
- Cabannes, J, 228
 Cabrera, P, 139
 Campbell, J S, 84, 221
 Capel, W H, 139
 Carter, N M, 27, 216
 Casimir, H, 199, 200
 Chalk, L, 149, 150
 Charola, F, 80
 Chenault, R L, 8
 Coeligh, M, 102
 Cohen, V W, 131, 200-1
 Compton, K T, 20
 Cotton, A, 84
 Condon, E U, 158-9
 Crookes, W, 236
 Curie, S, 127
 Curtis, W E, 7
- Dadiou, A, 228
 Darwin, C G, Paschen Back effect, 117, quantum mechanics, 211, Paschen-Back intensities, 118
 Darwin, K, 109-10
 Datta, S, 215
 Davis, B, 74-5
 de Boer, J H, 167, 172, 202
 de Broglie, L, 15
 de Bruin, T L, 36, 125
 de Gramont, A, 120
 de Haas, W J, 139
 Dennison, D M, 200
 Deutschbein, O, rare earths, 83, chrome phosphores, 230-5
 Dewar, J, 2
 Dickinson, R G, 201
 Dijkstra, H, 147
 Dirac, P A M, 93
 Dobson, G M B, 90
 Dorgelo, H B, 91-3, 109
 du Bois, H, 235
- Eastman, E D, 209
 Eckart, C, 167
 Ehrenfest, P, 82
 Elias, G J, 235
 Elason, A Y, 95, 150
 Epstein, P S, 147
 Evans, E J, 14, 168
 Evans, S F, 142

- Evert, H, 240
 Eymers, J G, 102
- Fagerberg, S, 240
 Fermi, E, 108, 203-7
 Ferschmin, A, 40
 Filippov, A, 107
 Fisher, R A, 104
 Foote, P D, 215, 8
 Fortrat, R, 68
 Foster, J S, 149-51, 155
 Fowler, A, helium, 14, series laws, 21, doublet series, 31, Moseley's law, 205, carbon, 28
 Fowler, H W, 27, 216
 Franck, J, 76, 81
 Frank, A, 139
 Fraser, R G J, 129-30, 143
 Fraunhofer, J, 68
 Frayne, J G, 218-19
 Freed, S, 139, 83-5
 French, R, intensities, 106, 121, auroral line, 221
 Fridrichson, J, 76
 Frisch, S, 40
 Fuchtbauer, C, 71, 107
- Gale, H G, 15
 Gebauer, R, 145
 Gehrcke, E, 84
 Gerlach, W, 129, 132-3, 138
 Gibbons, J J, 197
 Gibbs, R C, H₂, 43, He atom, 61, doublet laws, 208, 212-16, f electrons, 14, cerium, 80
 Gotze, R, 23, 1
 Goucher, F S, 74-5
 Goudsmit, S A, spinning electron, 32-4, fine structure of hydrogen, 40-5, He atom, 58, Paschen Back effect, 111, doublet laws, 209-10, 216, short periods, 44, long periods, 74, combination of electrons, 20, g , 126, 143, Γ sum, 130-2, jj coupling, 142, perturbation, 156, hyperfine structure, 169, 173, 180, 199, nuclear moments, 203-7, rare earth phosphores, 240
 Grace, N S, 43, 197
 Granath, L P, 188
 Grebe, L, 197
 Green, J B, Paschen-Back effect, 117, doublet laws, 216, perturbation, 155, hyperfine structure, 175-7, 183-6
 Griffith, I O, 90, 121
 Grotrian, W, hydrogen, 40, helium, 40, 58, aluminum, 68, indium, 70, Stark effect, 155, Moseley diagrams, 205, nebular lines, 217, 227
- Guthrie, A N, 143
- Haberlandt, H, 236
 Hansen, G, helium, 41, 60, hyperfine structure, 168
 Harkness, H W, 156, 160
 Harrison, D N, photometry, 90, 121
 Harrison, G R, intensities, 106, 135
 Heard, J F, 156, 160
 Heisenberg, W, hydrogen, 40, helium, 61, Paschen-Back effect, 117, 120, intensities, 91, 111, nuclear structure, 209
 Heitler, W, 174
 Hellmann, H, 202
 Herlhy, J, 108
 Hertz, G, 69, 76, 206
 Hesthal, C E, 106
 Hevesy, G v, 75, 81, 89
 Hill, E L, 185
 Hiyaama, S, 145
 Hoag, J B, 15
 Honl, H, 93, 111
 Hopfield, J J, 35
 Houston, W V, hydrogen, 41-3, helium, 60, Lorentz unit, 84, perturbed g factors, 155, 178, quadrupole radiation, 215, 219, 226
 Howes, H L, 241
 Hsieh, Y M, 43
 Hubner, H J, 107
 Huff, L D, 215, 226
 Hughes, D S, 167-8
 Hund, F, electronic structures, 193, energy rules, 15, combination of electrons, 20, short periods, 24-36, 44, inert gases, 42, long periods, 74, series limit, 160-3
- Idei, S, 200, 206
 Inglis, D R, perturbed, terms, 156, nucleus, 210, 212, 214
 Ishida, Y, 145
 Iwama, T, 147
 Iwanenko, D, 210
- Jack, R, 102
 Jackson, D A, 207
 Janes, R B, 143
 Jantsch, G, 236
 Jenkins, H G, 120
 Jevons, W, 174, 202, 201
 Jog, D S, 28
 Johnson, M H, 106, 185

- Jones, E G, 197-8, 207
 Joos, G, 107, 232
 Jordan, P, hydrogen, 40, energy levels, 81, Paschen Back effect, 117, 120, Zeeman intensities, 111
 Kaha, P N, 79
 Kallmann, H, hyperfine structure, 194-6, 210, 214
 Karhik, B, 236
 Karlson, P, 79
 Kassner, L, 147
 Kast, W, 157
 Kayser, H, 214
 Kellogg, J M B, 201
 Kent, N A, 41
 Keystone, J E, 171, 185-91, 193
 Kiess, C C, q factors, 90, 100, zirconium, 106, perturbation, 154, series limit, 161
 Kiess, H K, 106, 154
 King, A S, 46, 49, 79
 Klemm, W, 236
 Koch, J, 151
 Kohlrausch, K W F, 228
 Kohn, H, 107
 Kossell, W, 169, 176, 186
 Kramers, H A, 147, 154
 Kromer, E, 125
 Kronig, R de L, intensities, 93, 104, 150, Zeeman intensities, 111
 Kruger, P G, 61
 Kuhn, H, 207
 Kurt, E O, 134
 Ladenburg, R, 71, 155
 Lande, A, selection rule, 57, interval rule, 63, q factors, 90, Zeeman effect, 104, Γ sum, 125, doublet formula, 212, oxygen, 35, neon, 41, g sum, 125, nucleus, 210-14
 Lang, R J, 80, 161
 Langer, R M, 152
 Langevin, P, 127
 Langstroth, G O, 135, 156
 Laporte, O, interval rule, 64, paramagnetic ions, 141, *rare earths*, 121, g sum, 126, perturbation, 156, selection rule, 220, rare earth phosphores, 240
 Lenard, P, 73, 246
 Leu, A, 132-3
 Lewis, G N, 170
 Liveing, G D, 2
 London, F, 174
 Lorentz, H A, 82, 84
 Loring, R A, 155
 Lo Surdo, A, 145
 Lyman, T, 7, 58, 39
 McLennan, J C, 35, 216, 221
 McLeod, J H, 221
 Mack, J E, 162
 Masaki, O, 40
 Meggers, W F, q factors, 90, 100, intensities, 104, *rare earths*, 121, g sum, 125, Γ sum, 132, forbidden lines, 215
 Meissner, K W, caesium, 38, rubidium, 40, neon, 69, atomic rays, 130
 Mendeleeff, D, 162, 165
 Mensing, L, 117
 Metcalfe, E P, 70
 Meyer, S, 139
 Mihanczuk, B, 225
 Miller, G R, 126
 Millikan, R A, doublet laws, 204, 207 212, displaced terms, 6
 Minkowski, R, 147, 160
 Mishima, T, 168
 Mitchell, A C G, 81
 Mohler, F L, 79, 215
 Moll, W J H, 121
 More, K R, 197
 Moseley, H G J, 199, 203
 Mrozowski, S, 192
 Muliken, R S, 174
 Murakawa, K, 147
 Murphy, E J, 167
 Nagasaki, H, 168
 Newman, F H, 77
 Nichols, E L, 241
 Nutting, G C, 84, 232
 Ornstein, L S, intensities, 101-4, 106, Zeeman intensities, 108-10, photometry, 121
 Paschen, F, hydrogen, 7, serial numbers, 23, helium, 45, 58, Paschen-Back effect, 105-6, 115, oxygen, 35, neon, 39, 42, perturbed series, 151
 Pauli, W, matching terms, 119, g sum, 124, Stark effect, 158, exclusion principle, 201, g sum, 125, hyperfine structure, 166, 168
 Pauling, L, doublet laws, 209, 216, combination of electrons, 20, Γ sum, 132, jj coupling, 142
 Pearson, H, 41
 Pfluger, A, 71
 Pfund, A H, 7

- Phipps, T E, 134
 Piccardi, G, 192, 75
 Pickering, W H, 14
 Placzek, G, 228
 Planck, M, 10
 Poetker, A H, 7
 Pogány, B, 42, 145-7
 Preston, T, 85, 105
 Pringsheim, P, absorption spectra, 69,
 78, 81, fluorescence, 241, 244-6
 Przybram, K, 236

 Rabi, I I, atomic rays, 129, 131, 143,
 nuclear spin, 200-1
 Raman, C V, 228
 Ramb, R, 40
 Ramsauer, G, 155
 Rasetti, F, 107-8, 220
 Rawlins, F I G, 232
 Rayleigh, Lord, 80-1, 215
 Ritz, W, 3, 21, 24
 Rolla, L, 192, 75
 Roschdostwenski, D, 107-8
 Rosenthal, A H, 36
 Rowles, W, 155
 Ruark, A E, 180, 202
 Rubinowicz, A, 89, 227
 Ruedy, J E, 221
 Rumer, G, 174
 Rump, W, 79
 Runge, C, 58, 85, 214
 Russell, H N, series laws, 21, notation,
 34, unresolved patterns, 104,
 matching terms, 120, xenon, 158,
 displaced terms, 1, 5, notation, 4,
 combination of electrons, 10, euro-
 pium, 79, intensities, 93, 104-5,
 g sum, 125, Γ sum, 132, perturbed
 series, 151, 154
 Rydberg, J R, Rydberg constant, 2,
 series formula, 18, combination
 principle, 21, 24
 Ryde, J. W., 120
 Ryde, N, 156

 Saha, N K, 83
 Samburgsky, S, 108
 Sauer, H, 232-3
 Saunders, F A, 1, 5, 39
 Sawyer, R A, 7, 128, 151
 Scheffers, H, 130
 Schlapp, R, 151
 Schmidt, T, isotope displacements,
 195, hyperfine structure interval
 rule, 199, nuclear structure 209,
 fluorescence, 246

 Schnetzler, K, 232
 Schrodinger, E, 149
 Schuler, H, hyperfine structure,
 lithium, 166, vector model, 170-1,
 intensities, 185-7, isotope dis-
 placement, 189-97, perturbation,
 197-9, nucleus, 204-5, 209-10,
 213-14
 Schuster, A, 2
 Scribner, B F, 121
 Segré, E, nuclear moments, 203-7,
 quadrupole radiation, 222-6
 Selwyn, E W H, 205
 Seward, R S, 106
 Sexl, T, 200
 Shane, C S, 15, 43
 Shenstone, A G, unresolved patterns,
 104, notation, 34, 4, inverted terms,
 19, g sum, 125, Γ sum, 132, per-
 turbation, 151, 154-5, series limit,
 161
 Sherman, A, 202
 Shrum, G M, 27, 216
 Sidgwick, N V, 167, 172, 202
 Siegbahn, M, 200, 202, 228
 Slater, J C, 58, 174, 180
 Smith, D M, 120
 Snow, C P, 232
 Sommer, L A, 221
 Sommerfeld, A, intensity rule, 35, H I
 and He II, 40-5, Paschen-Back
 effect, 115, matching terms, 119;
 paramagnetism, 141, displacement
 law, 176, doublet laws, 210-11,
 216, intensities, 91-3
 Sowerby, A L M, 220
 Spedding, F H, H₂, 15, 43, para-
 magnetism, 139, rare earths, 83-9,
 chromium compounds, 232-4, rare
 earth phosphores, 238
 Spencer, J E, 89
 Stark, J, 144-5, 150, 160
 Stern, O, 129, 132, 210
 Stevenson, A F, 220
 Stokes, G G, 229
 Stoner, E C, 136, 143
 Stoney, G J, 2

 Takamine, T, 158-9, 8
 Tamm, I, 213
 Taylor, J B, 132
 Taylor, L B, 41
 Temple, G, 209
 Terenin, A, 80
 Terrien, J, 147
 Thomas, E, 168
 Thornton, R L, 149

- Tomaschek, R, rare earths, 83, fluorescence, 242-3, 246, rare earth phosphores, 236-40
Trautenberg, H R v, 145
Turner, L A, 34, 4
Twyman, F, 120

Uhlenbeck, G E, spinning electron, 32, H I and He II, 40-5, helium, 58, doublet laws, 210, *g* factor, 143
Unsold, A, 45
Urban, G, 236
Urey, H C, periodic system, 180, 202, combination of electrons, 20, hyperfine structure, 167

Van Arkel, A E, 167, 172, 202
Van Geel, W C, 117, 110, 116-21
Van Lohuizen, T, 87
Van Vleck, J H, 138-9, 143, 202
Van Wijk, W R, 101
Venkatesachar, B, 70
Voigt, W, 105-6, 108, 117
von Baeyer, A, 84

Weiss, P, 84

Wentzel, G, 196, 5
West, G D, 15
Weyl, A, 174
Whiddington, R, 13
White, H E, doublet laws, 208, 212, 216, combination of electrons, 14, cerium, 80, intensities, 95, 150, perturbation, 151, series limit, 163
Whitford, A E, 126
Wick, F G, 244
Wiersma, E C, 139
Wilber, D T, 14
Williams, J H, 43
Williams, S E, 108
Wolff, H W, 107
Wood, R W, 68, 79, 228
Wrede, E, 132
Wulff, J, hyperfine structure, 175-8, 183-7

Zacharias, J R, 201
Zahrahmicek, J, 211
Zeeman, P, Zeeman effect, 82, 104, aluminium, 36, hyperfine structure, 169, 180
Zemansky, M W, 81